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Optimization of Impulse Response for Borehole Gamma Ray Logging

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Abstract

The gamma-ray borehole logging plays a leading role in uranium exploration programs and more so in the estimation of the geological impulse response function in gamma ray intensity which is solely responsible to delineate the radioactive horizons. In the past several attempts have been made to estimate this function numerically, theoretically and heuristically too based on the controlled experiments.

However, the present work is an attempt to determine / estimate this geological function by directly using the data of radioactive borehole logging. It is interesting to note in this study that the results obtained by this approach are not only nearer to the results obtained by the earlier attempts but also encourage to study further in detail under different geological formations. The present work is based on the gamma-ray borehole logging data of Yellapur area, Nalgonda dist., A.P, India, for the estimation of the geological function.

Keywords: Gamma ray logging, uranium exploration, geological response, radioactive zones, exponential behavior, geological formations, exponential integral of first kind, Bessel Functions of first and second kind, gamma ray probe, gamma ray transport theory, field measurements

1. Introduction

The aim of quantitative interpretation of gamma-ray borehole data is to analyze it in terms of thickness and grades of homogenous and isotropic sub-surface plane layers of the earth. These sub-surface plane layers are anomalous zones above the cut-off grades –values such as 0.01, 0.02, 0.03 etc., of %eU_3O_8. Accordingly the product of the grade thickness is computed for further ore-grade computations.

The direct and indirect methods of interpretation of borehole data start either from GT product (G-Grade & T-Thickness of ore zones) or by approach given by Czubek (1972). The basic difference between these two approaches is that former resolved the problem using Gaussian like geological response function across barren and radioactive geological zones whereas Czubek (1978) used deconvolution of borehole data by employing a geological response function.

The major objective of these approaches has been the numerical computation of this geological response. Primarily this expression is represented as an infinite integral containing exponential integral, modified Bessel functions. This expression therefore does not lend itself to comfortable analytical integration. Its solution and evaluation is a major step in the direction of a better interpretation of borehole gamma-ray logging data which is the base for uranium ore deposit calculation. Due to this hurdle only approximate response function has been used to develop linear filters for deconvolving and interpreting the borehole gamma ray data in exploration programs.

Unfortunately, designing a filter requires extensive computations and attempt was not made for the derivation of this response function – the backbone of borehole logging data interpretation. This response function however can be estimated from practical borehole radiometric logging data.

In the present work an attempt has been demonstrated to evaluate this geological response function using directly the field data by practical radiometric borehole logging.

2. Formulation and statement of the problem

Assume a point radioactive source kept at a point P on the surface of a uniform non-radioactive matter with air above this non-radioactive surface makes semi-infinite geometry in geological terminology (Figure-1).

Let a particular gamma-ray photon of energy E emits from this source in a spherically symmetric geometry. The intensity (I) due to this gamma-ray source at any point Q at a distance (r) will therefore be

$$I = \frac{qke^{-\mu_a r}}{4\pi r^2} \tag{1}$$

where q = strength of the radioactive source.

μ_a= linear attenuation coefficient of photon of energy E in air.

k = the characteristic constant of a measurement device.

Similarly if the gamma-ray source were kept at any point P1 inside the non-radioactive medium, we can express the intensity I_g at point Q as

$$I_g = \frac{qke^{-\mu_a(E)r_a} \bullet e^{-\mu_e(E)r_e}}{4\pi(r_a + r_e)^2} \tag{2}$$

where μ_e(E) is linear attenuation coefficient of a gamma-ray energy E in the non-radioactive medium.

The distance traveled by the photon in the non-radioactive medium is r_e and

r_a is the distance traveled by photon to the measuring point Q.

From the expressions (1) and (2) one can infer that

$$I_g < I \tag{3}$$

The shape of the trace of these intensities along the boundary AA′, therefore will narrow down as the point P_i goes deeper and deeper in the non-radioactive medium. The peak intensity exactly over the source will also continue to decrease exponentially with the factor $e^{-\mu_e d}$, d being the depth of the point. Some of traces are depicted in re-2.

