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VISCOUS FLOW AND HEAT TRANSFER IN A VERTICAL CHANNEL WITH DEFORMABLE POROUS LAYER MEDIUM

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Abstract

Steady flow of a viscous fluid in a vertical deformable porous layer bounded by parallel plates is investigated. The vertical plates $y=0$ and $y=h$ are maintained at constant temperatures T_0 and T_w respectively. The solid displacement, the fluid velocity and the temperature distribution are obtained. The effects of various physical parameters such as ϕ^f and η on the velocity and displacement are discussed in detail. It is observed that the both velocity and displacement increase with the increasing drag in the deformable layer.

Keywords: natural convection; deformable porous layer; heat source.

Nomenclature

x, y Cartesian coordinates	Gr Grashoff number
μ_a the apparent viscosity of the fluid in the porous material	T_w Wall Temperature
K the drag coefficient	T_0 Ambient Temperature
μ the Lame constant	ρ Density
μ_f the coefficient of viscosity	C_p Specific heat at constant pressure
v the flow velocity	$\frac{\partial p}{\partial x}$ the pressure gradient
U the displacement	δ Viscous drag
h width of the wall	
θ Temperature	

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1. Introduction

Viscous flow through deformable porous materials with coupled fluid movement was initiated by Terzaghi [1]. Later Biot [2] developed a theory of soil consolidation and acoustic propagation. Atkin and Craine [3], Bowen [4] and Bedford and Drumheller [5] made some important contributions to the theory of mixtures. Applying this theory Jayaraman [6] discussed wall permeability of the arteries. Mow et al. [7] proposed a similar theory for the study of biological tissue mechanics. Using this theory, Holmes and Mow [8] discussed rectilinear cartilages. Barry [9] made a systematic study on the fluid flow over a deformable porous layer. Farina et al. [10] discussed moulding processes using the theory of deformable porous media. Ambrosi [11] investigated infiltration phenomena through deformable porous media.

Klubertanz et al. [12] are investigated multiphase flow in deformable porous media. Irfan Khan [13] made a numerical study of saturated deformable porous media. Sreenadh et al. [14] studied the Couette flow over a deformable permeable bed. In view of several important applications, it will be interesting to study fluid flow through and past deformable porous layers.

Ostrach [15] has studied laminar natural convection flow and heat transfer of fluids with and without heat sources in the channels with constant wall temperatures. Beckerman et al. [16] studied natural convection in vertical enclosures containing simultaneously fluid and porous layers. Tanda et al. [17] discussed natural convection in partially heated vertical channels. Kolar and Vafai [18] studied numerically free convection transpiration over a vertical plate. Holmes et al. [20] examined natural convection about a vertical plate embedded in porous medium. Vajravelu et al. [21] studied the influence of heat transfer on peristaltic transport of a Jeffrey fluid in a vertical porous stratum. Sanvicente et al. [22] investigated the natural convection flow and heat transfer in an open channel. Motivated by the above studies, steady flow and heat transfer in a vertical deformable porous layer is investigated. The solid displacement, the fluid velocity and the temperature distribution are obtained. The results are discussed for various physical parameters.

2. Mathematical formulation

Consider a steady, fully developed flow through a vertical deformable porous layer bounded by rigid walls as shown in Fig.1.Let the x-axis be taken along one of the plates. The plates are maintained at temperatures T_w and T_0 respectively and the $y-$axis is chosen perpendicular to $x-$axis. A pressure gradient $\frac{\partial p}{\partial x}$ is applied, producing an axially directed flow.

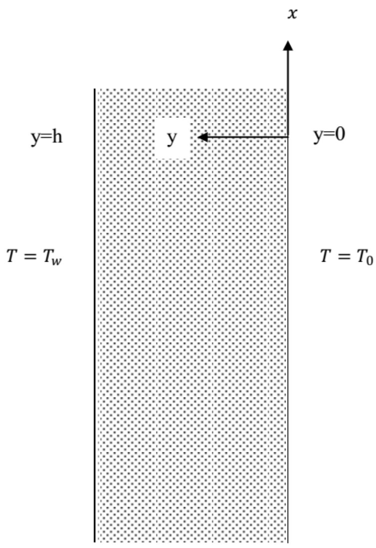

Fig.1 Physical Model

The basic equations of the problem are

$$\mu \frac{\partial^2 u}{\partial y^2} - (1-\varphi^f)\frac{\partial p}{\partial x} + Kv = 0$$
(1)

$$\frac{1}{\eta}\left(\frac{\mu_f}{\rho}\right)\frac{\partial^2 v}{\partial y^2} - \frac{\varphi^f}{\rho}\frac{\partial p}{\partial x} - \frac{Kv}{\rho} + g\beta(T-T_0) = 0$$
(2)

$$\frac{\partial^2 T}{\partial y^2} = 0 \qquad (3)$$

where μ_a is the apparent viscosity of the fluid in the porous material, K is the drag coefficient, μ is the Lame constant, μ_f is the coefficient of viscosity, v is the flow velocity, u is the solid displacement and $\frac{\partial p}{\partial x}$ is the pressure gradient .

It is convenient to introduce the following non-dimensional quantities:

$$y^* = \frac{y}{h},\; v^* = \frac{v}{U},\; \theta^* = \frac{T-T_0}{T_w-T_0},\quad p^* = \frac{hp}{\mu_f U},\; x^* = \frac{x}{h}, u^* = \frac{u\mu}{h^2 G_0},$$
$$\delta = \frac{Kh^2}{\mu_f}, Gr = \frac{g\beta(T_w-T_0)\rho h^2}{\mu_f U}, \eta = \frac{\mu_f}{2\mu_a}$$

In view of the above dimensionless quantities, the equations (1) – (3) take the following form. The asterisks (*) are neglected hereafter.

$$\frac{d^2u}{dy^2} = (1-\varphi^f)G - \delta v$$
(4)

$$\frac{d^2v}{dy^2} - \eta\delta v = (\varphi^f\eta)G - \eta Gr\theta$$
(5)

$$\frac{d^2\theta}{dy^2} = 0 \qquad (6)$$

Where $G = \frac{dp}{dx}$

The parameter δ is a measure of the viscous drag of the outside fluid relative to drag in the porous medium. The parameter η is the ratio of the bulk fluid viscosity to the apparent fluid viscosity in the porous layer.

The boundary conditions are

$u = 0$ at $y = 0$
$u = 1$ at $y = 1$
(7)

$v = 0$ at $y = 0$
$v = 1$ at $y = 1$
(8)

$\theta = 0$ at $y = 0$
$\theta = 1$ at $y = 1$
(9)

3. Solution of the problem

The governing equations (4) to (6) are coupled differential equations that can be solved by using the boundary conditions (7) and (9). The solid displacement, fluid velocity and temperature in the free flow region and porous regions are obtained as

$$u = \frac{a_2 y^2}{2} - \delta\left(\frac{Ae^{\alpha y}}{\alpha^2} + \frac{Be^{-\alpha y}}{\alpha^2} + \frac{a_0 y^2}{2} - \frac{a_1 y^3}{6}\right) + Cy + D \tag{10}$$

$$v = Ae^{\alpha y} + Be^{-\alpha y} + a_0 - a_1 y \tag{11}$$

$$\theta = y \tag{12}$$

where

$$\left.\begin{aligned}
&a_2 = (1-\phi^f)\frac{dp}{dx}, a_0 = \phi^f \eta \frac{dp}{dx},\\
&A = \frac{1}{e^{-\alpha} - e^{\alpha}}\left(-a_0\left(e^{-\alpha} - 1\right) - a_1 - 1\right), B = \frac{1}{e^{\alpha} - e^{-\alpha}}\left(-a_0\left(e^{\alpha} - 1\right) - a_1 - 1\right), \alpha = \sqrt{\delta\eta},\\
&C = 1 - \frac{a_2}{2} + \delta\left(\frac{Ae^{\alpha}}{\alpha^2} + \frac{Be^{\alpha}}{\alpha^2} + \frac{a_0}{2} - \frac{a_1}{6}\right), D = \delta\left(\frac{A+B}{\alpha^2}\right)
\end{aligned}\right\} \tag{13}$$

4. Results and discussions

In this paper, natural convection effects on the steady flow through a vertical deformable porous layer are investigated and the results are discussed for various physical parameters ϕ^f, Gr, δ and η.

The variation of solid displacement u with y is calculated from equation (10) for different values of δ, Gr and η and is shown in Figures 1, 2 and 3 for fixed $\phi^f = 0.6$ and $G = -1$. It is seen that the solid displacement increases with increasing drag δ or Grashoff number Gr or the viscous ratio parameter η.

The variation of solid displacement u with y is calculated for different values of φ^f and is shown in Figure 4 for fixed $G = -1, Gr = 5, \eta = 0.5$ and $\delta = 1.0$. It is observed that the velocity increases with the decreasing φ^f.

