Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.6, No.3, 2016

www.iiste.org

Design of Single Acceptance Sampling Plan Using Maximum Allowable Proportion Defective (MAPD) and Producer's Allowable Risk (PAR)

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Abstract

This paper focuses on single acceptance sampling designs on incoming quality with Maximum Allowable percent Defective (MAPD) along with a specified probability of acceptance referred to Producer's Allowable Risk (PAR) which is defined as the minimum probability of acceptance of the lot with a maximum allowable proportion defective. A statistical software (R) is used to obtain the table values and graphs. These tables and graphs are presented in order to compare the efficiency of this Single Sampling Plan so as to protect Acceptance Quality Level. Researchers thereafter suggest optimum criterion of sample size and acceptance number for a fixed ratio of PAR to MAPD within a reasonable sample size region.

Keywords: probability of acceptance (PA), operating characteristic, inflection point, Poisson distribution, optimum sampling plan, steepness angle.

1. Introduction

Maximum Allowable Percent Defective (MAPD) was suggested by Mayer (1956) and later developed by Mandelson (1962) below which the proportion of acceptance of the lot was expected to decline severely. Following Poisson distribution by the number of defectives, the ratio $\frac{c}{n}$ is efficient to divide the good and bad lots for the industrialist. There are some derivations on some basic operating procedure to locate single sampling plan on MAPD and its properties, (Norman (1953), Soundararajan (1975) Ramkumar and Suresh (1996)). Ramkumar (2010) also formed a criterion for developing Single Sampling Plan (SSP) on interval quality design in terms of MAPD in the "p" axis alike PAR in the Pa(p) axis. Also Ramkumar (2009) had suggested a sampling plan indexed through MAPD and discriminant distance, indicating the effectiveness of the Operating Characteristics (OC) curve on tangential distance and concept of optimum sampling plan under same operating ratio. PAR is the minimum probability of acceptance of a lot of quality having a maximum allowable proportion defective. In particular for a product with an incoming quality of MAPD <10%, more than 60% (PAR) lots are always accepted saving the interest of producer. Inflection point is the turning point of OC curve with steepest declination tangent indicating the sharpness of OC curve ensuring protection to the consumers. The steepness angle "q" is a sensitive measure to define the discrimination of the required OC curve. This angle will be wider for more stringent OC curve. Thus the parameters were capable of protecting both producer and consumer. So this plan is favorable in customer friendly, moderate costly, inspection oriented products like daily using items.
The aim and idea of this paper can be used practically when the producer had little information about the resultant quality, but he is ready to inspect a range of sample size for final decision.

2. Materials and Method

2.1 Maximum Allowable Percent Defective (MAPD)

MAPD is a key measure assessing to what degree the inflection point empowers the OC curve to discriminate between good and bad lots.
MAPD is therefore defined as the proportion of defective beyond which consumer won't be willing to accept the lot. The desirability for developing a system of sampling plan indexed by MAPD is $p^* = \frac{c}{n}$

2.2 Operating Characteristics (OC) Curve

An acceptance sampling plan is best described in graphical terms on an operating characteristic curve (OC curve), Baklizi, (2003). An OC curve is a plot of the actual number of nonconforming units in a lot (expressed as a percentage) against the probability that the lot will be accepted when sampled according to the plan. The shape of an OC curve is determined primarily by sample size, n, and acceptance number, c, although there is a small effect of lot size, N.
An important feature of a sampling plan is how it discriminates between lots of high and low quality. The ability of a sampling plan to discriminate is described by its operating characteristic (OC) curve

.2.3 Acceptance Quality Level

Acceptable quality level (AQL) is a quality measure fixing producers risk at 5, 10 and 1% in usual practice. But in an assembling of components, it is not good to prefix only one level of AQL at a fixed producer's risk. Also keeping at a level may exert pressure on the consumers as well as the producer because different items require different levels of acceptance. Thus fixed AQL sampling plans decline the confidence of both the producer and the customer. This is the limitation of all probability based indices with fixed levels as they are inadequate to specify the quality aspiration of the customer.

2.4 Selection and construction of the plan

Fix p* and Pa(p*) at suitable quality level in an OC curve.

Then $$Pa(p) = \sum_{r=0}^{c} \frac{e^{-c}c^{r}}{r!} \quad \textbf{(1)}$$

where the numbers of defectives follow Poisson distribution.

Therefore, $$d = 1 - P_a(p) = 1 - \sum_{r=0}^{c} \frac{e^{-c}c^{r}}{r!} \quad \textbf{(2)}$$

which is a function of c only, monotonically increasing, so that unique plan holds for each d. Find c matching with the given d (greater than or equal to the nearest d) But $c = np^*$.

So that $$n = \frac{c}{p^*} \quad \textbf{(3)}$$

Also one can find d from the steepness angle q opposite to distance d

$$tan\,\theta \;=\; \frac{d}{p^*} \quad \textbf{(4)}$$

Then $d = \; p^* tan\,\theta$. This value can be found for given MAPD and q.

$$\frac{d}{np^*} = \frac{d}{c} = \frac{\tan\theta}{n} \quad \textbf{(5)}$$

Therefore, from $Pa(p^*)$ or d and θ using equation (5) $\frac{\tan\theta}{n}$ is determined and substitute for $tan\,\theta$.
The value of n can then be equivalently be obtained as

$$n = \left(\frac{c}{d}\right)\tan\theta \quad \textbf{(6)}$$

3. Result and Data analysis

3.1 Sampling Plan Technique

Fixing various level of probability of acceptance at MAPD on the OC curve is called the producers allowable risk (PAR). PAR is probability liberalization to 70, 85 and 93% etc, instead of 95 and 100%. MAPD is an engineer's quality level with steepest declination beyond p*, so that the utmost quality of acceptance is prefixed by MAPD. Thus MAPD and PAR give a balanced design for both consumer and producer keeping information on steepness of OC. This is the only one point OC plan and may be flexible for various components. The steepness angle produced by d is a direct measure of efficiency of OC curve so the designs are possible in terms of required steepness.

Also in the composite production units or in the components, different MAPD and PAR can be fixed. Thus MAPD-PAR sampling plan is a flexible strategy of inspection of different components in a composite product with one point OC curves. Also angle q and PAR have a strong power of discrimination of lot as good and bad. Considering an example one can show the significance of this sampling plan. Fix the PAR=0.635, (d=0.365) and angle of steepness 80 °(1). Then c = 4 and n = 62 with MAPD=0.065 and AQL=0.032. For keeping the same AQL, with steepness angle 75 °, PAR will be 0.6767,
(d=0.3233) (2) and MAPD 0.083 satisfying a sampling plan (24.2) (Figure 2). Thus fixing AQL it is better to adjust PAR and the steepness angle so as to get required quality.

3.2 Construction of table

Table 1 is constructed by substituting $c = 1,2,..,10$ in equations (1) and (2). Table 2 represents Single Sampling Plans (SSPs) for various combinations of p* and d. Find c for each d from Table 1 and then n is determined by (3). Table 3 is the declination angle for various values p* and PAR. Finding d from (2), c is fixed from Table 1 and $tan\,\theta$ is found. It is helpful to identify (n,c) for given (p*, θ). Table 4 is the suitable sampling plans at various steepness angles and PA. For PAR and θ, n can then be evaluated by the relation $\frac{d}{c} = \frac{\tan\theta}{n}$ from Table 4.
For a defined PAR the decisive distance d can be calculated (Equation 2). Inspect the range in which calculated d or PAR falls uniquely and locates c (greater than or equal to d) from Table 1. Hence for a prefixed MAPD, $n = \frac{c}{p^*}$ The steepness angle θ subtended by d on the OC curve with the p axis at Pa(p*) and given MAPD can also be obtain using equation 4.

Table 1: Values of PAR for specified values of c =1 to 20.

c	1	2	3	4	5	6	7	8	9	10
Pa(p*)	0.7358	0.6767	0.6472	0.6289	0.6159	0.6063	0.5987	0.5926	0.5874	0.5830
d	0.2642	0.3233	0.3528	0.3711	0.3841	0.3937	0.4013	0.4074	0.4126	0.4170
c	11	12	13	14	15	16	17	18	19	20
Pa(p*)	0.5793	0.5759	0.5731	0.5704	0.5681	0.5659	0.5640	0.5623	0.5606	0.5591
d	0.4207	0.4241	0.4269	0.4296	0.4319	0.4341	0.4360	0.4377	0.4394	0.4409

