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# Results with Tripled Common Random Fixed points

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**Abstract**

In this paper, we present tripled coincidence point theorems for $F : X^3 \rightarrow X$ and $g : X \rightarrow X$ satisfying weak $(\mu, \varphi, \Psi)$contractions in partially ordered random metric spaces. Our results motivatedby Berinde and Borcut [22], Abbas et. al., Aydi et al. [23] Choudhary [30-32]. Gupta et. al.[33]

## 2. Introduction

Fixed point theorems are very important tools for providing evidence of the existence and uniqueness of solutions to various mathematical models. The literature of the last four decades flourishes with results which discover fixed points of self and nonself nonlinear operators in a metric space. The Banach contraction theorem plays a fundamental role in fixed point theory and has become even more important because being based on iteration, it can be easily implemented on a computer. This theorem provides a technique for solving a variety of applied problems in mathematical science and engineering. There are great number of generalizations of the Banach contraction principle. Bhaskar and Lakshmikantham [1] introduced the notion of coupled fixed point and prove some coupled fixed point results under certain conditions, in a complete metric space endowed with a partial order. Later, Lakshmikantham and Ciric [2] extended these results by defining the mixed g- monotone property. More accurately, they proved coupled coincidence and coupled common fixed point theorems for a mixed g- monotone mapping in a complete metric space endowed with partial order. Karapiner [3] generalized these results on a complete cone metric space endowed with a partial order. For other results on coupled fixed point theory, we refers [4 - 14].

Beside this, in [15] Alber and Guerre - Delabriere presented the generalization of Banach contraction principle by introducing the concept of weak contraction in Hilbert spaces. Rhoades [16] had shown the result of [15] is also valid in complete metric spaces. Khan et.al. [17] introduced the use of control function in metric fixed point problems. This function was referred to as 'Altering distance function' by the authors of [17]. This function and its extensions have been used in several problems of fixed point theory, some of them are noted in [18-21]. In recent times, fixed point theory has developed rapidly in partially ordered metric spaces, that is, in metric spaces endowed with a partial ordering . Using the control functions the weak contraction principle has been generalized in metric spaces [9] and in partially ordered metric spaces in [11].

Recently, Samet and Vetro [14] introduced the notion of fixed point of N- order, as natural extension of the coupled fixed point and established some new coupled fixed point theorems in complete metric spaces, using a new concept of F- invariant set. Later, Berinde and Borcut [22] obtained existence and uniqueness of triplet fixed point results in a complete metric space, endowed with a partial order.

3. **Privious Results:** Now we recall come privious known definitions and results which are as follows.

Again, let $(X, \leq)$ be a partially ordered set. The mapping $F: X^3 \rightarrow X$ is said to have the mixed monotone property if for any $x, y, z \in X$.

*i.* $x_1, x_2 \in X,\ x_1 \leq x_2 \Rightarrow F(x_1, y, z) \leq F(x_2, y, z)$ ,

*ii.* $y_1, y_2 \in X,\ y_1 \geq y_2 \Rightarrow F(x, y_1, z) \geq F(x, y_2, z)$ ,

*iii.* $z_1, z_2 \in X,\ z_1 \leq z_2 \Rightarrow F(x, y, z_1) \leq F(x, y, z_2)$

4. **Triplet fixed point** : An element $(x, y, z) \in X^3$ is called a triplet fixed point of F if $F(x, y, z) = x,\ F(y, x, y) = y,$ and $F(z, y, x) = z.$

Berinde and Borcut [22] proved the following theorem.

**5. Basic Theorems**

**Theorem 5.1.1:-** Let $(X, \leq)$ be a partially ordered set and (X,d) be a complete metric space. Let $F: X^3 \rightarrow X$ be a continuous mapping having the mixed monotone property on X. