

# On The Class of (K-N) Quasi-Normal Operators On Hilbert Space

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## Abstract

In this work, we introduce another class of normal operator which is (K-N) quasi-normal operator and given some properties of this concept as well as discussion the relation between this operator with another types of normal operators.

**Keyword:** (K-N) quasi-normal

## 1- Introduction and Terminologies

Let  $H$  be complex Hilbert space, and  $B(H)$  the space of all bounded linear operator from  $H$  in to  $H$ , the quasi-normal operator was introduced at first by A. Brown [2] in 1953 and given some properties of this operator, but this concept was generalized by researchers such as D. Senthilkumar and others introduced new types of quasi-normal operator is said to be K-quasi-normal operator with relationships between these types of operators. But in 2011 O. Ahmed [5], given another class of quasi-normal operator, which is n-power quasi-normal operator.

In this search, we introduce another generalize of quasi-normal operator which (K-N)quasi-normal operator and modification the result appear in [1] and [3] about this concept. As well as, given some basic properties of this operator with relation between (K-N) quasi-normal operator and another classes of quasi normal operators.

## 2- Basic Concepts

Here, we recall fundamental concepts of this work and in first we give the definition of normal operator.

### Definitions (2.2),[4]:

- (1) An operator  $T : H \rightarrow H$  is said to be normal operator if and only if  $TT^* = T^*T$ .
- (2) An operator  $T : H \rightarrow H$  is said to be quasi normal operator if  $T$  and  $T^*T$  are commute.

Next, we recall the generalized of normal operator by the following definition.

### Definition (2.4), [4]:

An operator  $T : H \rightarrow H$  is said to be n-power quasi normal operator if and only if  $T^n T^* T = T^* T T^n$ .

Also, we give the definition of (K-N) quasi-normal operator, this definition is generalized to definition appear in [4].

**Definition (2.5):**

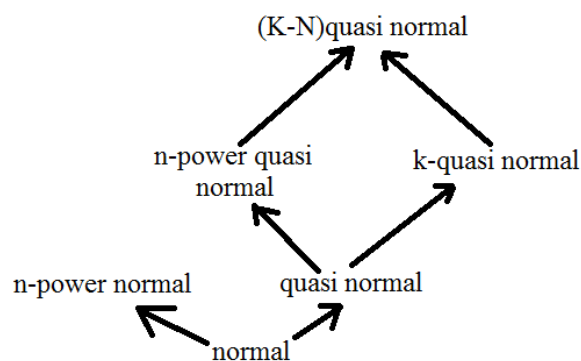
Let  $T$  be a bounded operator from a complex Hilbert space  $H$  to it self, then  $T$  is said to be (K-N) quasi normal operator if satisfy the condition  $T^K (T^*T) = N(T^*T)T^K$ , where K is positive integer and N is bounded operator from a complex Hilbert space  $H$  to it self.

Next, can be introduce the relation between (K-N) quasi normal operator and other classes by the following remark.

**Remarks (2.6):**

- 1- Its clearly that if  $K=1$ , we get  $T$  is (N) quasi normal operator, and if  $N=I$ , we get  $T$  is n-power- quasi normal, and we get  $T$  is quasi normal if  $N=I$  and  $K=1$ .
- 2- Every (N) quasi normal is (K-N) quasi normal

To illustrate this remarks, we will introduce the following digram.



**3- Properties of (K-N) quasi normal operator**

The following theorem give some properties of (K-N) quasi-normal operator.

**Theorem (3.1):**

Let  $T \in B(H)$  is an operator if  $C$  is commutes with  $U$  and  $V$ , and  $C^2T^K = NC^2T^K$  then  $T$  is (K-N) quasi normal.

Where,  $B^2 = TT^*$ ,  $C^2 = T^*T$ ,  $U = \text{Re}T = \frac{T + T^*}{2}$  and  $V = \text{Im}T = \frac{T - T^*}{2i}$

**Proof:**

Since  $CU = UC$ ,  $BV = VB$  so,  $C^2U = UC^2$ ,  $B^2V = VB^2$  thus  $C^2U^K = U^KC^2$ ,  $B^2V^K = V^KB^2$  then

$$C^2T^K + C^2(T^K)^* = T^KC^2 + (T^K)^*C^2$$

$$C^2T^K - C^2(T^K)^* = T^KC^2 - (T^K)^*C^2$$

This gives  $T^KC^2 = C^2T^K$

$T^K (T^*T) = (T^*T)T^K$ , and by using the condition  $B^2T^K = NB^2T^K$  so we get:  
 $T^K (T^*T) = N(T^*T)T^K$  then,  $T$  is (K-N) quasi normal.

more properties give by the following theorem.

