On Comparison of Some Imputation Techniques in Multivariate Data Analysis

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Abstract

Listwise or pairwise deletion as the method of handling missing data in multivariate data leads to loss of statistical power, biased results and underestimation of standard errors and P-values.Four imputation techniques namely Regression, Stochastic, Expectation-Maximization (EM) and Multiple Imputation (MI) were considered and compared in terms of preserving the original distribution of the (multivariate) data and the relationships among the variables before the techniques were applied. Results show that none of the techniques performed absolutely better than the rest leaving the choice of imputation techniques in any dataset on the objectives of the researcher.

Keyword: Imputation, missing data, Expectation Maximization, Multiple Imputation, Root Mean Square Error.

1.0 Introduction

Method of handling missing data in some statistical software like SAS, S-Plus and SPSS is listwise deletion. This involves dropping any case with missing value. Consequently, statistical power is lost, biased results are obtained and underestimation of standard errors and P-values are usually observed.

Alternatively, imputation can be applied which according toEveritt (2002) is a process of estimating missing values using the non-missing information available for the subject. It addresses the problem of reduction in statistical power. There are several imputation techniques which include mean imputation (the average value is filled in), regression imputation (a regression model is used to predict the missing value), hot deck imputation (which imputes new values from similar cases), Stochastic imputation and the more recent methods like Maximum Likelihood, Multiple Imputation, Expectation Maximization (EM), etc. However, only four techniques are considered in this work namely: Regression imputation, Stochastic imputation, Expectation-Maximization and the Multiple Imputation.

Von Hippel (2007) suggested that several factors should be considered before a technique is chosen. Such factors include type of parameter estimates that should be generated (biased or unbiased), non-response rate, nature of the missing data and availability of an auxiliary data that are correlated with characteristic of interest.

1.1 *Non-Response Rate*

For non-response rate, the researcher should consider percentage of missing data for each of the variable. ‘Small’ percentage of missing value is less problematic and may be corrected by simpler imputation techniques like overall mean or class-mean imputation technique where applicable. There’s no consistent definition of ‘small amount of missing data’. However, for Little and Rubin (2002) it ranges from 5% or less of values.

1.2 *Nature of Missing Data*

The researcher needs to establish whether data are missing completely at random (MCAR), missing at random (MAR) or missing not at random (MNAR). This factor determines the choice of the technique to choose as well as the case deletion method to adopt. If data are MCAR then listwise deletion yield unbiased results but if the data are not MCAR, listwise deletion will introduce bias because the sub-sample of case represented by the missing data are not representative of the original sample (and if the original sample was itself a representative sample of a population, then the complete cases are not representative of that population either. Brick and Kalton (2000)

1.2.1*Missing Completely at Random (MCAR)*

Here, the probability of response to a variable of interest, Y, is the same for all the units in the population. This means that the probability of response does not depend on the auxiliary variable(s), or the variable of interest, Y.Roughly speaking, the tendency for a data point to be missing is completely random. Pickles (2005) points out that for MCAR, the probability of missingness is a constant. This implies that $P_i = P(i \epsilon S_r) = P$. $\forall\ i \in U$. Where S_r is Sample of respondents for a given item and U is the target population of size N. Formally, when data are MCAR, evidently the set of objects with no missing data is also a random sample from the source population. Hence, most techniques for handling missing data under MCAR including listwise deletion yield unbiased results. According to Fellegi and Holt (1976), the imputed estimator, $\bar{y}^*$ for the population mean will be unbiased if data are MCAR. A statistical test for MCAR is a Chi-square test which is provided in the SPSS Missing Values Analysis (MVA) option. A significant value indicates that data are **not** MCAR.

1.2.2*Missing at Random (MAR)*

In this case, the probability of response to a variable of interest, Y is related to auxiliary variable(s) X. This implies that $P_i = P(i \epsilon S_r) = P(X_i)$. i.e. data are MAR if the probability of response to Y is not a function of its own value but a function of the values of the auxiliary variable(s). Generally, when data are MAR, a complete case analysis is no longer based on random sample from the source population. Hence, all simple techniques including the listwise deletion and overall mean imputations yield biasedresults. However, more advanced techniques like *Stochastic Regression and Multiple Imputation* techniques give unbiased results even when missing data are MAR.

The MAR test is a ‘‘Separate variance t-tests’’ which is also available on the SPSS Missing Values Analysis (MVA). A significant value indicates that data are MAR.

1.2.3*Missing Not at Random (MNAR)*

Often times, data are MNAR, implying that missingness is related to one or more of the outcome variables or that the missingness has a systematic pattern, Schaefer and Graham (2002). If there is a pattern to the missing values, the best decision is that they are MNAR. There are some simple ways researchers can examine their data to determine whether missing data follow a pattern. For instance, during instrument development like questionnaire, etc., response sets should include ‘don’t know’, ‘does not apply’ or ‘refused’ responses. These responses allow the researcher to distinguish among these ‘no answer’ responses. For example, if the majority of responses to an item asking for particular information which may be income, age, marital status, etc. are ‘refused’, the researcher will be confident that there is a pattern to the responses. When missing data are MNAR, valuable information is lost from the data and there is no universal way of handling the missing data properly.

Although, data imputation techniques typically assume that at a minimum, data are MAR, more advanced imputation techniques are robust and produce nearly as good results without strictly meeting this assumption. Little and Rubin (2002).