The variation of fluid velocity v with y is calculated from equation (11) for different values of δ, Gr and η is shown in Figures 5, 6 and 7 for fixed $\phi^f = 0.6$ and $G = -1$. It is observed that the velocity increases with the increasing drag δ or Grashoff number Gr or viscosity ratio parameter η.

The variation of fluid velocity v with y is calculated for different values of φ^f and is shown in Figure 8 for fixed $G = -1, Gr = 5, \eta = 0.5$ and $\delta = 2.0$. It is observed that the velocity decreases with the increasing φ^f.

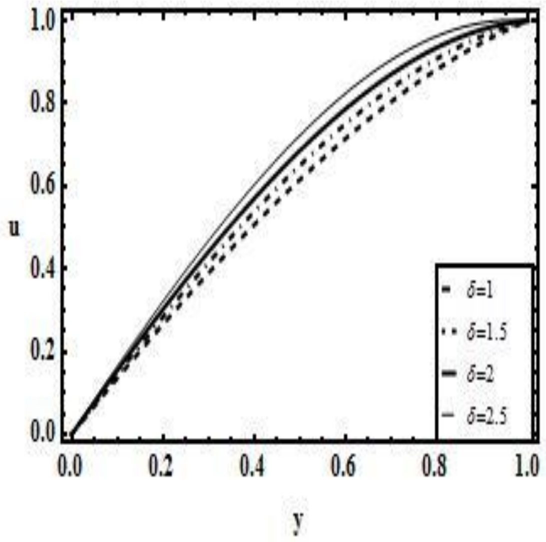

Figure 1.Displacement profiles for different values of δ .

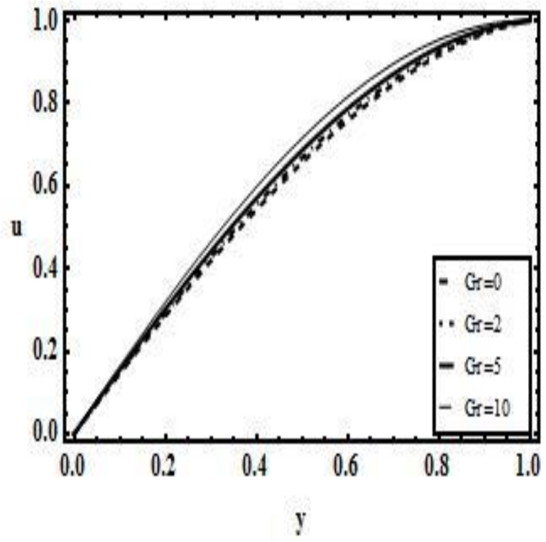

Figure 2.Displacement profiles for different values of Gr .

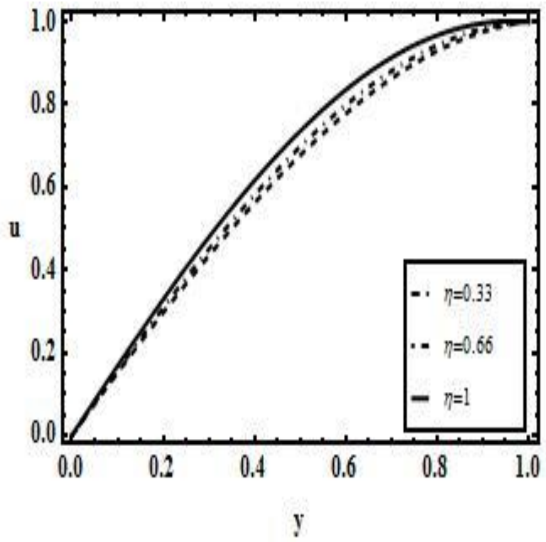

Figure 3.Displacement profiles for different values of η .

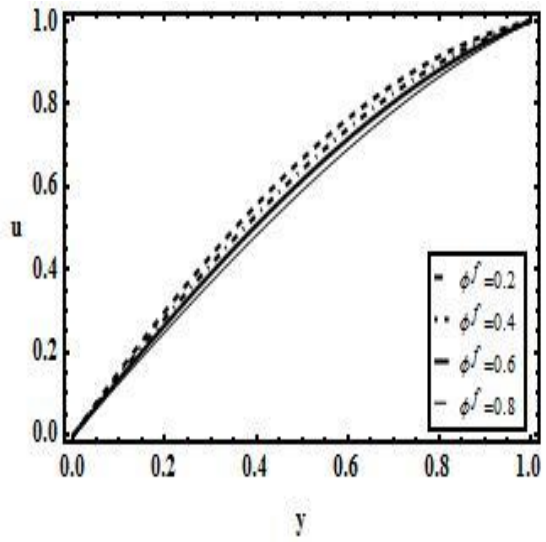

Figure 4.Displacement profiles for different values of ϕ^f .

