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Effect the Magnetic field over an inclined stretching sheet of three dimensional Maxwell fluid in obscene of Mixed convection

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Abstract

Three dimensional flow of Maxwell fluid with boundary condition is investigated. transformations are victimized to reduce the partial differential equations into ordinary differential equations. Effect of two parameters ,Magnatic field parameter and Deborah number parameter on non dimensional velocity are discussed ,homotopy analysis method(HAM) is used to solve the velocity equations.

Keywords: steady flow , Maxwell fluid , Magnetic field, HAM solution.

1. Introduction

The problem of non –Newtonian fluid are very important in many applications such as metallurgical process, wire drawing, polymer extrusion, food processing industry, and many others. Lost all the fluids occurring in industry and biomedicine are non – Newtonian.

In the recent years the flow of Maxwell fluid with magnatic field have been studied by some researchers, In [7] M.Qasim and S. Noreen studied the falkner –skan flow of Maxwell fluid with heat transfer and magnatic field, he used the homotopy method to solve the flow and heat equations . In another paper Vigendra Singh ,Shwet agranal in[14]discussed MHD flow and heat transfer for Maxwell fluid over exponentially stretching sheet,the implicit finite difference scheme is used to solve the problem .The flow of Maxwell fluid due to constantly moving flat radiative surface with convective condition under the influence of non uniform transverse magnetic field are studied by M. Mustafa [6].Mixed convection radiative flow of three dimensional Maxwell fluid over an inclined stretching sheet in presence of thermophoresis and convective condition investigation by[4]

In this paper we studied the effect the magnetic field of Maxwell fluid in three dimenational in the obscene the mixed convection raditive , homotopy methed is used to obtain the analysis solutions. This method is general and its power technique for the non linear differential equations.

2. **Homotopy analysis method** (HAM) [1], [2],[10]

In order to show the basic idea of HAM, consider the following differential equation

$$N[u(\tau)] = 0 \quad \text{.... (1)}$$

where N is a nonlinear operator, τ denote the independent variables and u is an unknown function. For simplicity, we ignore all boundary or initial conditions,

By means of the HAM, we construct the zeroth-order deformation equation.

$$(1-q)L[\Phi(\tau;q)-u_0(\tau)]=qhH(\tau)N[\Phi(\tau;q)] \quad ..(2)$$

where $q \in [0; 1]$ is the embedding parameter, $H(\tau \neq 0)$ is an auxiliary parameter, L is an auxiliary linear operator, $u_0(\tau)$is an initial guess . It is obvious that when the embedding parameter q = 0 and q = 1, it holds

$$\Phi(\tau; 0) = u_0(\tau);\ \Phi(\tau;1) = u(\tau); \quad ..(3)$$

Thus as q increases from 0 to 1, the solution varies from the initial guess $u_0(\tau)$ to the solution $u(\tau)$. Expanding $\Phi(\tau; q)$ in Taylor series with respect to q , one has

$$\Phi(\tau;q) = u_0(\tau)+\sum_{m=1}^{\infty} u_m(\tau)\ q^m \quad ..(4)$$

$$u_m = 1/m! \frac{\partial^m \Phi(t,q)}{\partial q^m}\Big|_{q=0}, \quad ..(5)$$

If the auxiliary linear operator, the initial guess, the auxiliary h, and the auxiliary function are so properly chosen, the series (4) converges at q=1 , then we have

$$u(\tau) = u_0(\tau) + \sum_{m=1}^{\infty} u_m(\tau) \quad ..(6)$$

define the vector

$$(\tau)=\{u_0(\tau), u_1(\tau), \ldots \ldots u_n(\tau)\}\vec{u}$$

Differentiating equation (2) m times with respect to the embedding parameter q and then setting q= 0 and finally dividing them by m!, we obtain the *m*th – order deformation equation

$$hH(\tau)R_m(\overrightarrow{u_{m-1}}) = L[u_m(\tau) - x_m u_{m-1}] \quad ..(7)$$

$$R_m(\overrightarrow{u_{m-1}}) = \frac{\partial^{m-1} N[\Phi(\tau;q)]}{\partial q^{m-1}}\Big|_{q=0} \quad ..(8)$$

$$\chi_m = \begin{cases} 0, m \le 1 \\ 1, m > 1 \end{cases} \quad ..(9)$$

Applying L^{-1} on both side of equation (7), we get

$$u_m(\tau) = x_m u_{m-1}(\tau) + hL^{-1}[H(\tau)R_m(\tau)]$$

In this way, it is easily to obtain u_m for m $\geq$ 1 ,at mth- order, we have

$$u(\tau) = \sum_{m=0}^{M} u_m(t) \quad ..(10)$$

when$M \to \infty$, we get an accurate approximation of the original equation (1).

3. Formulation of the problem

Consider, the flows are incompressible, steady MHD of Maxwell fluid in three dimensional the flow takes place in the domain z > 0. The mathematical statements for the Boundary layer problems are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad ..(11)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \upsilon\frac{\partial^2 u}{\partial y^2} - \lambda\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + w^2\frac{\partial^2 u}{\partial z^2} + 2uv\frac{\partial^2 u}{\partial x\,\partial y} + 2vw\frac{\partial^2 u}{\partial y\,\partial z} + 2uw\frac{\partial^2 u}{\partial x\,\partial z}\right) - \frac{\sigma\beta_0^2}{\rho}u \quad ..(12)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \upsilon\frac{\partial^2 v}{\partial y^2} - \lambda\left(u^2\frac{\partial^2 v}{\partial x^2} + v^2\frac{\partial^2 v}{\partial y^2} + w^2\frac{\partial^2 v}{\partial z^2} + 2uv\frac{\partial^2 v}{\partial x\,\partial y} + 2vw\frac{\partial^2 v}{\partial y\,\partial z} + 2uw\frac{\partial^2 v}{\partial x\,\partial z}\right) \quad ..(13)$$

Where u and v and w are the velocities in the x , y and z directions, respectively, ρ is the fluid density,$\upsilon = \frac{\mu}{\rho}$ is the kinematic viscosity , $\mu$is the dynamic viscosity, σ is the electric conductivity,$\beta_0$is the strength of magnatic field, λ is the relaxation time .

The boundary condition are given by

$u=u_e=ax$, $v=by$, $w=0$ at $z=0$

$u\to 0$, $v\to 0$, as $z \to\infty$

, ..(14)

$u \to 0,\ v \to 0$ as $z \to \infty$

In order to simplexes eqs (11-13),we introduce the new quantities:-

$u = axf'(\eta),\ v = ayg'(\eta),\ w = -\sqrt{a\,v}\ (f(\eta)+g(\eta)),\ \eta = z\ \sqrt{a/v},$

Now, in the above quantities Eq. (11) is satisfies automatically .While Eqs. (12),(13) are reduced as follows:

$$f''' + (f+g)f'' - f'^2 + \beta_1[2(f+g)f'f'' - (f+g)^2 f'''] - M f' = 0, \quad ..(15)$$

$$g''' + (f+g)g'' - g'^2 + \beta_1[2(f+g)g'g'' - (f+g)^2 g'''] = 0, \quad ..(16)$$

and the boundary conditions(14) reduce to

$f=0,\ g=0,\ f'=1,\ g'=\beta,$ at $\eta=0,$

$f' \to 0,\ g' \to 0$ as $\eta \to \infty,$

Where β_1. is the dimensionless Deborah number, β is ratio of rates parameter , ρ is the fluid density, g is the gravitational acceleration, M is the magnetic field parameter and prime shows the differentiation with respect to η. These are given by

$\beta_1 = \lambda_1 a,\ \beta = ba\ ,\ M = \frac{\sigma \beta_0^2}{a\rho}$

4. Method of solution

The homotopy analysis method is impieties to find the solutions of equation (15),(16) which are required the initial approximations and auxiliary linear operators are presented below i.e.

