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# Common Fixed Point Theorem for $\psi$-weakly commuting maps in L-Fuzzy Metric Spaces for integral type

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**Abstract**
In this paper, we proved a common fixed point theorem ψ-weakly commuting maps in L-Fuzzy Metric Spaces for integral type inequality.
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## 1. Introduction

In 1922, Let (X, d) be a complete metric space, c∈ (0, 1) and $f$: X→X be a mapping such that for each x, y ∈ X, d $(fx, fy) \le$ c d(x, y) Then f has a unique fixed point a ∈ X, such that for each x ∈ X, $\lim_{n\to\infty} f^n x = a$ by S. Banach [23].As a generalization of fuzzy sets introduced by L.A.Zadeh [14], K. Atanassov [13] introduced the idea of intuitionistic fuzzy set. Fixed point and common fixed point properties for mappings defined on fuzzy metric spaces by [5], [6], [8], [15], [16], Intuitionistic fuzzy metric spaces by [7], [21]. A. George and P.Veeramani [2] modified the concept of fuzzy metric space introduced by I. Kramosil and J. Michalek [10] and defined a Hausdorff topology on this fuzzy metric space by [12]. Most of the properties which provide the existence of fixed points and common fixed points are of linear contractive type conditions. L-fuzzy metric spaces have been studied by many authors [11], [24]. H. Adibi et al.[9] introduced the concept of compatible mappings and proved common fixed point theorems for four mappings satisfying some conditions in L-fuzzy metric spaces. In the sequel, we shall adopt the usual terminology, notation and conventions of L-fuzzy metric spaces introduced by R. Saadati et al. [19] which are a generalization of fuzzy metric spaces and intuitionistic fuzzy metric spaces [20]. R. Saadati, S.Sedghi and H. Zhou [22] by a common fixed point theorem $\psi$-weakly commuting maps in L-Fuzzy Metric Spaces

## 2. Preliminaries

**Definition 2.1 [1]:** Let (X, d) be a complete metric space, c∈ (0, 1) and f: X→X be a mapping such that for each x, y ∈ X,

$$\int_0^{d(fx,fy)} \varphi(t)dt \le c \int_0^{d(x,y)} \varphi(t)dt$$

where $\varphi$: [0,+∞) →[0,+∞) is a Lebesgue integrable mapping which is summable on each compact subset of [0,+∞) , non negative, and such that for each $\varepsilon$ > o, $\int_0^{\varepsilon} \varphi(t)dt,$ then f has a unique fixed point $a \in X$ such that for each $x \in X, \lim_{n\to\infty} f^n x = a.$

B.E.Rhoades [4], extending the result of Branciari by replacing the above condition by the following

$$\int_0^{d(fx,fy)} \varphi(t)dt \le c \int_0^{max\left\{d(x,y),d(x,fx),d(y,fy),\frac{d(x,fy)+d(y,fx)}{2}\right\}} \varphi(t)dt.$$

**Definition 2.2**[11] Let L = (L, $\le_L$) be a complete lattice, and U a nonempty set called a universe.

An L-fuzzy set A on U is defined as a mapping A : U → L. For each u in U, A(u) represents the degree (in L) to which u satisfies A.

**Lemma 2.1[8].** Consider the set $L^*$ and the operation $\le_{L^*}$ defined by:

 

$L^* = \{(x_1, x_2) : (x_1, x_2) \in [0,1]^2$ and $x_1 + x_2 \leq 1\}$, $(x_1, x_2) \leq_{L^*} (y_1, y_2) \leftrightarrow x_1 \leq y_1$ and $x_2 \geq y_2$, for every
$(x_1, x_2), (y_1, y_2) \in L^*$. Then $(L^*, \leq_{L^*})$ is a complete lattice.
Classically, a triangular norm T on $([0, 1], \leq)$ is defined as an increasing, commutative, associative mapping T: $[0,1]^2 \to [0,1]$ satisfying T(1, x) = x, for all x $\in$ [0, 1]. These definitions can be straightforwardly extended to any lattice L = (L, $\leq_L$) Define first $0_L$ = inf L and $1_L$ = sup L.

