

Some Fixed Point and Common Fixed Point Results in L – Space

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Abstract: There are several theorems are prove in L – space, using various type of mappings. In this paper, we prove some fixed point theorem and common fixed point theorem in L- space using different, symmetric rational mappings.

Keywords: Fixed point, Common Fixed point, L – space, Continuous mapping, Self Mapping, Weakly Compatible Mappings.

1. Introduction:

It was shown by Kasahara, S.[2] in 1976, that several known generalization of the Banach Contraction Theorem can be derived easily from a Fixed Point Theorem in an L – space. Iski [1] has used the fundamental idea of Kasahara to investigate the generalization of some known Fixed Point Theorem in L- space.

Let N be the set of natural number and X be a nonempty set. Then L – space is defined to be the pair (X, →) of the set X and a subset → of the set $X^N \times$ X, satisfying the following conditions:

L_1 – if x_n = x ∈ X for all n ∈ N, then $(\{x_n\}_{n\in N,x)})$ ∈ →

L_2 - if $(\{x_n\}_{n\in N,x)})$ ∈ →, then$\{ x_{n_i}\}_{i\in N}$

For every subsequence $\{x_{n_i}\}_{i\in N}$ of $\{x_n\}_{n\in N}$

In what follows instead of writing $(\{x_n\}_{n\in N,x)})$ ∈ →, we write $\{x_n\}_{n\in N}$ → x or x_n → x and read $\{x_n\}_{n\in N}$ converges to x. Further we give some definitions regarding L – space.

Definition1. Let (X, →) be an L- space. It is said to be 'separated' if each sequence in x converges to at most one point of X.

Definition 2. A mapping f on (X, →) into an L- space $(X',\to')$ is said to be continuous' if x_n → x implies f(x_n) →′ f(x) for some subsequence $\{x_{n_i}\}_{i\in N}$ for $\{x_n\}_{n\in N}$.

Definition3. Let d be a non negative extended real valued function on X × X: 0 ≤ d(x, y) ≤ ∞_i for all x, y ∈ X. The L – space is said to be d – complete if each sequence$\{x_n\}_{n\in N}$inX with $\sum_{i=0}^{\infty} d(x_i, x_{i+1}) < \infty$ converges to the atmost one point of X.

In this context Kasahara, S. proved a lemma, which as follows:

Lemma (Kasahara, S.): Let (X, →) be an L – space which is d – complete for a non negative real valued function d on X × X. If (X, →) is separated then:

d(x, y) = d(y, x) = 0 implies, x = y for all x, y ∈ X.

During the past few years many great mathematicians Yeh [7], Singh [6], Pathak and Dubey [4], Sharma and Agrawal [5], Patel, Sahu and Sao [3], worked for L- space. In this paper, we similar investigation for the study of Fixed Point Theorems in L- space are worked out. We find some more Fixed Point Theorem and Common Fixed Point Theorem in L – space.

Theorem 1: Let (X, →) be a separated L – space, which is d – complete for a non negative real valued function d on X × X with d(x, x) = 0, for each x in X. Let E, F, S and T be three continuous self mapping of X into itself, satisfying the following condition:

(1) E(X) ⊂ T (X) and F(X) ⊂ S(X),

(2)ET = TE, FS = FS

(3)$d(Ex,Fy) \le \alpha\left[\frac{[d(Sx,Ty)\{d(Sx,Ex) + d(Ty,Fy)\}]}{d(Sx,Fy) + d(Ty,Ex)}\right]$

$+\beta[d(Sx,Ex)+d(Ty,Fy)]+\gamma[d(Sx,Fy)$

$+d(Ty, Ex)] + \delta\, d(Sx, Ty)$

for all x, y in X where non negative α, β, γ, δ such that $0 \le \alpha + \beta + \gamma + \delta < 1$. With Sx = Ty. Then E, F, S, T have unique common fixed point.

Proof: Let $x_0 \in X$, since E(X) ⊂ T(X) we can choose a point $x_i \in X$, such that $S_{x_1} = E_{x_0} = y_0$ also F(X) ⊂ S(X), we can choose $x_2 \in X$ such that $T_{x_2} = F_{x_1} = y_1$. In general we can choose the point:

$S_{x_{2n+1}} = E_{x_{2n}} = y_{2n}$, $T_{x_{2n+2}} = F_{x_{2n+1}} = y_{2n+1}$.

Now consider,

$d(S_{x_{2n+1}}, T_{x_{2n+2}}) = d(E_{x_{2n}}, F_{x_{2n+1}}) \le \alpha\left[\frac{d(Sx_{2n},Tx_{2n+1})\{d(Sx_{2n},Ex_{2n})+d(Tx_{2n+1},Fx_{2n+1})\}}{d(Sx_{2n},Fx_{2n+1})+ d(Tx_{2n+1},Ex_{2n})}\right]$

$+ \beta[d(Sx_{2n}, Ex_{2n}) + d(Tx_{2n+1}, Fx_{2n+1})]$

$+\gamma\, [d(Sx_{2n}, Fx_{2n+1}) + d(Tx_{2n+1}, Ex_{2n})]$

$+ \delta\, d(Sx_{2n}, Tx_{2n+1})$

$d(y_{2n},y_{2n+1}) \le \alpha\left[\frac{[d(y_{2n-1},y_{2n})\{d(y_{2n-1},y_{2n}) + d(y_{2n},y_{2n+1})\}]}{d(y_{2n-1},y_{2n+1}) + d(y_{2n},y_{2n})}\right]$

$+\beta[d(y_{2n-1}, y_{2n})+d(y_{2n},y_{2n+1})]$ $+\gamma\, [d(y_{2n-1}, y_{2n+1}) + d(y_{2n}, y_{2n})]$ $+ \delta\, d(y_{2n-1}, y_{2n})$

$d(y_{2n}, y_{2n+1}) \le \alpha\, d(y_{2n-1}, y_{2n})$ $+ \beta\, d(y_{2n-1}, y_{2n+1})$ $+ \gamma\, d(y_{2n-1}, y_{2n+1})$

$+ \delta\, d(y_{2n-1}, y_{2n})$

$d(y_{2n}, y_{2n+1}) \le$ $\alpha\, d(y_{2n-1}, y_{2n})$ $+ \beta[\, d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n+1})]$

$+ \gamma\, [d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n+1})]$

$+ \delta\, d(y_{2n-1}, y_{2n})$

$(1 - \beta - \gamma)\, d(y_{2n}, y_{2n+1})$

$\le (\alpha + \beta + \gamma + \delta)\, d(y_{2n-1}, y_{2n})$

$d(y_{2n}, y_{2n+1}) \le \frac{(\alpha + \beta + \gamma + \delta)}{(1 - \beta - \gamma)}\, d(y_{2n-1}, y_{2n})$

$d(y_{2n}, y_{2n+1}) \le q\, d(y_{2n-1}, y_{2n})$

where $q = \frac{(\alpha + \beta + \gamma + \delta)}{(1 - \beta - \gamma)} < 1$

for n = 1, 2, 3, . . .

Whether $d(y_{2n}, y_{2n+1}) = 0$ or not

Similarly, we have

$d(y_{2n}, y_{2n+1}) \le q^n\, d(y_0, y_1)$,

for every positive integer, this means that,

$\sum_{i=0}^{\infty} d\,(Sx_{2i+1}, T\, y_{2i+2}) < \infty$.

Thus the d – completeness of the space implies that, the sequence $(S^n x_0)_{n \in N}$) and $(T^n x_0)_{n \in N}$ converges to some u.

$(E^n x_0)_{n \in N}$) and $(F^n x_0)_{n \in N}$ converges to some point u, respectively. Since E, F, S and T are continuous, there is a subsequence t of $(T^n x_0)_{n \in N}$ such that:

E[S(t) → E(u), S[E(t)] → S(u), F[T(t)] → F(u), T[F(t)] → T(u)

we have, E(u) = F(u) = S(u) = T(u)

Thus, we can write

S (Tu) = T(Su) = E(Tu) = T(Eu) = E(Su) = E(Eu) = E(Fu)= T(Fu) = S(Fu)) = F(Fu),

If E(u) ≠ F(Eu)

$$d(Eu,F(Eu)] \leq \alpha\left[\frac{[d(Su,T(Eu))\{d(Su,Eu) + d(T(Eu),F(Eu))\}]}{d(Su,F(Eu)) + d\,(T(Eu),Eu)}\right] + \beta[d(Su, Eu) + d(T(Eu), F(Eu))]$$

$+\gamma$ [d(Su, F(Eu)) + d(T(Eu), Eu)]

+ δ d(Su, T(Eu))

d(Eu, F(Eu)] ≤ (β +γ + δ)[d(Eu, , F(Eu))]

Thus we get a contradiction. Hence Eu = F(Eu)

we get Eu = F(Eu) = T(Eu) = E(Eu) = S(Eu). Hence Eu is a common fixed point of E, F, S and T.

Uniqueness: Let v is another fixed point of E, F, S and T different from u, then we have

$$d(u, v) = d(Eu, Fv) \leq \alpha\left[\frac{[d(Su,Tv)\{d(Sx,Eu) + d(Tv,Fv)\}]}{d(Su,Fv) + d\,(Tv,Eu)}\right]$$

+ β[d(Su, Eu) + d(Tv, Fv)]

$+\gamma$ [d(Su, Fv) + d(Tv, Eu)]

+ δ d(Su, Tv)

d(u, v) ≤ (2γ + δ) d(u, v)

which is a contradiction. Therefore u is unique fixed point of E, F, S and T in X.

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