$f_0(\eta) = (1 - e^{-\eta}),\ g_0(\eta) = \beta(1 - e^{-\eta}),$

$L_1 = f''' - f',\ L_2 = g''' - g',$

$L_1(C_1 + C_2 e^{\eta} + C_3 e^{-\eta}) = 0,\ L_2(C_4 + C_5 e^{\eta} + C_6 e^{-\eta}) = 0,$

where C_i (i = 1–10) are the arbitrary constants.

The zeroth order deformation equations are:

$$(1-q)L_1[f^*(\eta;q) - f_0(\eta)] = q\hbar_1 N_1[f^*(\eta;q), g^*(\eta;q)], \quad ..(17)$$

$$(1-q)L_2[g^*(\eta;q) - g_0(\eta)] = q\hbar_2 N_2[f^*(\eta;q), g^*(\eta;q)], \quad ..(18)$$

$f^*(0;q)=0,\ f^{*\prime}(0;q)=1,\ f^{*\prime}(\infty;q)=0,\ g^*(0;q)=0,\ g^{*\prime}(0;q)=\beta,\ g^{*\prime}(\infty;q)=0$

$$N_1[f^*(\eta;q), g^*(\eta;q)] = \frac{\partial^3 f^*(\eta;p)}{\partial\eta^3} - \left(\frac{\partial f^*(\eta;p)}{\partial\eta}\right)^2 + (f^*(\eta;p) + g^*(\eta;p))\frac{\partial^2 f^*(\eta;p)}{\partial\eta^2} + \beta^*[2(f^*(\eta;p) + g^*(\eta;p))\frac{\partial f^*(\eta;p)}{\partial\eta}\frac{\partial^2 f^*(\eta;p)}{\partial\eta^2} - ((f^*(\eta;p) + g^*(\eta;p))^2 \frac{\partial^3 f^*(\eta;p)}{\partial\eta^3}] - M\frac{\partial f^*(\eta;p)}{\partial\eta} \quad ..(19)$$

$$N_2[f^*(\eta;q), g^*(\eta;q)] = \frac{\partial^3 g^*(\eta;p)}{\partial\eta^3} - \left(\frac{\partial g^*(\eta;p)}{\partial\eta}\right)^2 + (f^*(\eta;p) + g^*(\eta;p))\frac{\partial^2 g^*(\eta;p)}{\partial\eta^2} + \beta^*[2(f^*(\eta;p) + g^*(\eta;p))\frac{\partial g^*(\eta;p)}{\partial\eta}\frac{\partial^2 g^*(\eta;p)}{\partial\eta^2} - ((f^*(\eta;p) + g^*(\eta;p))^2 \frac{\partial^3 g^*(\eta;p)}{\partial\eta^3}] \quad ..(20)$$

Where q is an embedding parameter, $\hbar_1$, $\hbar_2$, are the non-zero auxiliary parameters and N_1, N_2, the nonlinear operators. When $q = 0$ and $q = 1$

$f^*(\eta;0) = f_0(\eta),\ f^*(\eta;1) = f(\eta),$

Clearly when q is increased from 0 to 1 then $f(\eta,\ q),\ g(\eta,\ q)$ vary from $f_0(\eta),\ g_0(\eta)$, to $f(\eta), g(\eta)$,). By Taylor's expansion we have

$f(\eta,q) = f_0(\eta)\ \sum_{m=1}^{\infty}\ f_m(\eta)q_m,\ f_m(\eta) = \frac{1}{m!}\ \frac{\partial^m f(\eta;q)}{\partial\eta^m}]_{q=0}$

..(21)

$g(\eta, q)=g_0(\eta)\sum_{m=1}^{\infty} g_m(\eta)q^m$, $gm(\eta)=\frac{1}{m!}\frac{\partial^m g(\eta;q)}{\partial\eta^m}]_{q=0}$

Where the convergence of above series strongly depends upon $\hbar_1, \hbar_2$. Considering that $\hbar_1, h_2$ are selected properly so that the series(21) converge at $q=1$ then we can write

$f(\eta)=f_0(\eta)+\sum_{m=1}^{\infty} f_m(\eta)$,

$g(\eta)=g_0(\eta)+\sum_{m=1}^{\infty} g_m(\eta)$,

The resulting problems at mth order deformation can be constructed as follows:

$L_1[f_m(\eta)-\chi_m f_{m-1}(\eta)]=\hbar_1 R_1^m(\eta)$

..(22)

$L_2[g_m(\eta)-\chi_m g_{m-1}(\eta)]=\hbar_2 R_2^m(\eta)$

,

$f_m(0)=f'_m(0)=f'_m(\infty)=0$, $g_m(0)=g'_m(0)=g'_m(\infty)=0$

$$R_1^m(\eta)= D[f_{m-1},\eta,\eta,\eta]-(M*D[f_{m-1},\eta])-\sum_{k=0}^{m-1}(D[f_{m-1-k},\eta]*D[f_k,\eta])+\sum_{k=0}^{m-1}((f_{m-1-k}*D[f_k,\eta,\eta])+(g_{m-1-k}*D[f_k,\eta,\eta]))+\beta\sum_{k=0}^{m-1}\sum_{i=0}^{k}(2(f_{m-1-k}+g_{m-1-k})*D[f_{k-i},\eta]*D[f_i,\eta,\eta])-(((f_{m-1-k}*f_{k-i})+(g_{m-1-k}*g_{k-i}))+(2*(f_{m-1-k}*g_{k-i})))*D[f_i,\eta,\eta,\eta]$$

..(23)

$$R_2^m(\eta)= D[g_{m-1},\eta,\eta,\eta]-\sum_{k=0}^{m-1}(D[g_{m-1-k},\eta]*D[g_k,\eta])+\sum_{k=0}^{m-1}((f_{m-1-k}*D[g_k,\eta,\eta])+(g_{m-1-k}*D[g_k,\eta,\eta]))+\beta\sum_{k=0}^{m-1}\sum_{i=0}^{k}(2(f_{m-1-k}+g_{m-1-k})*D[g_{k-i},\eta]*D[g_i,\eta,\eta]-(((f_{m-1-k}*f_{k-i})+(g_{m-1-k}*g_{k-i}))+(2*(f_{m-1-k}*g_{k-i})))*D[g_i,\eta,\eta,\eta]$$

..(24)

By using software mathematic it is found that:

$f_0=1\text{-Exp}[-\eta]$ ..(25)

$$f_1=\frac{1}{24}e^{-3\eta}(h1\beta(-1+\beta^2)-4e^{\eta}h1\beta(1+2\beta+2\beta^2)-6e^{2\eta}(h1(M+\beta(2+2\beta+\beta^2))(3+2\eta)+4(\tfrac{1}{24}(24-6h1M-23h1\beta-28h1\beta^2-19h1\beta^3))))$$ ..(26)

$$f_2=\frac{1}{2880}e^{-5\eta}(h1^2\beta^2(3+32\beta+22\beta^2-32\beta^3-25\beta^4)+8e^{\eta}h1^2\beta(-1-14\beta-20\beta^2+13\beta^3+53\beta^4+29\beta^5)+5e^{2\eta}h1\beta\left(96(-1+\beta^2)+h1\left(-26+9\beta^5(-15+4\eta)+4\beta^3(-80+9\eta)+8\beta^4(-43+9\eta)-2\beta^2(91+36\eta)-\beta(73+72\eta)+2M(-18(1+\eta)+\beta^2(-7+6\eta))\right)\right)$$

..(27)

$$f_3=\frac{1}{2419200}e^{-7\eta}(5h^3\beta^3(-155-1408\beta-3967\beta^2-2880\beta^3+2695\beta^4+4288\beta^5+1427\beta^6)-4e^{\eta}h1^3\beta^2(-240-4751\beta-24522\beta^2-39788\beta^3+36\beta^4+67749\beta^5+67862\beta^6+20054\beta^7)-7e^{2\eta}h1^2\beta(840\beta(-3-32\beta-22\beta^2+32\beta^3+25\beta^4)$$