**Definition 2.3[19].** A triangular norm (t-norm) on L is a mapping T: $L^2 \to L$ satisfying the following conditions:
(i) $(\forall x \in L)(T(x, 1_L) = x)$; (boundary condition)
(ii) $(\forall (x, y) \in L^2)(T(x, y) = T(y, x))$; (commutativity)
(iii) $(\forall (x, y, z) \in L^3)(T(x, T(y, z)) = T(T(x, y), z))$; (associativity)
(iv) $(\forall (x, x', y, y') \in L^4)(x \leq_L x'$_ and $y \leq_L y' \Rightarrow T(x, y) \leq_L T(x', y'))$;(monotonicity)
A t-norm $\mathcal{T}$ on L is said to be continuous if for any x, y $\in$ X and any sequences$\{x_n\}$ & $\{y_n\}$ which converge to x and y we have

$$\lim_{n\to\infty} \mathcal{T}(x_n, y_n) = \mathcal{T}(x, y)$$

**Definition 2.4 [7].**A t-norm $\mathcal{T}$ on $L^*$ is called t-represent able if and only if there exist a t-norm T and a t-co norm S on [0, 1] such that, for all $(x_1, x_2), (y_1, y_2) \in L^*$. $\mathcal{T}(x, y) = (T(x_1, y_1), S(x_2, y_2))$.

**Definition 2.5[19].** A negation on L is any decreasing mapping N: L $\to$ L satisfying N $(0_L) = 1_L$ and N $(1_L) = 0_L$. If N (N(x)) = x, for all x $\in$ L, then N is called an involutive negation.

**Definition 2.6[19].** The 3-tuple (X, M, $\mathcal{T}$) is said to be an L-fuzzy metric space if X is an arbitrary (non-empty) set, $\mathcal{T}$ is a continuous t–norm on L and M is an L-fuzzy set on $X^2 \times (0, \infty)$satisfying the following conditions for every x, y, z in X and t, s in $(0, \infty)$:
(a) M(x, y, t) $>_L 0_L$;
(b) M(x, y, t) = $1_L$ for all t > 0 if and only if x = y;
(c) M(x, y, t) = M(y, x, t);
(d) $\mathcal{T}$(M(x, y, t), M(y, z, s)) $\leq_L$ M(x, z, t + s);
(e) M(x, y, ·) : $(0, \infty) \to$ L is continuous.
Let (X, M, $\mathcal{T}$) be an L-fuzzy metric space. For t $\in (0, \infty)$, we define the open ball B(x, r, t) with center x $\in$ X and radius r $\in L \setminus \{0_L, 1_L\}$, as B(x, r, t) = {y $\in$ X :M(x, y, t) $>_L$ N(r)}. A subset A $\subseteq$ X is called open if for each x $\in$ A, there exist t > 0 and r $\in L \setminus \{0_L, 1_L\}$ such that B(x, r, t) $\subseteq$ A. Let $\mathcal{T}_M$ denote the family of all open subsets of X. Then $\mathcal{T}_M$ is called the topology induced by the L-fuzzy metric M.

**Example 2.1 [21].** Let (X, d) be a metric space. Denote $\mathcal{T}(a, b) = (a_1 b_1, \min(a_1 + b_2, 1))$ for all
$a = (a_1, a_2)$ and $b = (b_1, b_2)$ in $L^*$ and let M and N be fuzzy sets on $X^2 \times (0,\infty)$ be defined as follows:
$M_{M,N}(x, y, t) = (M(x, y, t), N(x, y, t)) = \left(\frac{t}{t+md(x,y)}, \frac{d(x,y)}{t+d(x,y)}\right)$, in which $m > 1$.Then (X, $M_{M,N}$,$\mathcal{T}$) is an intuitionistic fuzzy metric space.

**Example 2.2 [19].** Let X = N. Define $\mathcal{T}(a, b) = (\max(0, a_1 + b_1 - 1), a_2 + b_2 - a_2 b_2)$ for all $a = (a_1, a_2)$ and $b = (b_1, b_2)$ in $L^*$, and let M *(x, y, t)* on $X^2 \times (0,\infty)$ be defined as follows:

$$M(x, y, t) = \begin{cases} \left(\frac{x}{y}, \frac{y-x}{y}\right) & \text{if } x \leq y \\ \left(\frac{x}{y}, \frac{x-y}{x}\right) & \text{if } y \leq x \end{cases}$$

for all x, y $\in$ X and t $>$ 0. Then (X, M, $\mathcal{T}$) is an L-fuzzy metric space.
Let (X, M, $\mathcal{T}$) be an L-fuzzy metric space. For t$\in (0, \infty)$, we define the open ball B(x, r, t) with center x $\in$ X and radius r $\in L \setminus \{0_L, 1_L\}$, as B(x, r, t) = {y $\in$ X :M(x, y, t) $>_L$ N(r)}. A subset A $\subseteq$ X is called open if for

each x ∈ A, there exist t > 0 and r ∈ L \ $\{0_L, 1_L\}$ such that B(x, r, t) ⊆ A. Let $\mathcal{T}_M$ denote the family of all open subsets of X. Then $\mathcal{T}_M$ is called the topology induced by the L-fuzzy metric M.

**Lemma 2.2 [9].** Let (X, M, $\mathcal{T}$) be an L-fuzzy metric space. Then M(x, y, t) is non decreasing with respect to t, for all x, y in X.

**Definition 2.7[19]**. A sequence $\{x_n\}_{n\epsilon N}$in an L-fuzzy metric space (X, M, $\mathcal{T}$) is called a Cauchy sequence, if for each ε ∈ L \ $\{0_L\}$ and t > 0, there exists $n_0$ ∈ N such that for all m ≥ n ≥ $n_0$ (n ≥ m≥ $n_0$),

M ($x_m$, $x_n$, t) $>_L$ N(ε). The sequence $\{x_n\}_{n\epsilon N}$ is said to be convergent to x ∈ X in the L-fuzzy metric space (X, M, $\mathcal{T}$) if $M(x_n, x, t) = M(x, x_n, t) = 1_L$ whenever $n \to \infty$ for every t > 0. A L-fuzzy metric space is said to be complete if and only if every Cauchy sequence is convergent.

**Definition 2.8 [22]** Let (X, M,$\mathcal{T}$) be an L-fuzzy metric space. M is said to be continuous on $X^2 \times(0,\infty)$ if

$$\lim_{n\to\infty} M(x_n, y_n, t_n) = M(x, y, t)$$

whenever a sequence $\{(x_n, y_n, t_n)\}$ in $X^2 \times(0,\infty)$ converges to a point (x, y, t) ∈ $X^2 \times(0,\infty)$ i.e.,

$$\lim_{n\to\infty} M(x_n, x, t) = lsim_{n\to\infty} M(y_n, y, t) = 1_L \;\&\; \lim_{n\to\infty} M(x, y, t_n) = M(x, y, t).$$

**Lemma 2.3 [22]** Let (X, M, $\mathcal{T}$) be an L-fuzzy metric space. Then M is continuous function on $X^2 \times(0,\infty)$.

**Definition 2.9[22]** Let $A$ and $B$ be maps from an L-fuzzy metric space (X, M, $\mathcal{T}$) into itself. The maps $f$ and $g$ are said to be weakly commuting if $M(ABx, BAx, t) \geq_L M(Ax, Bx, t)$ for each $x$ in X & $t > 0$.

**Definition 2.10[22].**Let $A$ and $B$ be maps from an L-fuzzy metric space (X, M, $\mathcal{T}$) into itself. The maps $A$ and $B$ are said to be $\psi$-weakly commuting if there exists a positive real function $\psi: (0,\infty) \to (0,\infty)$ such that $M(ABx, BAx, t) \geq_L M(Ax, Bx, \psi(t))$ for each $x$ in X and $t > 0$.

**Example 2.3[22]**. Let X = R. Let $\mathcal{T}(a, b) = (a_1 b_1, \min(a_1 + b_2, 1))$ for all $a = (a_1, a_2)$ and $b = (b_1, b_2)$ in $L^*$ and let M and N be fuzzy sets on $X^2 \times(0,\infty)$ be defined as follows:

$$M_{M,N}(x, y, t) = \left( (e^{\frac{|x-y|}{t}})^{-1}, \frac{\left(e^{\frac{|x-y|}{t}}\right)-1}{(e^{\frac{|x-y|}{t}})} \right),$$

for all $t > 0$. Then (X, $M_{M,N}$,$\mathcal{T}$) is an intuitionistic fuzzy metric space. Define $A(x) = 2x - 1, B(x) = x^2, then$

$$M_{M,N}(x, y, t) = \left( (e^{\frac{|x-y|}{t}})^{-1}, \frac{\left(e^{\frac{|x-y|}{t}}\right)-1}{(e^{\frac{|x-y|}{t}})} \right)$$

$$= \left( (e^{\frac{2|x-y|^2}{t}})^{-1}, \frac{\left(e^{\frac{2|x-y|^2}{t}}\right)-1}{(e^{\frac{2|x-y|^2}{t}})} \right)$$

$$= \left( (e^{\frac{2|x-y|^2}{t/2}})^{-1}, \frac{\left(e^{\frac{2|x-y|^2}{t/2}}\right)-1}{(e^{\frac{2|x-y|^2}{t/2}})} \right) = M_{M,N}(Ax, Bx, t/2)$$

$$<_{L^*}\left((e^{\frac{|x-y|^2}{t}})^{-1}, \frac{\left(e^{\frac{|x-y|^2}{t}}\right)-1}{(e^{\frac{|x-y|^2}{t}})}\right) = M_{M,N}(Ax, Bx, t)$$

. Therefore, for $\psi(t) = {}^t/_2$, $A\ and\ B\ are\ \psi$ weakly commuting. But $A\ and\ B\ are$ not weakly commuting since the exponential function is strictly increasing.

**3. Main Results**

**Theorem 3.1.** Let (X, M, $\mathcal{T}$) be a left L-fuzzy metric space and let $A$ and $B$ be $\psi$ weakly commuting self-mappings of X satisfying the following conditions:

(3.1.1) $A(X) \subseteq B(X)$;

(3.1.2) either $A$ or $B$ is continuous;

(3.1.3)

$$\int_0^{M\,(Ax,Ay,t)} \xi(t)dt \geq_L \int_0^{C\{M(Bx,By,t),M(Bx,Ax,t),M(Ax,By,t)\}} \xi(t)dt$$

where $C: L \to L$ is a continuous function such that $C(a) >_L a$ for each $a \in L \setminus \{0_L, 1_L\}$, for every x, y in X .Then $A$ and $B$ have a unique common fixed point in X.

Proof. Let $x_0 \in X$ be an arbitrary point in X. By (3.1.1), there exists $x_1 \in X$ such that $Ax_0 = Bx_1$. In general choose $x_{n+1}$ such that $Ax_n = Bx_{n+1}$. Then for t > 0,

$$\int_0^{M(Ax_n,Ax_{n+1},t)} \xi(t)dt \geq_L \int_0^{C\{M(Bx_n\,,Bx_{n+1}\,,t),M(Bx_n\,,Ax_n\,,t),M(Ax_n\,,Bx_{n+1}\,,t)\}} \xi(t)dt$$

$$\geq_L \int_0^{C\{M(Ax_{n-1}\,,Ax_n\,,t),M(Ax_{n-1}\,,Ax_n\,,t),M(Ax_n\,,Ax_n\,,t)\}} \xi(t)dt$$