2.0 Methodology

Procedures for data imputation using regression, stochastic, Expectation-Maximization (EM) and Multiple Imputation (MI) were described. The descriptive statistics, the statistical relationships and the normal Q-Q plots of the four variables were obtained and examined before and after imputation. The SPSS and R were used for the analysis.

2.1 Regression Imputation

This assumes a linear relationship between the variables used in the regression equation when there may not be one. We used the variable with the missing data as the dependent variable. Cases with the complete data for the predictor variables were used to generate the regression equation. Predictors of the missing values were identified using a correlation matrix. The best predictors were the variables with the highest correlations and were therefore used as the independent variables in a regression equation to predict missing values for the incomplete cases. That is for a variable Y_i with missing values, a model

$$Y_i = \beta_0 + \beta_1 Y_1 + \beta_2 Y_2 + \cdots + \beta_{(i-1)} Y_{(i-1)} \quad 1.1$$

is fitted for the non-missing observations.

Whereβ_0,$\beta_1, \dots, \beta_{(i-1)}$ are the estimated parameters of the model andY_1 ,Y_2 ,...,$Y_{(i-1)}$are the covariates used as predictors of the missing value. Multicollinearity was also considered.

2.2 Stochastic Imputation

We introduced an error term to Equation 1.1 because regression imputation doesn’t supply ‘uncertainty’ about the predicted value. Consequently, imputed data in 1.1 do not have an error term included in their estimation, thus the estimates fit perfectly along the regression line without any residual variance. This causes relationships to be over identified and underestimation of variance.

From the regression model in 1.1 above, the parameter estimates and the associated Variance-Covariance matrix of the fitted model are given by

β_0,* β_1 *, ..., $\beta_{(i-1)}$ * and $\sigma^2 Q_i$respectively. Where Q_i is the usual $\mathrm{X}^\mathrm{I}\mathrm{X}$ matrix from the intercept and the variables Y_1 ,Y_2 ,...,$Y_{(i-1)}$. For each imputation, new parametersβ_0*, β_1*,..., $\beta_{(i-1)*}$ and $\sigma^{2}_{i}{}^{*}$ are drawn from the posterior predictive distribution of the missing data. Then a new model for the estimation of missing data is given by

$$Y_i = \beta_0 + \beta_1 Y_1 + \beta_2 Y_2 + \cdots + \beta_{(i-1)} Y_{(i-1)} + rmse * \varepsilon_i \quad 1.2$$

Whereε_i~ N (0,1) , and rmse is the Root Mean Square Error.

Clearly from 1.2, the error term for each model is a function of its Root Mean Square Error (rmse)

The procedure used here is summarized as follows:

(i) Regress the variable with the missing data on other variables.
(ii) Obtain the root mean square error (Standard Error) of the estimate.
(iii) Compute the predicted value of the missing data given the values of other variables in the equation.
(iv) Add random variability (which is simply a product of the rmse and ϵ_i) to the predicted value Y_i

2.3 Expectation–Maximization (EM) Imputation

The EM technique is a maximum likelihood based approach that works with the relationship between the unknown parameters of the data model and the missing data. We assumed a multivariate normal model. The procedure here is summarized as follows:
We first estimate the model parameters, then estimate the missing values, then use the filled-in dataset to re-estimate the parameters, then *use the re-estimated parameters to estimate missing values, and so on. When the process finally converges on stable estimates the iterative process ends*. Schaefer and Olsen (1998)
Recall that regression imputation underestimates the true variability in the data because there is no error associated with the imputed observations. Howell, D.C. (2008) points out that EM like Stochastic imputation corrects that problem by estimating variances and covariances that incorporate the residual variance from the regression. For instance, assume that we impute values for missing data on $Y_1$from data on Y_2, $Y_3$and Y_4, to find the estimated mean of Y_1we simply take the mean of that variable. Now that we have a new set of parameterestimates, we repeat the imputation process to produce another set of data. From that new set we re-estimate our parameters as above, and then impute yet another set of data. We continue this process in an iterative fashion until the estimates converge.

2.4 Multiple Imputation.

MI uses a stochastic linear model.

$$Y_i = \beta_0 + \beta_1 Y_1 + \beta_2 Y_2 + \cdots + \beta_{(i-1)} Y_{(i-1)} + \varepsilon_i \qquad 1.3$$

Where$\varepsilon_i \sim$ N(0,1) and β_0,$\beta_1, \dots, \beta_{(i-1)}$are the parameters of the model.

From the term ‘multiple’, clearly we are required to impute several times. This implies that we shall have multiple complete dataset before we proceed with analysis. We then combine the results of those analyses and make inference. The process involves three different stages namely:

(i) Impute for the missing data *k* times using the above model.
(ii) The *k* completed datasets are analysed for the parameter estimates of interest.
(iii) The results from the *k* completed datasets are combined for inference.

Step (iii) is broken down as follows:

Suppose *k* imputations were done, from each analysis, we calculate and save the estimates. The overall estimate is the *mean* of the individual estimates.

For the overall standard error for each variable, the within-imputation variance was first calculated which is the *mean* of the variances of the variable in the five dataset given below as

$$\bar{y} = \frac{1}{k}\sum_{i=1}^{k} W_{\mathrm{I}} \qquad 1.4$$

W_i i = 1,…,k are the variances for each dataset.