Table 2: Confident SSP for given PAR (or d) and MAPD.

c	Pa(p*)	d	p*									
			0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.09	0.10	0.11
1	0.7358	0.2642	100	50	33	25	20	17	13	11	10	9
2	0.6767	0.3233	200	100	66	50	40	34	25	22	20	9
3	0.6472	0.3528	300	150	100	75	60	50	38	34	30	18
4	0.6289	0.3711	400	200	133	100	80	67	50	45	40	27
5	0.6159	0.3841	500	250	167	125	100	84	62	55	50	36
6	0.6063	0.3937	600	300	200	150	120	100	75	66	60	45
7	0.5987	0.4013	700	350	234	175	140	116	87	77	70	55
8	0.5926	0.4074	800	400	267	200	160	133	100	88	80	64
9	0.5874	0.4126	900	450	300	225	180	150	112	100	90	73
10	0.5830	0.4170	1000	500	333	250	200	166	125	111	100	82

Table 3: Confident parametric combinations of PAR, MAPD and Angle θ

c	d	PAR	p*									
			0.01	0.02	0.05	0.06	0.08	0.1	0.11	0.12	0.15	0.2
1	0.2642	0.7358	θ=87.8	85.67	79.2	77.20	73.15	69.26	67.3	65.57	60.4	52.87
2	0.3233	0.6767	88.22	86.69	81.20	79.48	76.10	72.8	71.2	69.6	65.1	58.25
3	0.3528	0.6472	88.37	86.75	81.93	80.34	77.22	74.1	72.6	71.2	66.96	60.45
4	0.3711	0.6289	88.45	86.55	82.32	80.8	77.8	74.9	73.4	72.08	67.99	61.67
5	0.3841	0.6159	88.50	87	82.58	81.12	78.2	75.4	74	72.6	68.6	62.49
6	0.3937	0.6063	88.52	87.09	82.76	81.3	78.5	75.7	74.3	73.04	69.1	63.06
7	0.4013	0.5987	88.57	87.14	82.89	81.49	78.7	76	74.6	73.35	69.5	63.5
8	0.4074	0.5926	88.59	87.18	83	81.62	78.8	76.2	74.8	73.58	69.78	63.8
9	0.4126	0.5874	88.61	87.2	83.09	81.72	79.02	76.3	75.07	73.78	70.02	64.13
10	0.4170	0.5830	88.62	87.25	83.16	81.8	79.1	76.5	75.22	73.9	70.2	64.23

Table 4: Values of n for given PAR (or d) and angle θ.

c	1	2	3	4	5	6	7
d	0.2642	0.3233	0.3528	0.3711	0.3841	0.3937	0.4013
Pa(p*)	0.7358	0.6767	0.6472	0.6289	0.6159	0.6063	0.5987
tanθ/n	0.2642	0.1616	0.1176	0.0927	0.0768	0.0656	0.0573
c/d	3.7850	6.1862	8.5034	10.7787	13.0174	15.2400	17.4433
c	8	9	10	11	12	13	14
d	0.4074	0.4126	0.417	0.4207	0.4241	0.4269	0.4296
Pa(p*)	0.5926	0.5874	0.583	0.5793	0.5759	0.5731	0.5704
tanθ/n	0.0509	0.0458	0.0417	0.0382	0.0353	0.0328	0.0306
c/d	19.6367	21.8128	23.9808	26.1468	28.2952	30.4520	32.5884
c	15	16	17	18	19	20	21
d	0.4319	0.4341	0.436	0.4377	0.4394	0.44	0.4419
Pa(p*)	0.5681	0.5659	0.564	0.5623	0.5606	0.56	0.5581
tanθ/n	0.0287	0.0271	0.0256	0.0243	0.0231	0.022	0.0210
c/d	34.7302	36.8578	38.9908	41.1240	43.2407	45.4545	47.5220

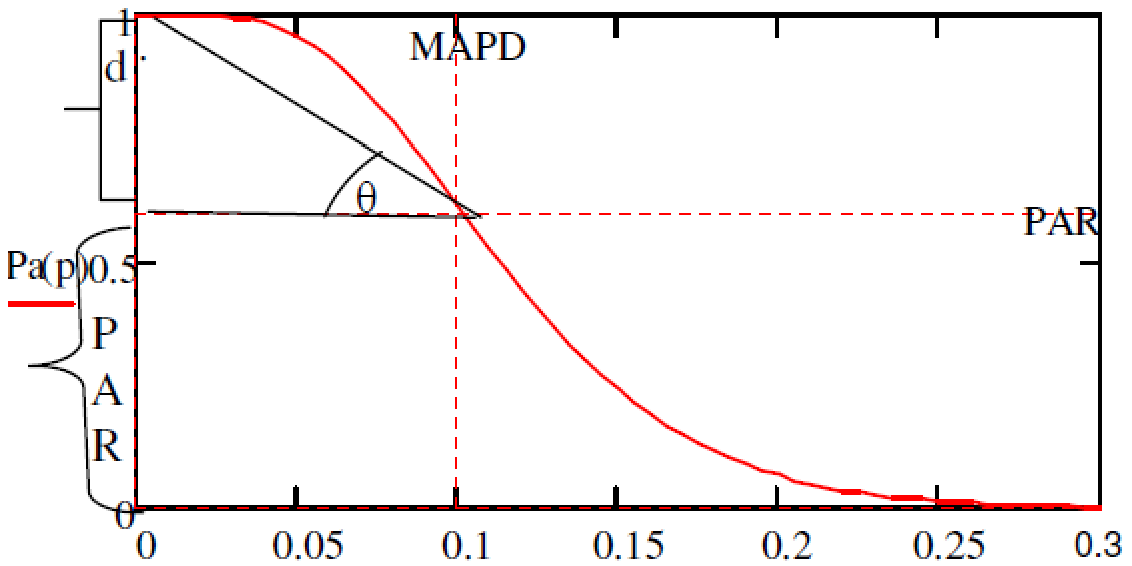

Figure 1: Operating curve (OC) curve showing maximum allowable proportion defective (MAPD) and producer’s allowable risk (PAR).

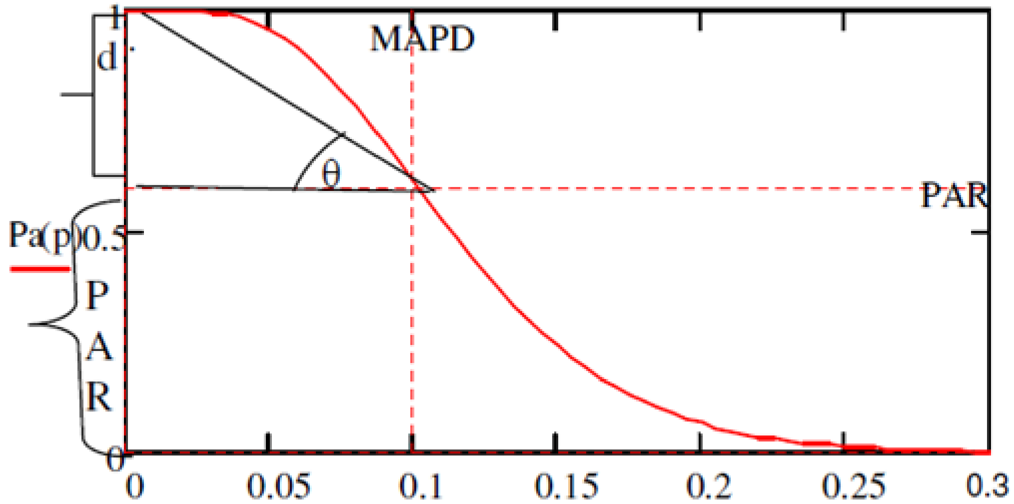

Figure 2: Operating characteristics (OC) curves for fixed acceptable quality level (AQL) defined on producer's allowable risk (PAR) and steepness angle.