$$= \int_0^{M(Ax_{n-1}\,,Ax_n\,,t)} \xi(t)dt$$

Thus,$\{M(Ax_n, Ax_{n+1}, t); n \geq 0\}$is an increasing sequence in L and therefore, tends to a limit $a \leq_L 1_L$. we claim that $a = 1_L$. For if $a <_L 1_L$, when $n \to \infty$ in the above inequality we get $a \geq_L C(a) >_L a$ a contradiction. Hence $a = 1_L$, i.e.

$\lim_{n\to\infty} M(Ax_n, Ax_{n+1}, t) = 1_L$.

If we define (2.9) $c_n(t) = M(Ax_n, Ax_{n+1}, t)$ then $\lim_{n\to\infty} c_n(t) = 1_L$. Now, we prove that $\{Ax_n\}$ is a Cauchy sequence in $A(X)$. suppose that $\{Ax_n\}$ is not a Cauchy sequence in $A(X)$. For convenience, Let $y_n = Ax_n$ for n =1,2,3......Then there is an $\epsilon \in L \setminus \{0_L, 1_L\}$ such that for each integer k, there exists integers $m(k) and\ n(k)$ with $m(k) > n(k) \geq k$ such that

(2.10) $d_k(t) = M(y_{n(k)}, y_{m(k)}, t) \leq N(\epsilon)$ for k = 1, 2, 3.....

We may assume that

Example.2.3 $M\left(y_{n(k)}, y_{m(k)-1}, t\right) > N(\epsilon)$,

by choosing $m(k)$ to be the smallest number exceeding $n(k)$ for which (2.10) holds. Using (2.9), we have

(3.1)

$$N(\epsilon) \geq d_k(t) \geq \mathcal{T}\left(M\left(y_{n(k)}, y_{m(k)-1}, {}^t/_2\right), M\left(y_{m(k)-1}, y_{m(k)}, {}^t/_2\right)\right) \geq \mathcal{T}\left(c_k\left(\frac{t}{2}\right), N(\epsilon)\right)$$

Hence, $d_k(t) \to N(\epsilon)$ for every $t > 0$ as $k \to \infty$.
We know that

$$\begin{aligned} d_k(t) &= M\left(y_{n(k)}, y_{m(k)}, t\right) \\ &\geq \mathcal{T}^2\{M\left(y_{n(k)}, y_{m(k)+1}, {}^t/_3\right), M\left(y_{n(k)+1}, y_{m(k)+1}, {}^t/_3\right), M\left(y_{n(k)+1}, y_{m(k)+1}, {}^t/_3\right)\} \\ &\geq \mathcal{T}^2\left\{\left(c_k({}^t/_3), C\left(M\left(y_{n(k)}, y_{m(k)+1}, {}^t/_3\right)\right), c_k({}^t/_3)\right)\right\} \\ &= \mathcal{T}^2\left\{\left(c_k({}^t/_3), C\left(d_k({}^t/_3)\right), c_k({}^t/_3)\right)\right\} \end{aligned}$$

Thus, as $k \to \infty$ in the above inequality we have $N(\epsilon) \geq C\big(N(\epsilon)\big) > N(\epsilon)$ which is a contradiction. Thus, $\{Ax_n\}$ is a Cauchy and by the completeness of X, $\{Ax_n\}_n$ converges to z in X. Also $\{Bx_n\}_n$ converges to z in X. Let us suppose that the mapping A is continuous. Then $\lim_{n\to\infty} AAx_n = Az$ and $\lim_{n\to\infty} ABx_n = Az$. Further we have since $A$ and $B$ be $\psi$ weakly commuting

$$M\,(ABx, BAx, t) \;\geq_L\; M\,(Ax, Bx, \psi(t))$$

On letting $n \to \infty$ in the above inequality we get $\lim_{n\to\infty} BAx_n = Az$, by lemma (2.3). We now prove that $z = Az$. Suppose $z \neq Az$ then $M(z, Az, t) <_L 1_L$. By (3.1.3)

$$\int_0^{M(Ax_n,AAx_n,t)} \xi(t)dt \geq_L \int_0^{C\{M(Bx_n\,,BAx_n\,,t),M(Bx_n\,,Ax_n\,,t),M(Ax_n\,,BAx_n\,,t)\}} \xi(t)dt$$