..(28)

$$f_4=-\frac{1}{174182400}e^{-9\eta}h1^4\beta^4(-17619-213424\beta-917030\beta^2-1745360\beta^3-1162248\beta^4+859184\beta^5+1874270\beta^6+1099600\beta^7+222627\beta^8)+\frac{1}{304819200}e^{-8\eta}h1^4\beta^3(-41910-1003527\beta-7123122\beta^2-22075329\beta^3-30357442\beta^4-4581577\beta^5+40482514\beta^6+52695097\beta^7+27280216\beta^8+5205080\beta^9))$$

..(29)

$g_0=\beta(1-\text{Exp}[-\eta])$; ..(30)

$$g_1 = \frac{1}{24} e^{-3\eta}(-h2\beta^2(-1+\beta^2) - 4e^{\eta}h2\beta(1+2\beta+2\beta^2) - 6e^{2\eta}(h2\beta^2(2+2\beta+\beta^2)(3+2\eta) + 4(\frac{1}{24}(24\beta - 8h2\beta - 25h2\beta^2 - 28h\beta^3 - 9h2\beta^4)))) + \frac{1}{12}(12\beta - 2h2\beta + 9h2\beta^2 + 8h2\beta^3 + 5h2\beta^4)$$
..(31)

$$g_2 = \frac{1}{2880} e^{-5\eta}(h2^2\beta^3(-25-32\beta+22\beta^2+32\beta^3+3\beta^4) - 8e^{\eta}h2^2\beta^2(-10-38\beta-47\beta^2 - 2\beta^3+29\beta^4+8\beta^5) - 5e^{2\eta}h2\beta(96\beta(-1+\beta^2) + h2(16+\beta^2(265-72\eta)+\beta^3(386-72\eta) + 12\beta^4(23+3\eta)+8\beta^5(5+9\eta)+\beta^6(3+36\eta)+\beta(94-50M-24M\eta)))$$
..(32)

$$g_3 = \frac{1}{2419200} e^{-7\eta}(-5h2^3\beta^4(-1427 - 4288\beta - 2695\beta^2 + 2880\beta^3 + 3967\beta^4 + 1408\beta^5 + 155\beta^6) + 4e^{\eta}h2^3\beta^3(-5361 - 34054\beta - 73844\beta^2 - 57204\beta^3 + 16603\beta^4 + 46010\beta^5 + 19338\beta^6 + 2112\beta^7)$$
..(33)

$$g_4 = \frac{1}{174182400} e^{-9\eta}h2^4\beta^5(-222627 - 1099600\beta - 1874270\beta^2 - 859184\beta^3 + 1162248\beta^4 + 1745360\beta^5 + 917030\beta^6 + 213424\beta^7 + 17619\beta^8) - \frac{1}{304819200} e^{-8\eta}h2^4\beta^4(-1145481 - 10557722\beta - 35222367\beta^2 - 54589726\beta^3 - 32677927\beta^4 + 14826314\beta^5 + 33985495\beta^6 + 19665294\beta^7 + 4820536\beta^8 + 415584\beta^9)$$
..(34)

5.Result and discussion

Its clear that the convergence of homotopic depends upon the parameters $h1, h2$. In this section we show the graphical results of velocity ,for this purpose figurs [1,2] explain the effect the magnatic field on profile velocity f when the magnatic parameter M=[1.2,2,2.7] and ratio of rates parameter β =0.3,1.5, its noted that when the magnatic field increasing and β =0.3 the velocity profile f and momentum boundary layer thickness is decreasing see figure(1)also when M are increasing , $\beta >= 1.3$the velocity is increasing see figure(2).

Figures[3,4] shows the influence of stretching ratio parameter β on velocity profile f , its clear that at figure3 the increasing in β at M=1.7 make the velocity profile and momentum boundary layer thickness is decreasing this is due to the fact that within the increase of ratio of rates parameter βrelaxation time increases as a result the velocity and boundary layer thickness decreases .fig[4] explain the effect β on profile velocity f ,when M= 3,it noted that the result as similar as figure3 . In fig [5] show the effect the magnetic field M on velocity g its noted that as the M increase , where M=[0,0.5,1] , β =0.3the velocity g is decreasing. In case take M=[0,0.7,1.3], β=1.4 then the velocity is increase . Figure [6,7] discuss the effect stretching ratio parameter β on the velocity g its noted that at M=0.5, 1, β=[0.2,0.3,0.6] the velocity is decreasing with increase the magnetic field.