**Theorem(3.2):**

If  $T \in B(H)$  is an operator such that  $C^2U^K = \frac{1}{N}U^K C^2$ ,  $C^2V^K = \frac{1}{N}V^K C^2$  then  $T$  is (K-N) quasi normal.

**Proof:**

Since  $C^2U^K = \frac{1}{N}U^K C^2$ ,  $C^2V^K = \frac{1}{N}V^K C^2$  then we have

$$C^2(U + iV)^K = \frac{1}{N}(U + iV)^K C^2 \text{ and we have } C^2T^K = \frac{1}{N}T^K C^2 \text{ therefore;}$$

$$(T^*T)T^K = \frac{1}{N}T^K (T^*T) \text{ so, } T^K (T^*T) = N(T^*T)T^K \text{ then, } T \text{ is (K-N) quasi normal.}$$

The operation on (K-N) quasi normal have been given by the following theorem.

**Theorem(3.3):**

Let  $T_1, T_2$  be two (K-N) quasi normal from  $H$  to  $H$ , such that  $T_1^K T_2^* = T_2^K T_1^* = T_1^* T_2 = T_2^* T_1 = 0$  then  $T_1 + T_2$  is (K-N) quasi normal.

**Proof:**

$$\begin{aligned} (T_1 + T_2)^K [(T_1 + T_2)^* (T_1 + T_2)] &= (T_1 + T_2)^K [(T_1^* + T_2^*)(T_1 + T_2)] \\ &= (T_1 + T_2)^K (T_1^*T_1 + T_1^*T_2 + T_2^*T_1 + T_2^*T_2) \\ &= (T_1 + T_2)^K (T_1^*T_1 + T_2^*T_2) \\ &= (T_1^K + T_2^K)(T_1^*T_1 + T_2^*T_2) \\ &= T_1^K T_1^* T_1 + T_1^K T_2^* T_2 + T_2^K T_1^* T_1 + T_2^K T_2^* T_2 \\ &= T_1^K T_1^* T_1 + T_2^K T_2^* T_2 \\ &= N((T_1 T_1^*) T_1^K) + N((T_2 T_2^*) T_2^K) \end{aligned}$$

Hence  $T_1 + T_2$  is (K-N) quasi normal.

From above theorem, we can get the corollary its proof easy can be omitted it.

**Corollary (3.4):**

Let  $T_1, T_2$  be two (K-N) quasi normal, such that  $T_1^K T_2^* = T_2^K T_1^* = T_1^* T_2 = T_2^* T_1 = 0$  then  $T_1 - T_2$  is (K-N) quasi normal.

**Theorem(3.5):**

Let  $T_1$  be (K-N) quasi normal operator and  $T_2$  (K-power) quasi normal operator. Then there product  $T_1 T_2$  is (K-N) quasi normal operator if the following conditions are satisfied

- (i)  $T_1 T_2 = T_2 T_1$
- (ii)  $T_1 T_2^* = T_2^* T_1$

**Proof:**

$$\begin{aligned}
 (T_1 T_2)^K (T_1 T_2)^* (T_1 T_2) &= (T_1^K T_2^K)(T_2^* T_1^*)(T_1 T_2) \\
 &= (T_1^K T_2^K)(T_1^* T_2^*)(T_1 T_2) \\
 &= T_1^K (T_2^K T_1^*)(T_2^* T_1) T_2 \\
 &= T_1^K (T_1^* T_2^K)(T_1 T_2^*) T_2 \\
 &= T_1^K T_1^* (T_2^K T_1) T_2^* T_2 \\
 &= (T_1^K T_1^* T_1)(T_2^K T_2^* T_2) \\
 &= N((T_1^* T_1) T_1^K) (T_2^* T_2) T_2^K \\
 &= N(T_1^* T_1 (T_1^K T_2^*) T_2 T_2^K) \\
 &= N(T_1^* T_1 T_2^* T_1^K T_2 T_2^K) \\
 &= N[T_1^* (T_1 T_2^*) (T_1^K T_2) T_2^K] \\
 &= N[(T_1^* T_2^*) (T_1 T_2) (T_1^K T_2^K)] \\
 &= N[(T_2 T_1)^* (T_1 T_2) (T_1 T_2)^K] \\
 &= N[(T_1 T_2)^* (T_1 T_2) (T_1 T_2)^K]
 \end{aligned}$$

Hence, the product  $T_1 T_2$  is (K-N) quasi normal operator.

**4- Reference :**

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