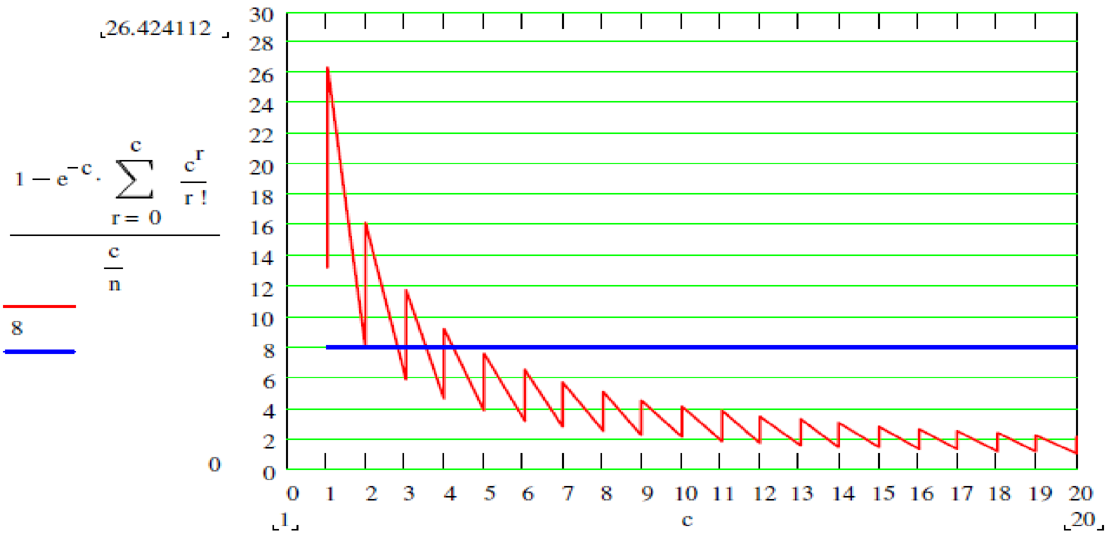

Figure 3: Feasible acceptance numbers for a fixed OR=8.

4. Discussion and Practical Examples

Figure 1 shows the operating characteristics (OC) curves stringent or moderate at the same quality of AQL which can be finalized on PAR and steepness angle. When PAR is less and angle is more there exists a more discriminating OC curve than higher PAR and lower angle. Also for such parameters MAPD increases on PAR increase. Figure 2 is the feasible acceptance numbers within a sample size 50 to 100 keeping operating ratio $\frac{d}{p^*}$ a constant, from which maximum and minimum efficient OC curves can be derived. Figure 3 shows the OC curves satisfying the defined OR=8.0 so that switching rules could be implemented.

4.1 Practical Examples

Practically this plan can be put to use when the producer had little prior knowledge about the resulting quality, but he is ready to inspect a range of sample size for final decision. Pa(p*) or d is a good measure of efficiency of OC curve. The MAPD is an appropriate quality measure so that a ratio is fixed as constant and this sampling plan is developed on these criteria.

Example 1

The quality of an electronic is fixed at maximum allowable proportion defective 8% and PAR = 0.70. The decisive distance d = 1 - 0.70 = 0.30. From Table 1, $0.2642 \leq d < 0.3233$ for c=2.
Sampling plan (n, c) is (25. 2). And the steepness angle θ = 76.10.

Example 2

A wheel-barrow producing company fixes the MAPD =10% and steepness angle of OC curve as 74 and 68° respectively. Then decisive distances d_1=p*.tanθ_1 = 0.1 × 3.487 =0.3487 and d_2=p*.tan θ_2 =0.1x2.475 = 0.2475 on OC curve with sampling plans from Table 2 will be (30. 3) and (100.1).

Example 3

If the same wheel-barrow producing company fixes a quality index OR = 8 based on MAPD and decisive distance, provided they are ready to bear an inspection cost of 50 to 100 units. Then what will be the optimum sampling plan? Since the quality index is fixed at R=8, for a range of sample n = 50 to 100 and c = 1 to 20. Then the various sampling plans obtained were (50.2), (67.3) and (85.4), (Fig. 4). Among these sampling plans, optimum is reached at (85.4), and they can start with (50.2) and switch over to (67.3) on successive completion of fixed number of lots. From the OC curves (85. 4), has an optimum probability of acceptance at maximum allowable proportion defective (Figure 4).

5.0 Conclusion

This sampling plan therefore suggested to be efficient to contain a Maximum Allowable Percent Defective (MAPD) oriented Acceptance Quality Level (AQL) protection satisfactory for both consumer and producer (i.e reducing both producer and consumer's risk). Decisive distance and declining angle is a concept to exhibit the quality of Operating Characteristics (OC) curve. Adopting these sampling plans will not highly affect the OC curve, but can also reduce inspection cost.

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