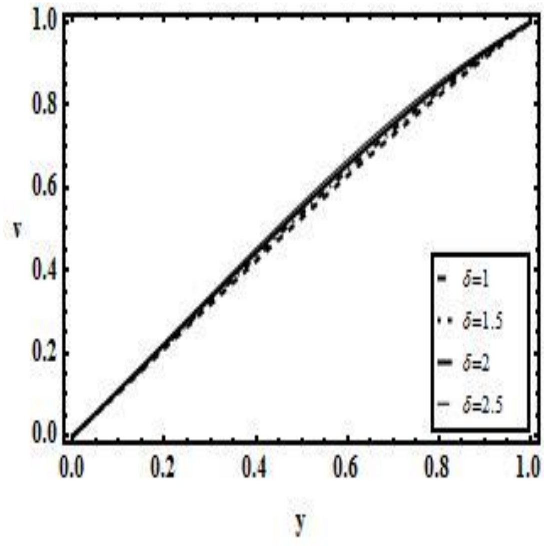

Figure 5. Velocity profiles for different values of δ.

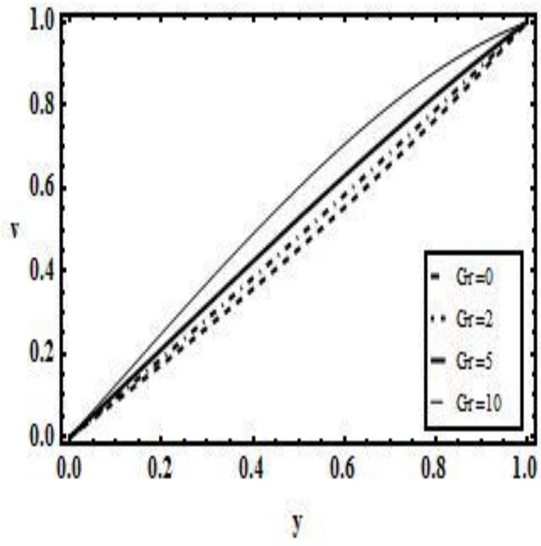

Figure 6. Velocity profiles for different values of Gr.

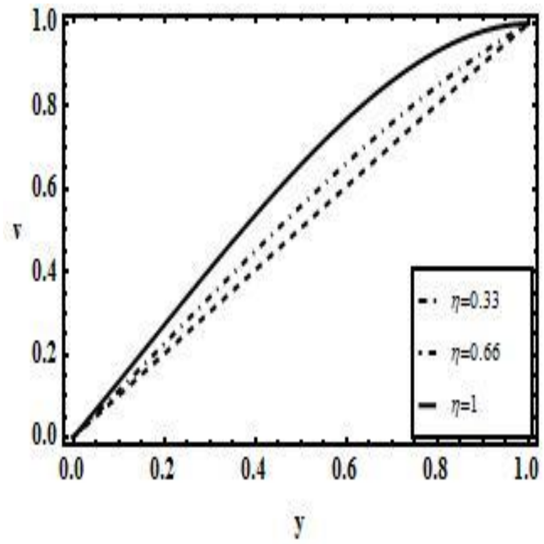

Figure 7.Velocity profiles for different values of η.

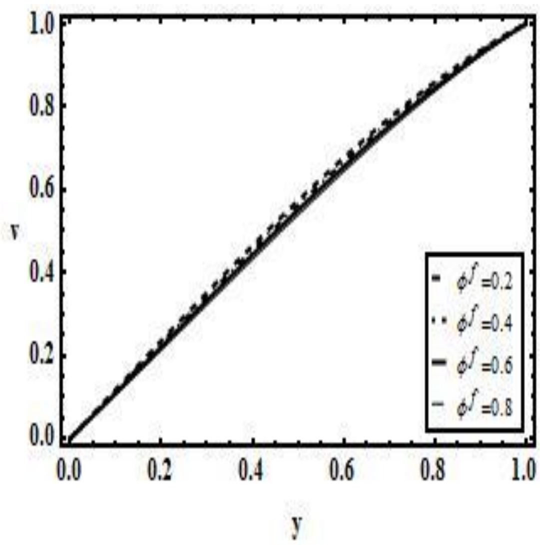

Figure 8. Velocity profiles for different values of ϕ^f.

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