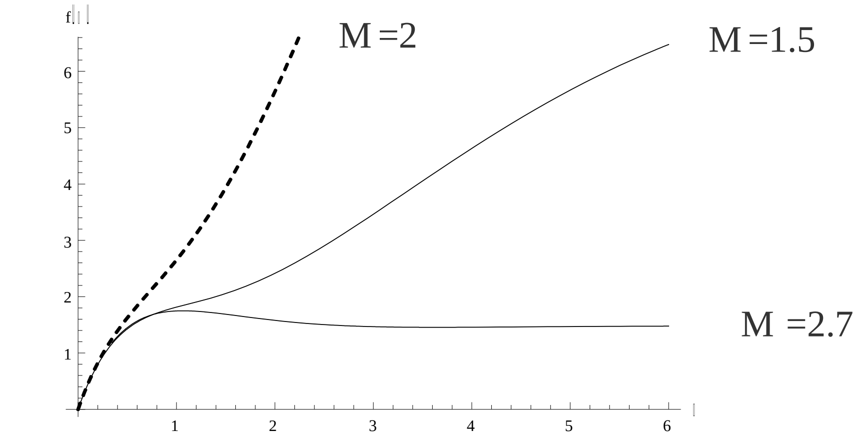

Figure1: Effect of M on velocity profile f ,M= 1.5,2,2.7 and β=0.3,h=-1.5

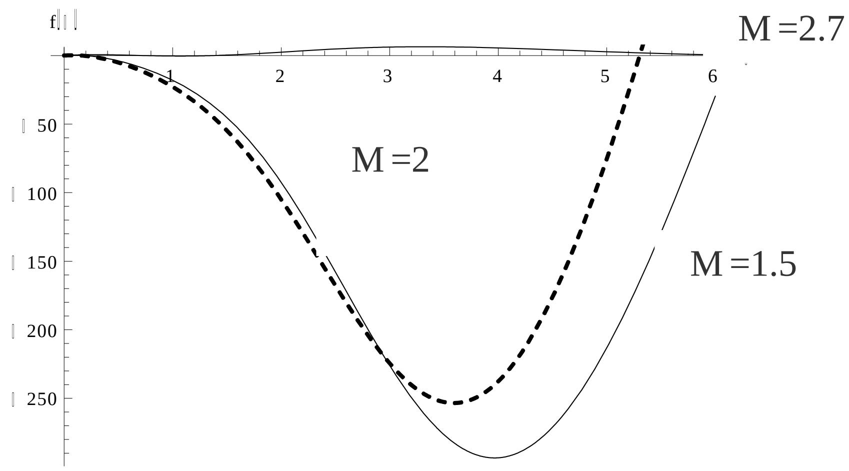

Figure2. Effect M on velocity profile f ,M= 1.5,2,2.7 and β=1.5, h=-1.5

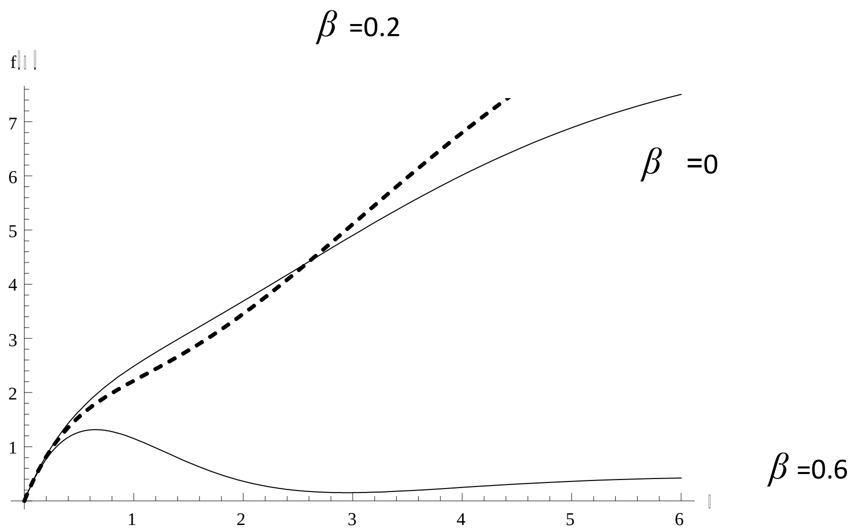

Figure 3. Effect β on velocity profile f , β= 0,0.2,0.6 and M=1.7, h=-1.5