If all point sources similar to P are placed along the vertical line P1, P2 and the intensity I_g of the gamma-ray energy E is measured along AA′ then a composite profile shall be sum total of intensities ranging from (1), (2). In such situation, the radioactive sources along P1, P3 line extending up to infinity can be summarized as a thin line of radioactive source embedded in a non radioactive medium.

If the gamma-ray intensity (I_g/qk) = Φ(μz) is very near to surface source at any point z, for a unit radioactive concentration, then Φ(μz) (neglecting the attenuation μ_a of air is referred to as an unit response function or shape function Φ(z) of a gamma-ray anomaly and is also referred to as geological impulse response function in gamma-ray borehole logging terminology. The shape of this response function Φ(z) (for a point like detector) for an infinitely thin layer of a unit radioactive source concentration (say, uranium, thorium, potassium or any other source) can be approximately written as follows:-

$$\Phi(z) \cong A[\mu_e(E), R]e^{-\alpha(E)|z|} \tag{4}$$

where A(μ_e(E), R) is a function of linear attenuation coefficient for the energy E of gamma-ray in barren material.

α(E) is attenuation coefficient-like factor in the medium for a gamma-ray of energy E.

R is a radius of borehole in which a thin layer has been assumed to be present perpendicular to depth of borehole taken along a z-axis in cylindrical coordinate system.

Obviously the factor α(E) determines the width of the response function and depends mainly upon the nature of the material around the borehole.

Now suppose such thin sheets are stacked along the boundary AA′ as shown in figure-3, the individual response of all such sheets will look like R_1, R_2, ---- R_n but the combined picture looks like S as shown in figure-3.

The combined gamma-ray response S can be shown as follows:

$$S = R_1 + R_2 + R_3 + \cdots + R_n \tag{5}$$

where n the number of the thin zones that tends to infinity.

If all thin sheets were identical, of width Δz, then

$S = n\Phi\,\Delta z$ Or, $\Phi = (S/n)\,\Delta z = \Delta s/\Delta z$

When this number n of the thin sheets are very large while $\Delta z \rightarrow 0$,

$\Phi = dS/dz$, first derivative of the response S.

This shows that the geological response is first derivative of the total anomaly S with respect to depth z.

S is referred as the step response function.

2.1 Methodology of the solution

When one takes into account the gamma-ray log theory given by Czubek (1961) an analytical form for the geological response (Figure-4) can be easily given. The expression for the shape Φ(z) of gamma ray anomaly for an infinitely thin layer of a unit uranium/thorium ore grade is approximately given as

$$\Phi(z) = \frac{\alpha(E)}{2} e^{-\alpha(E)|z|} \tag{6}$$

where z is the depth of the borehole from some reference and α is a coefficient, which determines the width of the symmetrical function as shown in Figure4.

Czubek (1978) has derived the expression for α(E) -coefficient based upon the gamma-ray transport theory as follows:

$$\alpha(E) = \frac{E_1(\mu R)}{R\left[(K_1(\mu R) - \int_{\mu R}^{\infty} K_0(x)dx\right]} \tag{7}$$

where $E_1(\mu R)$ is exponential integral of first kind (Abramowitz, M. and Stegun, I.A, 1972 and Gradishyten I.S. and Ryzhik. I.M., 1965) and is expressed as follows,

$$E_1(\mu R) = \int_1^{\infty} \frac{e^{-\mu R t}}{t} dt \tag{8}$$

R = radius of the borehole,

$K_1(\mu R)$ = modified Bessel function of first order,

$K_0(\mu R)$ = modified Bessel function of second order,

μ = Attenuation coefficient.

