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On The Class of (K-N) Quasi-Normal Operators On Hilbert Space

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Abstract

In this work, we introduce another class of normal operator which is (K-N) quasi-normal operator and given some properties of this concept as well as discussion the relation between this operator with another types of normal operators.

Keyword: (K-N) quasi-normal

1- Introduction and Terminologies

Let H be complex Hilbert space, and $B(H)$ the space of all bounded linear operator from H in to H, the quasi-normal operator was introduced at first by A. Brown [2] in 1953 and given some properties of this operator, but this concept was generalized by researchers such as D. Senthhilkumar and others introduced new types of quasi-normal operator is said to be K-quasi-normal operator with relationships between these types of operators. But in 2011 O. Ahmed [5], given another class of quasi-normal operator, which is n-power quasi-normal operator.

In this search, we introduce another generalize of quasi-normal operator which (K-N)quasi-normal operator and modification the result appear in [1] and [3] about this concept. As well as, given some basic properties of this operator with relation between (K-N) quasi-normal operator and another classes of quasi normal operators.

2- Basic Concepts

Here, we recall fundamental concepts of this work and in first we give the definition of normal operator.

Definitions (2.2),[4]:

(1) An operator $T : H \to H$ is said to be normal operator if and only if $TT^* = T^*T$.

(2) An operator $T : H \to H$ is said to be quasi normal operator if T and T^*T are commute.

Next, we recall the generalized of normal operator by the following definition.

Definition (2.4), [4]:

An operator $T : H \to H$ is said to be n-power quasi normal operator if and only if $T^nT^*T = T^*TT^n$.

Also, we give the definition of (K-N) quasi-normal operator, this definition is generalized to definition appear in [4].

Definition (2.5):

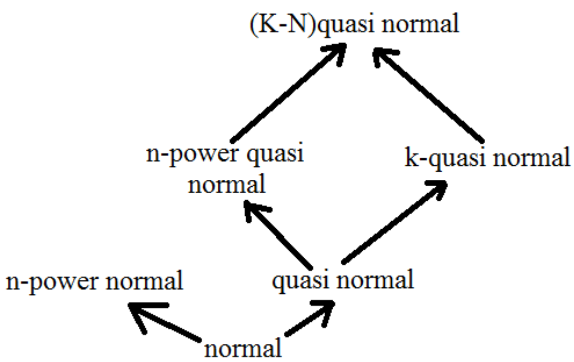
Let T be a bounded operator from a complex Hilbert space H to it self, then T is said to be (K-N) quasi normal operator if satisfy the condition $T^K(T^*T) = N(T^*T)T^K$, where K is positive integer and N is bounded operator from a complex Hilbert space H to it self.

Next, can be introduce the relation between (K-N) quasi normal operator and other classes by the following remark.

Remarks (2.6):

1- Its clearly that if K=1, we get T is (N) quasi normal operator, and if N=I, we get T is n-power- quasi normal, and we get T is quasi normal if N=I and K=1.
2- Every (N) quasi normal is (K-N) quasi normal

To illustrate this remarks, we will introduce the following digram.



3- Properties of (K-N) quasi normal operator

The following theorem give some properties of (K-N) quasi-normal operator.

Theorem (3.1):

Let $T \in B(H)$ is an operator if C is commutes with U and V, and $C^2T^K = NC^2T^K$ then T is (K-N) quasi normal.

Where, $B^2 = TT^*, C^2 = T^*T$, $U = \operatorname{Re} T = \dfrac{T+T^*}{2}$ and $V = \operatorname{Im} T = \dfrac{T-T^*}{2i}$

Proof:

Since $CU = UC$, $BV = VB$ so, $C^2U = UC^2$, $B^2V = VB^2$ thus $C^2U^K = U^KC^2$, $B^2V^K = V^KB^2$ then

$$C^2T^K + C^2(T^K)^* = T^KC^2 + (T^K)^*C^2$$

$$C^2T^K - C^2(T^K)^* = T^KC^2 - (T^K)^*C^2$$

This gives $T^KC^2 = C^2T^K$

$T^K(T^*T)=(T^*T)T^K$, and by using the condition $B^2T^K = NB^2T^K$ so we get: $T^K(T^*T)=N(T^*T)T^K$ then, T is (K-N) quasi normal.

more properties give by the following theorem.

Theorem(3.2):

If $T \in B(H)$ is an operator such that $C^2U^K = \frac{1}{N}U^K C^2$, $C^2V^K = \frac{1}{N}V^K C^2$ then T is (K-N) quasi normal.

Proof:

Since $C^2U^K = \frac{1}{N}U^K C^2$, $C^2V^K = \frac{1}{N}V^K C^2$ then we have

$C^2(U+iV)^K = \frac{1}{N}(U+iV)^K C^2$ and we have $C^2T^K = \frac{1}{N}T^K C^2$ therefore;

$(T^*T)T^K = \frac{1}{N}T^K(T^*T)$ so, $T^K(T^*T)=N(T^*T)T^K$ then, T is (K-N) quasi normal.

The operation on (K-N) quasi normal have been given by the following theorem.

Theorem(3.3):

Let T_1, T_2 be two (K-N) quasi normal from H to H, such that $T_1^K T_2^* = T_2^K T_1^* = T_1^* T_2 = T_2^* T_1 = 0$ then $T_1 + T_2$ is (K-N) quasi normal.

Proof:

$$
\begin{aligned}
(T_1+T_2)^K[(T_1+T_2)^*(T_1+T_2)] &= (T_1+T_2)^K[(T_1^*+T_2^*)(T_1+T_2)] \\
&= (T_1+T_2)^K(T_1^*T_1 + T_1^*T_2 + T_2^*T_1 + T_2^*T_2) \\
&= (T_1+T_2)^K(T_1^*T_1 + T_2^*T_2) \\
&= (T_1^K + T_2^K)(T_1^*T_1 + T_2^*T_2) \\
&= T_1^K T_1^* T_1 + T_1^K T_2^* T_2 + T_2^K T_1^* T_1 + T_2^K T_2^* T_2 \\
&= T_1^K T_1^* T_1 + T_2^K T_2^* T_2 \\
&= N\left((T_1T_1^*)T_1^K\right) + N\left((T_2T_2^*)T_2^K\right)
\end{aligned}
$$

Hence $T_1 + T_2$ is (K-N) quasi normal.

From above theorem, we can get the corollary its proof easy can be omitted it.

Corollary (3.4):

Let T_1, T_2 be two (K-N) quasi normal, such that $T_1^K T_2^* = T_2^K T_1^* = T_1^* T_2 = T_2^* T_1 = 0$ then $T_1 - T_2$ is (K-N) quasi normal.

Theorem(3.5):

Let T_1 be (K-N) quasi normal operator and T_2 (K-power) quasi normal operator. Then there product $T_1 T_2$ is (K-N) quasi normal operator if the following conditions are satisfied

(i) $T_1 T_2 = T_2 T_1$

(ii) $T_1 T_2^* = T_2^* T_1$

Proof:

$$
\begin{aligned}
(T_1T_2)^K (T_1T_2)^* (T_1T_2) &= (T_1^K T_2^K)(T_2^* T_1^*)(T_1T_2) \\
&= (T_1^K T_2^K)(T_1^* T_2^*)(T_1T_2) \\
&= T_1^K (T_2^K T_1^*)(T_2^* T_1)T_2 \\
&= T_1^K (T_1^* T_2^K)(T_1 T_2^*)T_2 \\
&= T_1^K T_1^* (T_2^K T_1) T_2^* T_2 \\
&= (T_1^K T_1^* T_1)(T_2^K T_2^* T_2) \\
&= N\big((T_1^* T_1)T_1^K\big)(T_2^* T_2)T_2^K \\
&= N\big(T_1^* T_1 (T_1^K T_2^*) T_2 T_2^K\big) \\
&= N(T_1^* T_1 T_2^* T_1^K T_2 T_2^K) \\
&= N[T_1^* (T_1 T_2^*)(T_1^K T_2) T_2^K] \\
&= N[(T_1^* T_2^*)(T_1 T_2)(T_1^K T_2^K)] \\
&= N[(T_2 T_1)^* (T_1 T_2)(T_1 T_2)^K] \\
&= N[(T_1 T_2)^* (T_1 T_2)(T_1 T_2)^K]
\end{aligned}
$$

Hence, the product $T_1 T_2$ is (K-N) quasi normal operator.

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