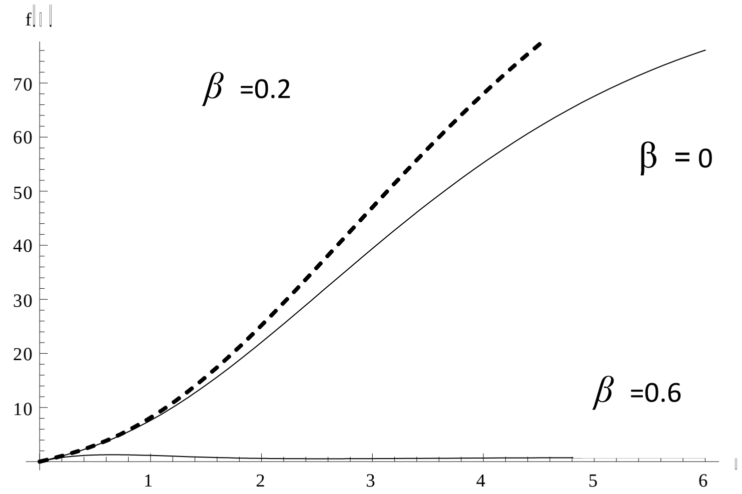

Figure4.Effect β on velocity profile f , β= 0,0.2,0.6 and M=3, h=-1.5

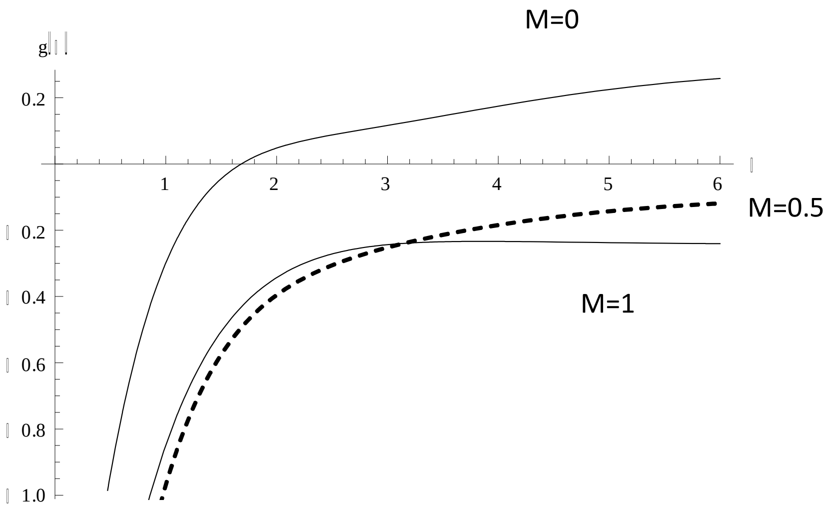

Figure5. Effect M on velocity profile g ,M= 0,0.5,1 and β=0.3,h=-1.4

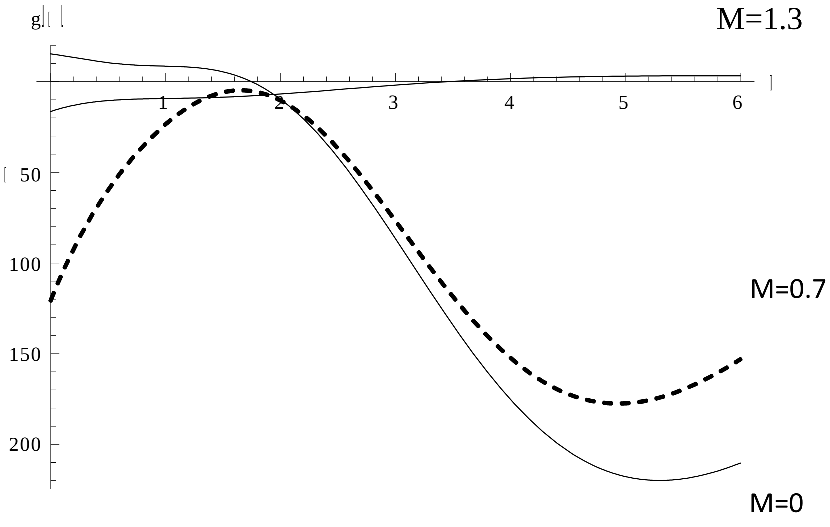