Having assumed the shape of the anomaly as given by expression (1), it remains a challenge to determine α. If approximation to the shape were not made, then the shape of elementary anomaly for zero length borehole probe according to gamma-ray transport theory is given as

$$\Phi_0(\mu z) = \frac{E_1(\mu\sqrt{R^2 + Z^2})}{2\left[1 - G(\infty, \mu R)\right]} \tag{9}$$

where $$G(\infty, \mu R) = 1 - \mu R\left[K_1(\mu R) - \int_{\mu R}^{\infty} K_0(x)dx\right]$$

Two more approximations to response function $\phi(Z)$ have been discussed and are as follows:

$$\Phi_{02} = \frac{\beta}{\mu\pi} \bullet \frac{1}{\cosh\left[(\frac{\beta}{\mu}) \bullet \mu z\right]} \tag{10}$$

and

$$\Phi_{03} = \frac{\gamma}{\mu\sqrt{\pi}} e^{-(\gamma/\mu)^2 (\mu z)^2} \tag{11}$$

where β, γ are coefficients like α as described before for expression (1) .

The shape of the unit anomaly Φ can be determined in practice on a controlled experiment basis such as the standards borehole configuration situated in the building complex of AMD(Atomic Minerals Directorate for Exploration and Research) at Hyderabad, India.

The configuration (Figure 5) of the standard borehole used for calibrating the instruments in AMD has 1.5 meter of 0.069 %eU3O8 and rest is filled with quartz.

Assuming the thickness of this borehole as infinite, the spherical zone of influence around the probe can be taken as a true simulation of natural borehole. Under these assumptions the radiation intensity I (z) growth curve in figure-2 can be assumed to be actually the locus generated by the function.

$$I(z) = K\int_{-\infty}^{+\infty} c\left|(z - z')\right|\Phi(z - z')q(z')dz' \tag{12}$$

where K is calibration factor, which is the intensity at the middle of the ore body divided by its grade. C is a correction factor for borehole conditions and can be considered unity under most of the practical situations of boreholes. q(z′) is ore grade (i.e. eU3O8%) at depth z′.

2.2 Treatment of borehole data from Yellapur (Dist., Nalgonda, AP) and Estimation of α:

In order to apply this methodology, we have selected boreholes in Yellapur area as shown in Table-1. This choice was made because we observed a range of variations of radiometric values along the depth, which is useful for modeling of the geological response function. The selection was particularly based on availability of ore zones with sharp peak activity and tapering off on both sides with no overlap of neighboring zones. The rising portion and the falling portion of the same profile were taken as resource data for the present work (table-2). The data was exponential in nature and was likewise therefore regressed. The graphs are plotted with the exponential function on y-axis and the depth on x-axis with arbitrary starting depth as shown in (Figures 6 to 25). The coefficient of the exponent function (the **α** factor of the response function) is plotted separately with different boreholes. The optimization of this factor (**α(E)**) is done by plotting (Figure 26) all the values of **α** obtained from a large number of boreholes. The maximum value of **α** is found to be near to the value obtained from standard borehole data (made in AMD complex, Hyderabad) and it is also near to the value computed theoretically by Czubek (1978).