Letting $n \to \infty$ in the above inequality we get

$$\int_0^{M(z,Az,t)} \xi(t)dt \geq_L \int_0^{C\{M(z\,,Az\,,t),M(z\,,Az\,,t),M(z\,,Az\,,t)\}} \xi(t)dt >_L \int_0^{M(z\,,Az\,,t)} \xi(t)dt$$

a contradiction. Therefore, $z = Az$. Since $A(X) \subseteq B(X)$ we can find $z_1$ in X such that $z = Az = Bz_1$. Now,

$$\int_0^{M\,(AAx_n,Az_1,t)} \xi(t)dt \geq_L \int_0^{C\{M(BAx_n,Bz_1,t),M(BAx_n,AAx_n,t),M(AAx_n,Bz_1,t)\}} \xi(t)dt$$

Letting $n \to \infty$ in the above inequality we get

$$\int_0^{M(Az,Az_1,t)} \xi(t)dt \geq_L \int_0^{C\{M(Az\,,Bz_1\,,t),M(Az\,,Az\,,t),M(Az\,,Bz_1\,,t)\}} \xi(t)dt \geq_L \int_0^{C(M(Az\,,Bz_1\,,t))} \xi(t)dt$$

Since $C(1_L) = 1_L$, this implies that $Az = Az_1, i.e.\ z = Az = Az_1 = Bz_1$. also for any $t > 0$, $M(Az, Bz, t) = M(ABz_1, BAz_1, t) \geq_L M\big(Az_1, Bz_1, \psi(t)\big) = 1_L$ which again implies that $Az = Bz$. thus $z$ is a common fixed point of A and B. Now, to prove uniqueness suppose $z \neq \dot{z}$ is another common fixed point of A and B. Then there exists $t > 0$ such that $M(z, \dot{z}, t) <_L 1_L$ and

$$\begin{aligned} \int_0^{M\,(z,\dot{z},t)} \xi(t)dt &= \int_0^{M\,(Az,A\dot{z},t)} \xi(t)dt \\ &\geq_L \int_0^{C\{M(Bz,B\dot{z},t),M(Bz,Az,t),M(Az,B\dot{z},t)\}} \xi(t)dt \\ &= \int_0^{C\{M(z,\dot{z},t)\}} \xi(t)dt >_L \int_0^{M(z,\dot{z},t)} \xi(t)dt \end{aligned}$$

Which is contradiction. Therefore, $z = \dot{z}$. i.e, $z$ is a unique common fixed point A and B.

 

**Example 2.4[22]**.Consider example 2.1 in which $X = [0 ,1]$.

Define $A(x) = 1$ and $B(x) = \begin{cases} 1, & if\ x\ is\ rational \\ 0, & if\ x\ is\ irrational \end{cases}$ on X. It is evident that $A(X) \subseteq B(X)$, A is continuous

and B is discontinuous. Define $C: L^* \to L^*$ by $C(a) = (\sqrt{a_1}, {a_2}^2)$, then $C(a) = (\sqrt{a_1}, {a_2}^2) >_{L^*} (a_1, a_2) = a$ for $0 < a_i < 1, i = 1,2$ and $\mathrm{M}(Ax, Ay, t) \geq_{L^*} \mathrm{C}(\mathrm{M}(Bx, By, t))$ for all $x, y$ in X, $A$ and $B$ be $\psi$ weakly commuting. Thus all the conditions of last theorem are satisfied and 1 is a common fixed point of A and B.

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