Figure6.Effect M on velocity profile g ,M= 0,0.7,1.3and β=1.4,h=-1.4

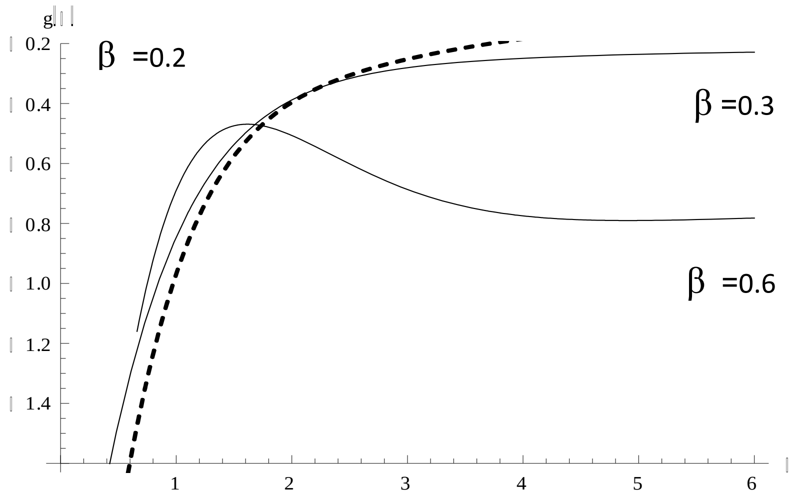

Figure7. Effect M on velocity profile g , β= 0.2,0.3,0.6and M=0.5 ,h=-1.4

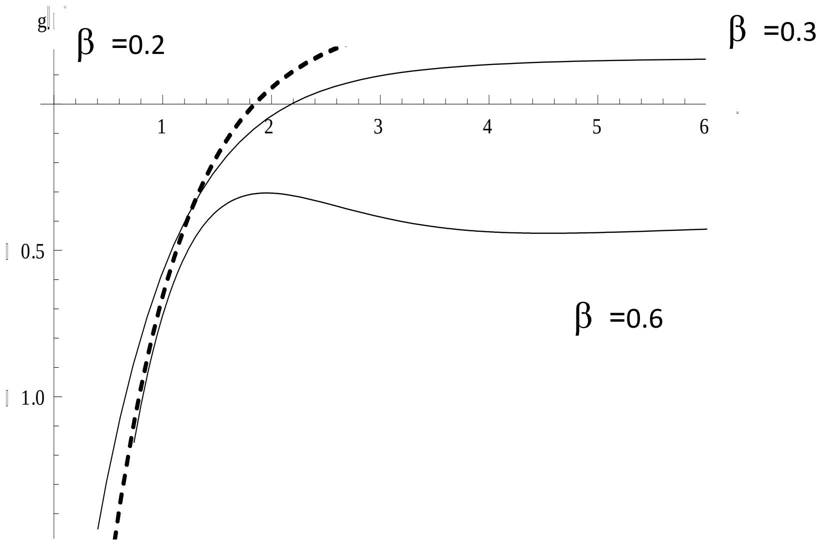

Figure8. Effect M on velocity profile g , β= 0.2,0.3,0.6 and M=1, h=-1.4

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