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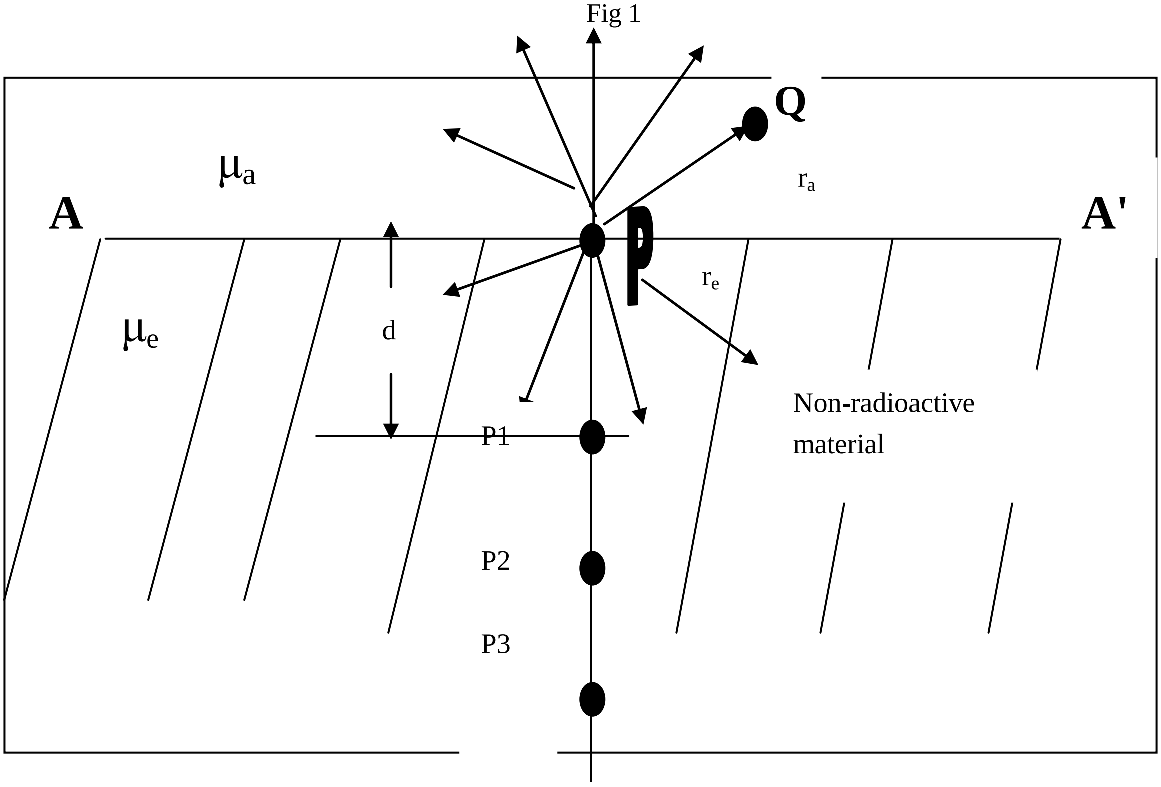
Fig 1



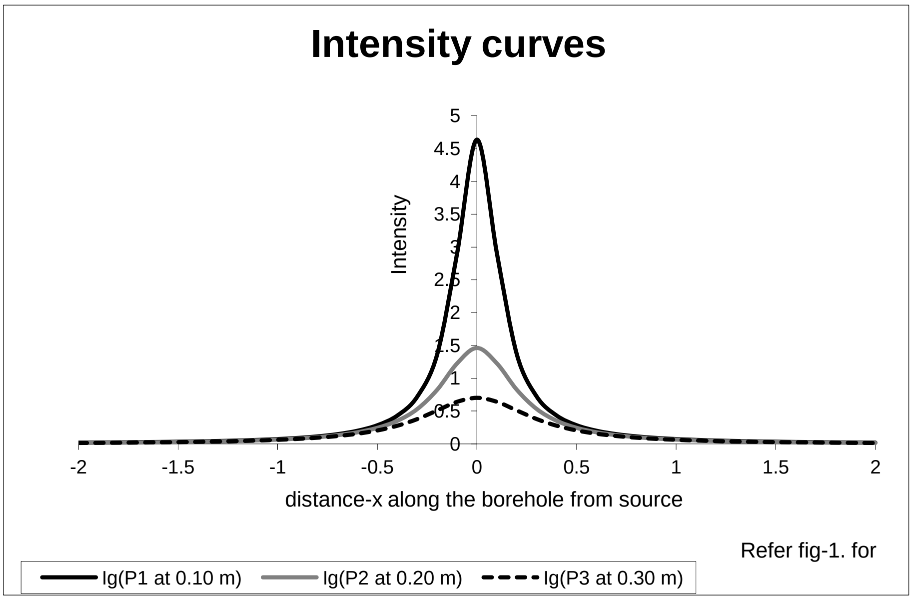

Intensity curves
Intensity
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
distance-x along the borehole from source
Refer fig-1. for
Ig(P1 at 0.10 m)
Ig(P2 at 0.20 m)
Ig(P3 at 0.30 m)

Fig 2

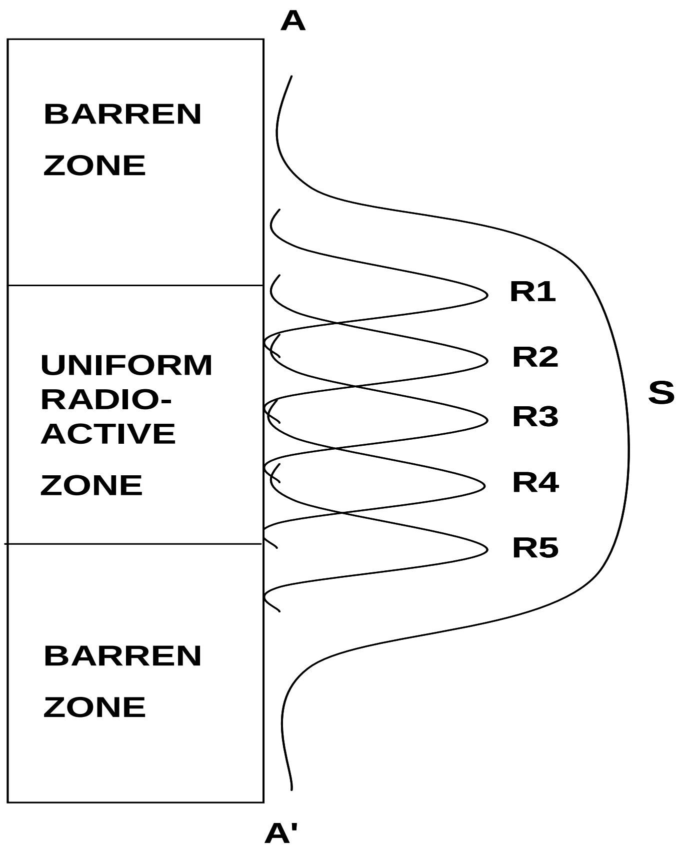

A
BARREN ZONE
UNIFORM RADIO-ACTIVE ZONE
BARREN ZONE
R1
R2
R3
R4
R5
S
A'