The between-imputation variance which is the *variance* of the variances of the variable in the k dataset is given below as:

$$B = \frac{1}{k-1}\sum_{i=1}^{k-1}(W_i - \bar{y})^2 \qquad 1.5$$

The total variance is given by

$$\mathrm{T} = \bar{y} + (1+ \frac{1}{n})\mathrm{B} \qquad 1.6$$

Where $(1 + \frac{1}{n})$ is a correction factor. Then the overall standard error is the square root of T, $(\mathrm{T})^{1/2}$

3.0 Example/ Analysis

We illustrate with a multivariate data of four variables obtained from the Records department of Bishop Shananhan Hospital, Nsukka, Enugu State, Nigeria. The variables are Age of the patients,(Y_{1},), Number of days they spent in the hospital (Y_2), Blood pressure (Systolic)Y_3 and their Weight (Y_4). It is on 40 patients, with each patient providing information on each variable. There are 20 missing observations out of 160 across the data implying a non-response rate of 12.5%. Little’s Chi-Square test for MCAR and the ‘Separate Variance t-test’ for MAR were used to determine the nature of the missing data. Using the MVA(Missing Value Analysis) option on SPSS, under EM estimation, the Little’s MCAR test has the following values: Chi-Square = 14.006, degree of freedom = 14 and P-value = 0.449. At 5% level of significance, the null hypothesis that data are MCAR was accepted which implies that the probability of missingness is a constant and the set of items with no missing data is a random sample from the source population. *Conclusion*: Data are MCAR (5% level of significance).

Like in all practical cases and for a valid inference, the assumption of linear dependency amongst the variables was not taken for granted. We obtained a correlation matrix and tested for linear dependency among the variables. The essence of this was to determine how the four variables are correlated amongst themselves and subsequently determine the independent variable(s) that should be used in the imputation model for each variable. Using

H_0: $\rho_{ij} = 0 Vs$ H_1: $\rho_{ij} \neq 0$ $\mathrm{i} \neq \mathrm{j}$ $\forall ii$ =1,2,3,4: $\forall j,\ j$ = 1, 2,3,4 where ρ_{ij} signifies correlation between Y_i and $Y_{j.}$

In Table 1 below, the values outside the bracket are the Pearson correlation coefficient while the values in the bracket are the P-values for the tests of hypothesis above.

Table 1. Correlation before imputation

	Age	Days	BP	Weight
Age (P-Value)	1	0.437 (0.016)*	0.575(0.001)**	-0.453(0.014)*
Days	0.437(0.016)*	1	0.449(0.011)*	-0.233(0.207)
BP	0.575(0.001)**	0.499(0.011)*	1	-0.347(0.060)
Weight	-0.453(0.014)*	-0.233(0.207)	-0.347(0.060)	1

** Correlation significant at 1% level of significance.

* Correlation significant at 5% level of significance.

Table 2 Descriptive Statistics before Imputation

	N	**Mean**	**SE(**Mean)	**Variance**	**Skewness**
Age	35	54.0000	2.77534	269.588	0.061
Days	35	11.2857	0.81845	23.445	0.133
BP	36	123.3333	1.92106	132.857	-0.515
Weight	34	64.7857	1.52675	79.253	0.252

Clearly from Table 1, at 5% level of significance, Only Y_2, Y_3 and Y_4 are correlated with Y_1. Using regression imputation to impute for Y_1, it was discovered that regressing Y_1 on Y_2, Y_3 and Y_4 yielded a model with a low (adjusted) Coefficient of DeterminationR^2 which is very poor for the purpose of prediction and even a high standard error of the estimate. Consequently, Y_4 was dropped and a "better" model emerged. Y_2, Y_3 and Y_4 were also imputed using the same procedure. Using Table 1 and considering the (adjusted) R^2, we shall have the following imputation models,

$$Y_{1i} = \beta_0 + \beta_1 Y_{2i} + \beta_2 Y_{3i} \tag{1.6}$$
$$Y_{2i} = \beta_0 + \beta_1 Y_{1i} + \beta_2 Y_{3i} \tag{1.7}$$
$$Y_{3i} = \beta_0 + \beta_1 Y_{1i} + \beta_2 Y_{4i} \tag{1.8}$$
$$Y_{4i} = \beta_0 + \beta_1 Y_{1i} + \beta_2 Y_{3i} \tag{1.9}$$

The parameters were estimated and the following models emerged:

,
$$Y_{1i} = -31.419 + 0.898Y_{2i} + 0.620Y_{3i} \tag{2.0}$$
$$Y_{2i} = -11.282 + 0.100Y_{1i} + 0.134Y_{3i} \tag{2.1}$$
$$Y_{3i} = 148.860 + 0.190Y_{1i} - 0.519Y_{4i} \tag{2.2}$$
$$Y_{4i} = 118.360 - 0.378Y_{1i} - 0.105Y_{3i} \tag{2.3}$$

The Variance Inflation factor (VIF) is an indicator of multicollinearity. It's an index that measures how much the variance of an estimated regression coefficient is inflated due to multicollinearity. Kutner (2004) suggests that VIF > 5 signifies high multicollinearity. The VIF is given by

$$VIF = 1/1\text{-}R^2 \qquad 2.4$$

where R^2 is the Adjusted coefficient of determination of the model.
For the models in 2.0, 2.1, 2.2 and 2.3, the VIF were computed as 1.278, 1.442, 1.299 and 1.284 respectively. These values are less than 5. So, there was no problem of multicollinearity.

For Y_1, there are missing cases in items 3, 7, 17, 25 and 36 and there imputed values are 42, 66,53, 42 and 62 respectively.
The complete dataset from regression imputation included the other variables is tabulated on the Appendix.

For Y_1 , the regression model before imputation is given in 2.0 as

$$Y_{1i} = -31.419 + 0.898Y_{2i} + 0.620Y_{3i}$$

While the regression model after imputation is given by:

$$Y_{1i} = -38.632 + 0.904Y_{2i} + 0.672Y_{3i} 2.4$$

3.1 Stochastic Regression:

Using the normal variates, ε_{ij} ~N(0,1) generated by the SPSS. We have the following models:

$$Y_{1i} = -31.419 + 0.898Y_{2i} + 0.620Y_{3i} + 13.784 * \varepsilon_{ij} \qquad 2.5$$

$$Y_{2i} = -11.282 + 0.100Y_{1i} + 0.134Y_{3i} + 4.946 * \varepsilon_{ij} 2.6$$

$$Y_{3i} = 148.860 + 0.190Y_{1i} - 0.519Y_{4i} + 8.660 * \varepsilon_{ij} 2.7$$

$$Y_{4i} = 118.360 - 0.378Y_{1i} - 0.105Y_{3i} + 7.248 * \varepsilon_{ij} 2.8$$

The rmse for the imputation models in 2.5, 2.6, 2.7 and 2.8 are 13.820, 4.624, 9.309 and 7.428 respectively. For Y_2, the missing cases are in items 6, 23, 26, 30 and 33 and the imputed values are 18.1, 9.6 ,5.1, 8.6 and 9.5 respectively.

The descriptive statistics after stochastic imputation and the new correlation matrix with their significance levels are tabulated below

3.2 Expectation-Maximization.

This technique is stochastic in nature. The imputation models used were those of the regression imputation but with the EM algorithm introducing an error term which is a normal variate, ϵ_j ~ N(0,1) (*not a function of the rmse*) Using the EM Algorithm for Missing Value Analysis (MVA)options,we used 25 iterations which is the default number of iterations. This implies that the algorithm estimated the model parameters, then estimated the missing values, then used the filled-in dataset to re-estimate the parameters with the process occurring 25 times before the values converged. The estimates are Maximum Likelihood Estimates

For the four variables, we used the following imputation models:

$$Y_{1i} = -31.419 + 0.898Y_{2i} + 0.620Y_{3i} + \varepsilon_{ij} \quad 2.9$$

$$Y_{2i} = -11.282 + 0.100Y_{1i} + 0.134Y_{3i} + \varepsilon_{ij} \quad 3.0$$

$$Y_{3i} = 148.860 + 0.190Y_{1i} - 0.519Y_{4i} + \varepsilon_{ij} \quad 3.1$$

$$Y_{4i} = 118.360 - 0.378Y_{1i} - 0.105Y_{3i} + \varepsilon_{ij} \quad 3.2$$

3.3 Multiple Imputation.

Graham,et al(2007) recommends at least five imputations. From each analysis, we calculated and saved the estimates.

The overall estimate is the *average* of the individual estimates.

$$\overline{W} = \frac{1}{n}\sum_{i=1}^{n} W_{\mathrm{I}} \quad 3.3$$

Where $W_{i:}$ i = 1,…,5 are the individual estimates.

For the overall standard error, the within imputation variance as given by 1.4, the between-imputation variance by 1.5 and the total variance by 1.6 were all computed.

Then the overall standard error is the square root of T while a significance test of null hypothesis is performed using the test statistic below

$$\mathrm{T} = \frac{\overline{W}}{\sqrt{(T)}} \sim \text{t–distribution.} \quad 3.4$$

where$\overline{W}$ is the average of the estimate and $\sqrt{(T)}$ is the overall standard error. For Y_1, the within-imputation variance $\bar{Y}_1$ = 291.089 and the between imputation variance B_1 = 393.243.Therefore, the total variance T_1 (as computed using equation 1.6) = 694.163. The overall standard error of Y_1 is simply given by $(694.416)^{1/2}$ = 26.34697. The total variance and overall standard error of other variables were computed in a similar manner.

*Table 3*__Correlations before Imputation__

	Age	Days	BP	Weight
Age (P-Value)	1	0.437 (0.016)*	0.575(0.001)**	-0.453(0.014)*
Days	0.437(0.016)*	1	0.449(0.011)*	-0.233(0.207)
BP	0.575(0.001)**	0.499(0.011)*	1	-0.347(0.060)
Weight	-0.453(0.014)*	-0.233(0.207)	-0.347(0.060)	1

*Table 4***After Regression Imputation**

	Age	Days	BP	Weight
Age (P-Value)	1	0.481 (0.02)*	0.590(0.000)**	-0.443(0.004)**
Days	0.481 (0.02)*	1	0.447(0.004)**	-0.202(0.212)
BP	0.590(0.000)**	0.447(0.004)**	1	-0.358(0.023)*
Weight	-0.443(0.004)**	-0.202(0.212)	-0.358(0.023)*	1

*Table 5***After Stochastic Imputation**

	Age	Days	BP	Weight
Age (P-Value)	1	0.492 (0.001)**	0.558(0.000)**	-0.426(0.006)*
Days	0.492 (0.001)**	1	0.478(0.002)**	-0.243(0.130)
BP	0.558(0.000)**	0.478(0.002)**	1	-0.296(0.063)
Weight	-0.426(0.006)**	-0.243(0.130)	-0.296(0.063)	1

*Table 6***After EM Imputation**

	Age	Days	BP	Weight
Age (P-Value)	1	0.477 (0.002)**	0.631(0.000)**	-0.423(0.007)**
Days	0.477 (0.002)**	1	0.452(0.003)**	-0.276(0.084)
BP	0.631(0.000)**	0.452(0.003)**	1	-0.278(0.083)
Weight	-0.423(0.007)**	-0.276(0.084)	-0.278(0.083)	1

*Table 7***After Multiple Imputation**

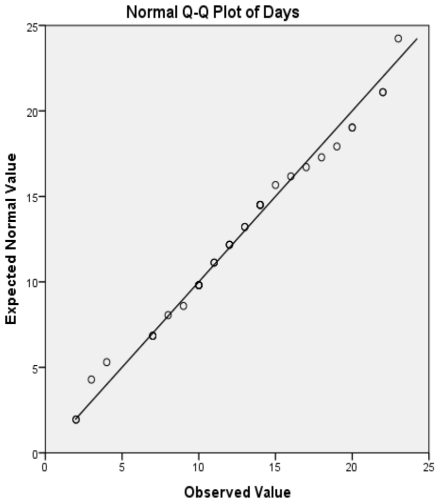
	Age	Days	BP	Weight
Age (P-Value)	1	0.443 (0.016)*	0.595(0.000)**	-0.305(0.014)*
Days	0.443 (0.016)*	1	0.409(0.011)*	-0.228(0.207)
BP	0.595(0.001)**	0.409(0.011)*	1	-0.337(0.060)
Weight	-0.305(0.014)*	-0.228(0.207)	-0.337(0.060)	1

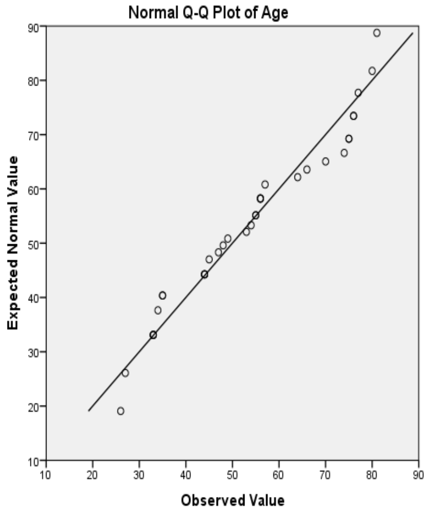
* Correlation significant at 5% level of significance.
**Correlation significant at 1% level of significance

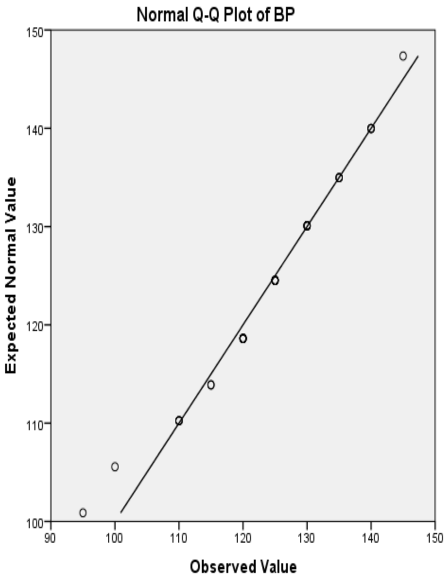
*Table 8***Descriptive Statistics before Imputation**

	N	**Mean**	**SE(**Mean)	**Variance**	**Skewness**
Age	35	54.0000	2.77534	269.588	0.061
Days	35	11.2857	0.81845	23.445	0.133
BP	36	123.3333	1.92106	132.857	-0.515
Weight	34	64.7857	1.52675	79.253	0.252

The Normal QQ Plot of the four variables *before* imputation are given below:







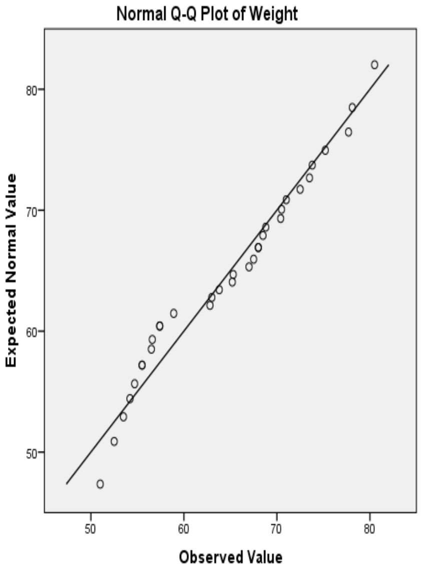
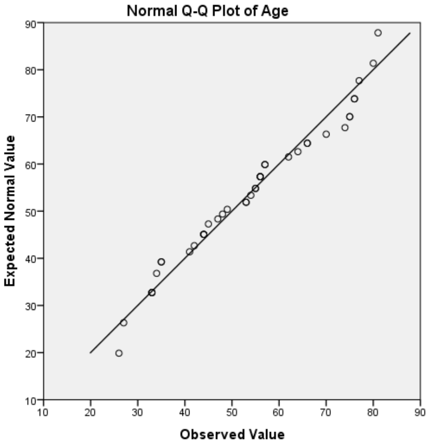
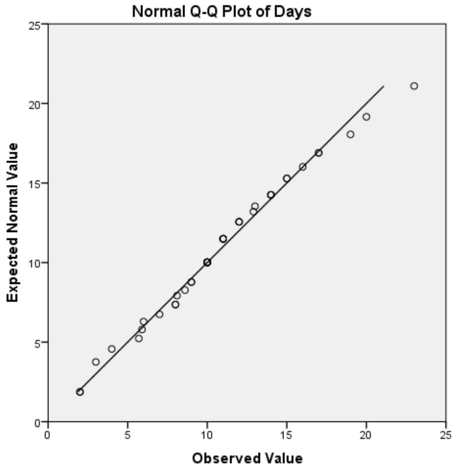
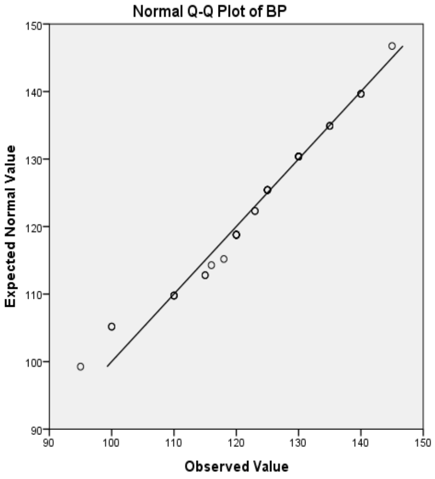


*Table 9***Descriptive Statistics after regression Imputation**

	N	**Mean**	**SE(**Mean)	**Variance**	**Skewness**
Age	40	53.8250	2.49192	248.387	0.078
Days	40	10.9050	0.74744	22.347	0.282
BP	40	123.0000	1.74091	121.347	-0.443
Weight	40	64.5725	1.31076	68.724	0.328







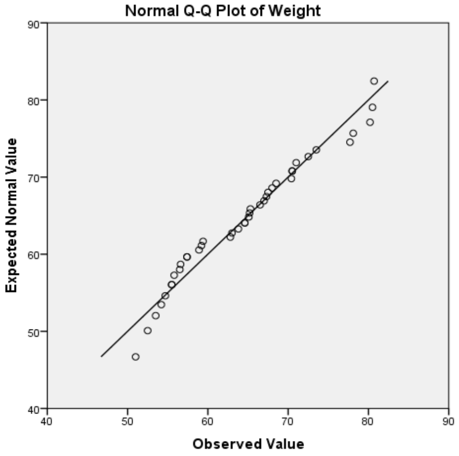
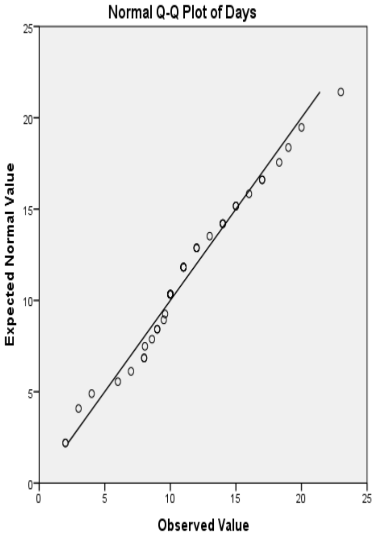
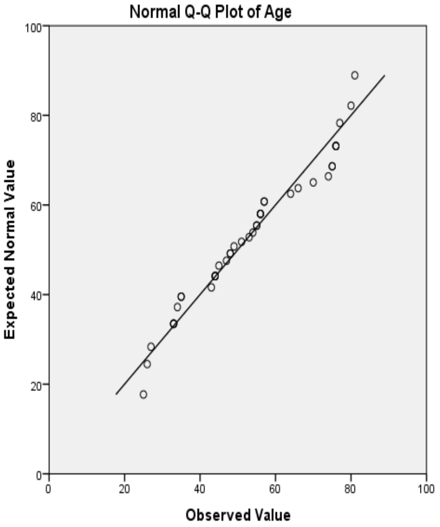


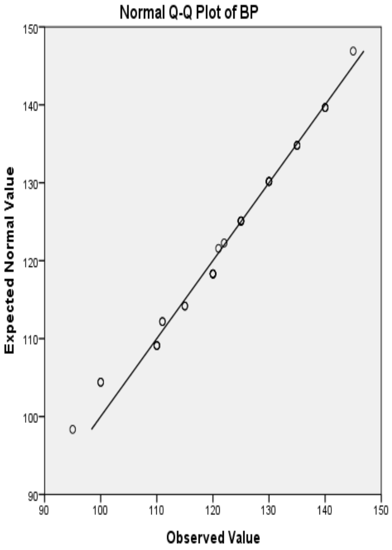
*Table 10***Descriptive Statistics after Stochastic imputation**

	N	**Mean**	**SE(**Mean)	**Variance**	**Skewness**
Age	40	53.3250	2.61146	272.789	0.089
Days	40	11.2265	0.74672	22.304	0.236

BP	40	122.6250	1.97976	126.702	-0.366
Weight	40	64.3800	1.40524	78.988	0.008







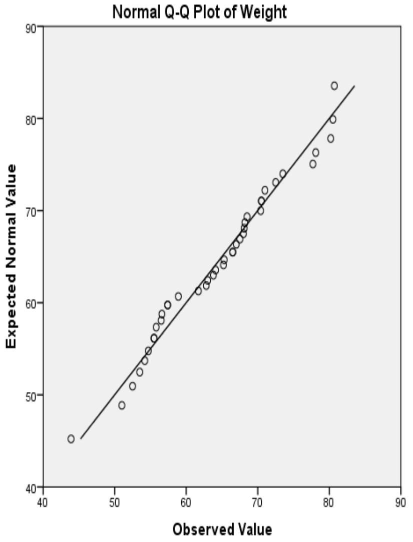
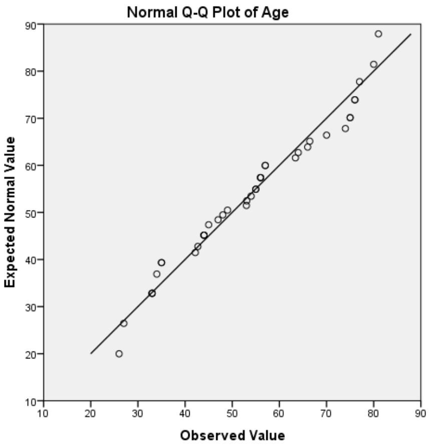
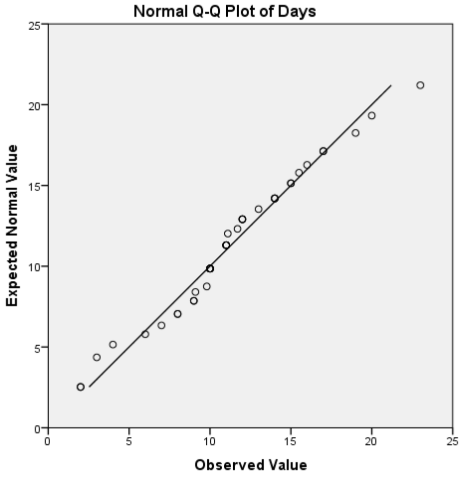
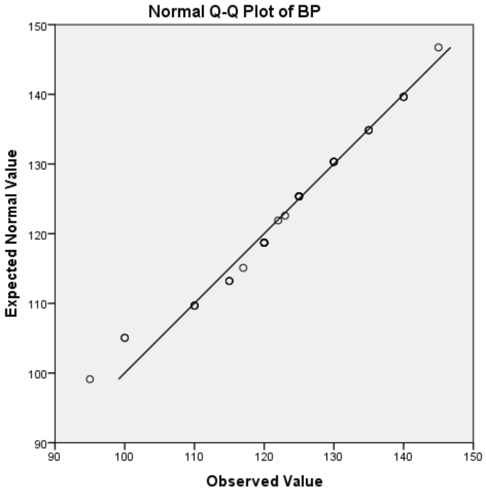


*Table11***Descriptive Statistics after EM Imputation.**

	N	**Mean**	**SE(**Mean)	**Variance**	**Skewness**
Age	40	53.5750	2.70484	292.646	0.074
Days	40	11.3050	0.80582	25.973	0.138
BP	40	122.8732	1.89100	143.035	-0.410
Weight	40	64.8541	1.50293	90.352	0.242







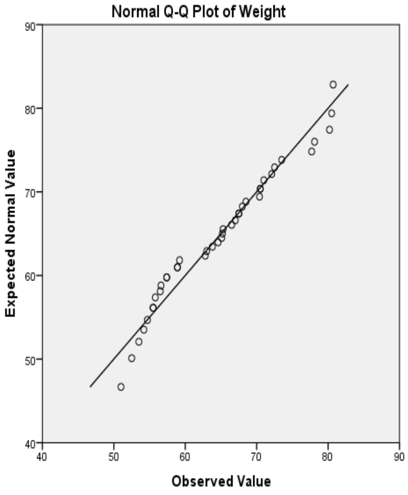


*Table 12***Descriptive Statistics after MI imputation**

	N	**Mean**	**SE(**Mean)	**Variance**	**Skewness**
Age	**40**	53.33816	26.34697	694.163	-0.0108
Days	**40**	11.40202	4.89959	24.006	0.1106
BP	**40**	122.92726	14.40951	207.634	-0.4628
Weight	**40**	64.70972	12.57911	158.234	0.2428

4.0 DISCUSSION OF RESULTS AND FINDINGS

Regression imputation underestimated the standard error of the four variables as shown in *Table 9*, From269.588 to 248.387 for Y_1, 23. 445 to 22.347 for Y_2, 132.857 to 121.347 for Y_3 and 79.253 to 68.724 for Y_4 . Also, from Table 4, we see that the P-values in the test for correlation between the variables, Y_1, Y_2 , Y_3 and Y_4 are underestimated. At 5% level of significance, ρ_{34} was not significant *before* imputation with P-value of 0.060 but significant *after* imputation with a p-value of 0.023. However, at 1% level of significance, ρ_{14}, and ρ_{23} which were not significant*before* imputation with p-values 0.014 and 0.011 respectively became significant*after* imputationwith p-values 0.004 and 0.004. This shows that regression imputation is not robust enough to preserve the relationships among the variables and may lead to Type 1 error – reporting significance in hypothesis testing when there is none. This, of course leads to erroneous inference. Also in Table 9, the changes in values for skewness (a distribution's departure from symmetry) are very minimal which suggests that there may not be any deviation from the original distribution of the dataset before imputation.

Stochastic imputation underestimated standard errors but not as much as Regression imputation. Table 10 reveals that the variance for the four variables increased. This is clearly due to the introduction of the error term, rmse*$\epsilon_{ij}$where ϵ_{ij}~ N(0,1) in the models which increases the variability in the dataset. Table 5 shows that at 5% level of significance the statistical relationships are preserved. However, at 1% level of significance, ρ_{21}, ρ_{32} and ρ_{41} which were not significant*before* imputation with p-values 0.016, 0.011 and 0.014 respectively became significant*after* imputation.with p-values 0.001, 0.002 and 0.006 respectively. Also, changes in values for skewness are not much as shown in Table 10.

EM imputation only records a slight underestimation of standard errors. A study of Tables 8 and 11 reveals the underestimation after EM imputation as follows: 2.77534 to 2.70484, 0.81845 to 0.80582, 1.92106 to 1.89100 and 1.52675 to 1.50293 for S.E(Y_1), S.E(Y_2), S.E(Y_3) and S.E(Y_4) respectively. This underestimation is very minimal and Table 6 shows that relationships at 5% level of significance are still maintained. This verifies the claim of Enders, C.K. (2010) that "EM imputations preserve the relationships with other variables, which is extremely vital if the researcher is going into Factor Analysis and Regression." Interestingly, at 1% level of significance as revealed in Table 6 , ρ_{12}, ρ_{23} and ρ_{14} which were not significantat 5% became significant with with p-values 0.002, 0.007 and 0.003 respectively. The skewness for the four variables remains almost the same with the ones before imputation. From 0.061 to 0.074 for Y_1, 0.133 to 0.138 for Y_2, -0.515 to -0.410 for Y_3 and 0.252 to 0.242 for Y_4. These values are close to zero showing that their distributions still remain approximately symmetric.

Multiple imputation, MI is the only technique that preserved the relationships of the variables both at 5% and 1% levels of significance. This is evident in Table 7. This makes it "better" than the EM technique especially in studies where the statistical relationships amongst the variables are paramount. A study of Table 12 reveals that there is no underestimation of standard errors rather they were *inflated*. The S.E.(Y_1) moved from 2.77534 to 26.43697, 0.81845 to 4.89959 for S.E.(Y_2), 1.92106 to 14.40951 for S.E.(Y_3) and 1.52675 to 12.57911 for S.E.(Y_4). This could be a disturbing development when testing hypotheses about the variables especially for Y_1 which records the highest margin. Clearly, MI doesn't underestimate standard errors but may create the problem of Type II error- not reporting significance when they actually exist. Now, because of the inverse relationship between the probability of Type II error, β and statistical power (1- β), we can say that the MI has not been able to appropriately tackle the issue of reduced power in this case. Though, some statisticians would rather risk Type II error than Type I error because in hypothesis testing, the practice is usually to consider Type I error more seriously than Type II error. In other words, any technique that would minimise Type I error would surely be preferred.

Once again, values in skewness did not change significantly showing that the distribution of the dataset is still maintained. The results of MI done well are unbiased parameter estimates and no underestimation of standard errors and P-values. However, MI created by an incorrect model can lead to erroneous decisions.

Descriptive statistics before Imputation

Descriptive statistics after MI

	N	Mean	SE(Mean)	Variance	Skewness	N	Mean	SE(Mean)	Variance	Skewness
Age	35	54.0000	2.77534	269.588	0.061	**40**	53.33816	26.34697	694.416	-0.0108
Days	35	11.2857	0.81845	23.445	0.133	**40**	11.40202	4.89959	24.006	0.1106
BP	36	123.3333	1.92106	132.857	-0.515	**40**	122.92726	14.40951	207.634	-0.4628
Weight	34	64.7857	1.52675	79.253	0.252	**40**	64.70972	12.57911	158.234	0.2428

5.0 Conclusion

The existence of missing data creates problems that can never be completely solved but managed using some *good* imputation techniques.

The underestimation of standard errors and P-values are highest in the regression technique, moderate in the stochastic, minimal in the EM technique and inflated in the MI. At 5% level of significance, correlations were maintained only by the EM and the MI techniques with only the MI still maintaining correlations at 1% level of significance but with inflated standard errors, exposing the researcher to the risk of Type II error. Unlike Multiple Imputations, the regression imputation, stochastic imputation and EM techniques are not robust enough to preserve relationships among variables at 1% level of significance. These techniques should be dropped in vital statistics, epidemiology, and government budgeting where most decisions are based on hypothesis testing at 1% level of significance

An attempt to compare these techniques shows that none of them is *universally better than* the other. While regression, stochastic and the EM were relatively better than the MI in maintaining the distribution of the original dataset. MI performed better than them in maintaining the relationships. Some underestimate standard errors and P-values thereby creating the problems of Type I error while others like the MI inflate standard errors and P-values, creating the problem of Type II error.

The EM has the advantage that even when the assumption of multivariate normal distribution of observations is in error, the algorithm seems to work remarkably well. But because it still underestimates standard errors, it is only advisable if the percentage of missing data is under 5%. Enders (2010)

Luengo, *et al* (2011) suggested that the EM and the MI techniques should be adopted in fields of knowledge like Bioinformatics, Climatic science and Medicine.

However, there is no universal imputation technique that performs ‘best’ for all cases. It all depends on the objectives of the researcher.

6.0 References

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