Fig-3

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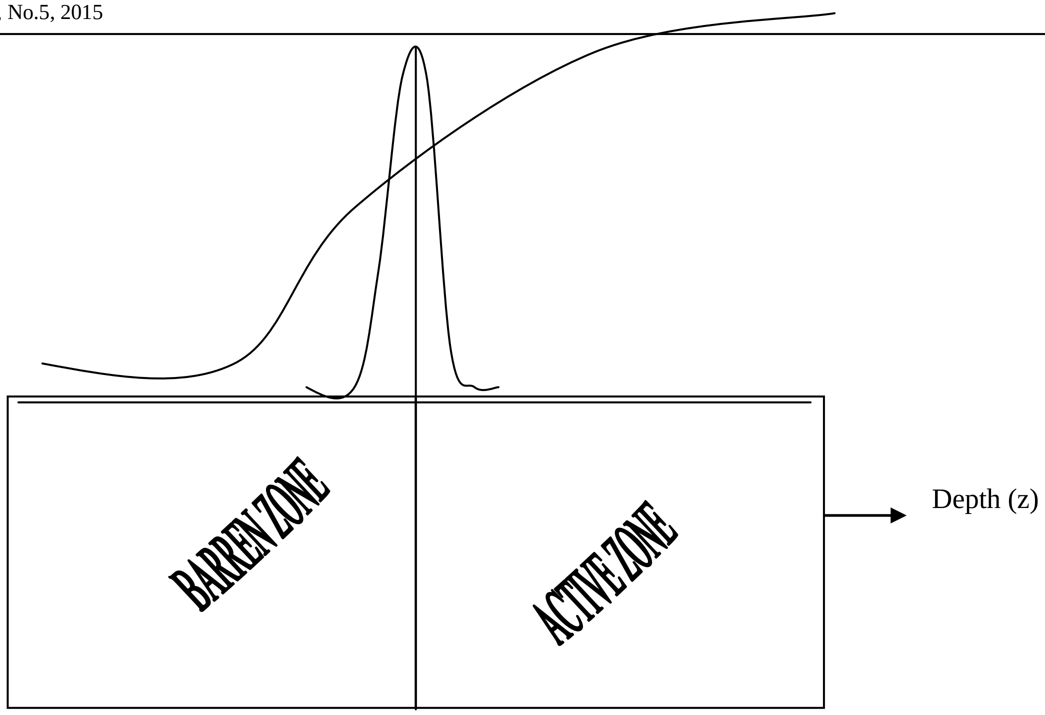
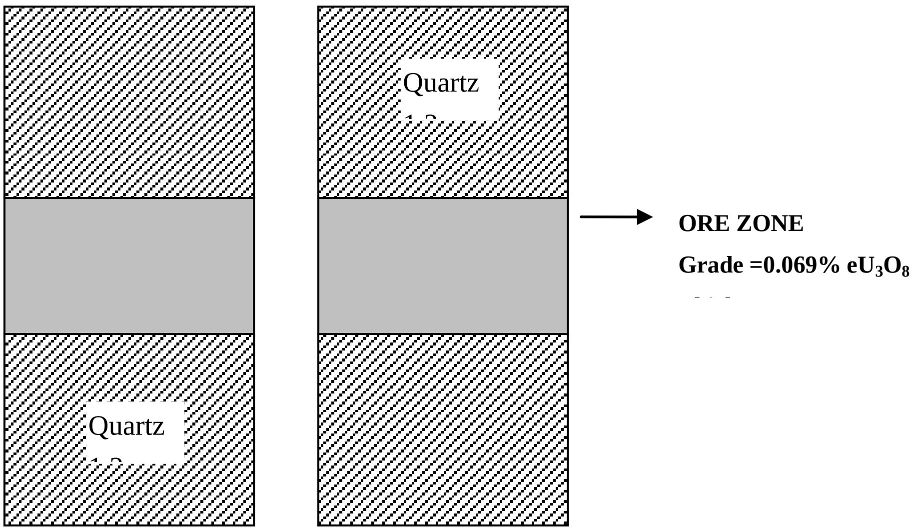


Fig-4



Standard Borehole

Fig-5

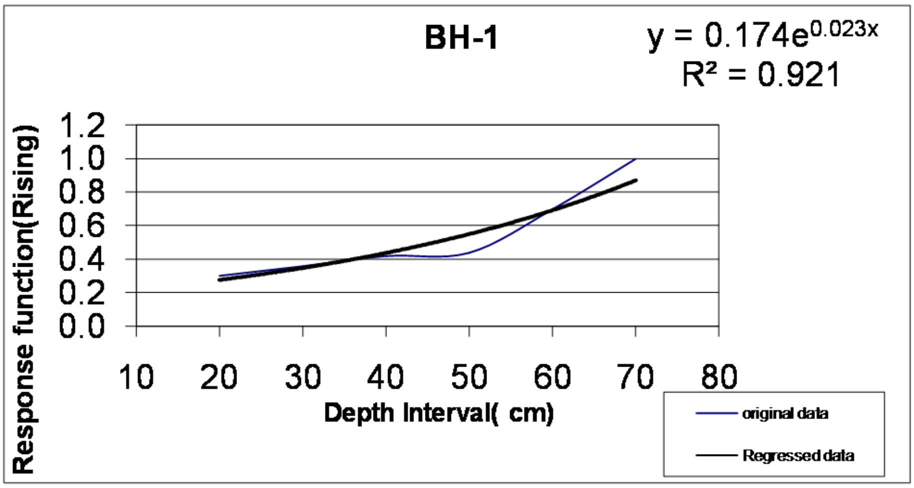

BH-1
y = 0.174e^0.023x
R² = 0.921
Response function(Rising)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
10 20 30 40 50 60 70 80
Depth Interval(cm)
original data
Regressed data

Fig 6

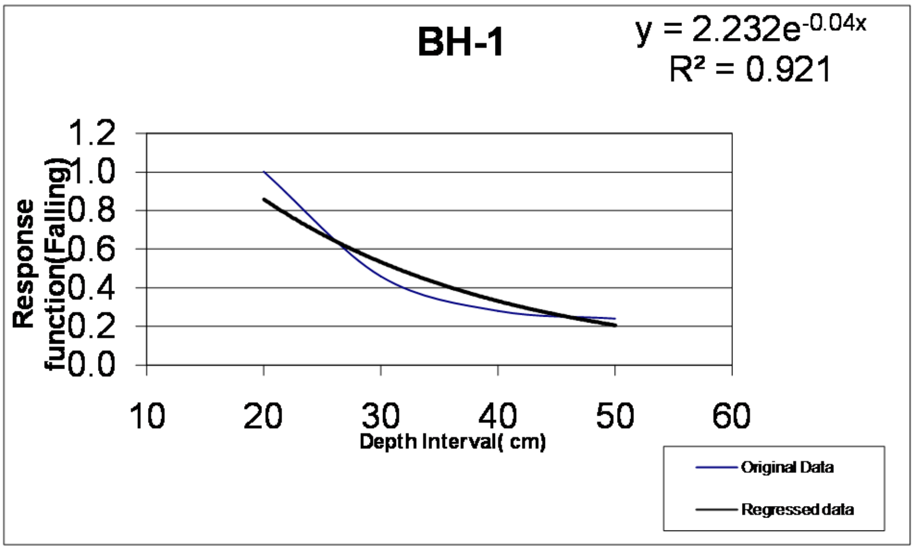

BH-1
y = 2.232e^-0.04x
R² = 0.921
Response function(Falling)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
10 20 30 40 50 60
Depth Interval(cm)
Original Data
Regressed data

Fig 7

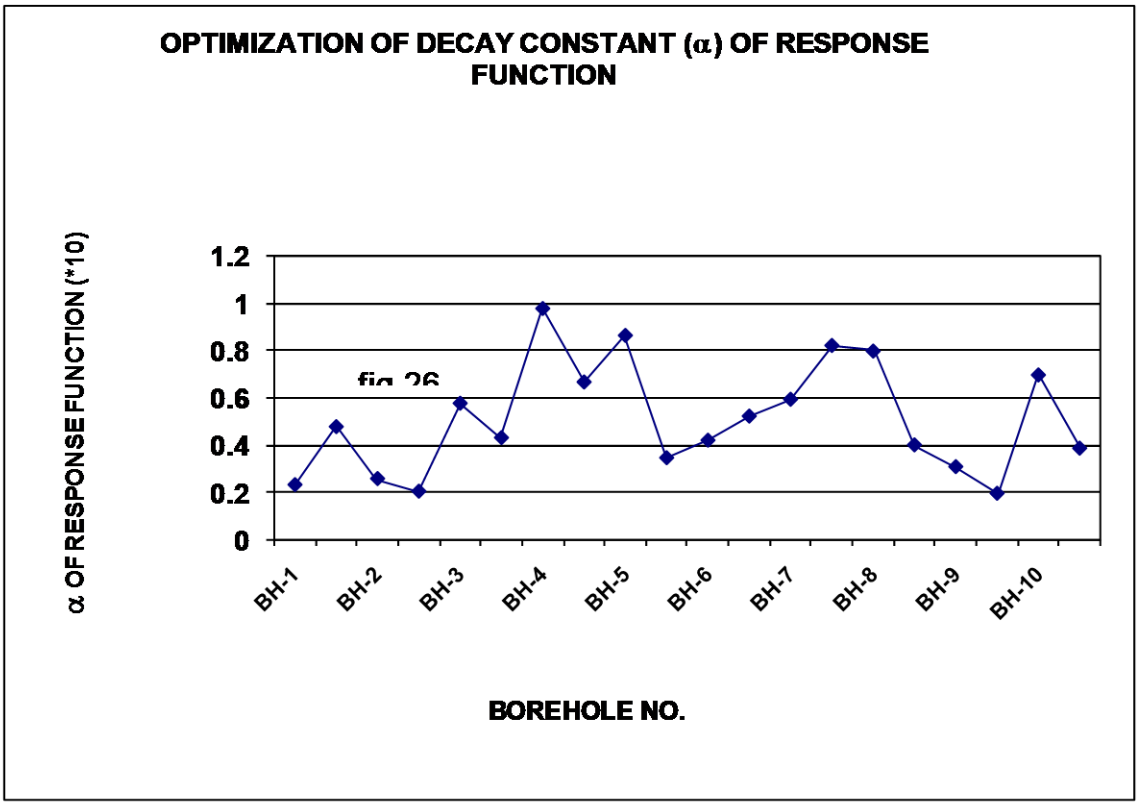

OPTIMIZATION OF DECAY CONSTANT (α) OF RESPONSE FUNCTION
α OF RESPONSE FUNCTION (*10)
1.2
1
0.8
0.6
0.4
0.2
0
fig 26
BH-1
BH-2
BH-3
BH-4
BH-5
BH-6
BH-7
BH-8
BH-9
BH-10
BOREHOLE NO.

Table 1. %eU_3O_8 values along the depth at 10 cm interval

BH-1	BH-2	BH-3	BH-4	BH-5	BH-6	BH-7	BH-8	BH-9	BH-9 Contd..	BH-10
0.015	0.010	0.011	0.014	0.010	0.013	0.010	0.010	0.011	0.015	0.011
0.014	0.011	0.026	0.020	0.020	0.018	0.015	0.020	0.014	0.016	0.017
0.016	0.011	0.052	0.060	0.054	0.023	0.018	0.036	0.040	0.015	0.040
0.015	0.014	**0.060**	0.240	0.140	0.031	0.033	0.087	0.044	0.014	**0.085**
0.014	0.015	**0.060**	**0.550**	**0.290**	0.054	0.084	0.280	0.040	0.018	**0.085**
0.017	0.014	0.040	0.300	0.190	0.110	0.230	**0.470**	0.065	0.014	0.042
0.028	0.019	0.032	0.120	0.054	0.150	**0.250**	0.360	0.090	0.015	0.021
0.027	0.020	0.016	0.044	0.031	0.200	0.170	0.210	**0.100**	0.014	0.017
0.022	0.018	0.011	0.019	0.032	**0.330**	0.071	0.120	0.095	0.012	0.013
0.020	0.022	0.013	0.015	0.031	0.320	0.023	0.110	0.060	0.022	0.012
0.020	0.020	0.015	0.011	0.028	0.290	0.011	0.063	0.050	0.020	0.012
0.022	0.018	0.016	0.014	0.017	0.150		0.034	0.045	0.014	0.014
0.025	0.018	0.016		0.014	0.048		0.024	0.042	0.015	0.012
0.022	0.016	0.019			0.022		0.020	0.044	0.012	0.012
0.030	0.018	0.028			0.020		0.013	0.040	0.013	0.021
0.015	0.022	0.036			0.016		0.010	0.040		0.030
0.018	0.038	0.025			0.019		0.010	0.030		0.025
0.021	0.028	0.012			0.016		0.011	0.032		0.016
0.022	0.024				0.013		0.010	0.026		0.010
0.035	0.032				0.015		0.012	0.028		
0.050	**0.040**				0.019		0.012	0.026		
0.023	0.032				0.011		0.010	0.024		
0.014	0.022							0.020		
0.012	0.020							0.025		
	0.018							0.030		
	0.014							0.028		
	0.016							0.020		
								0.022		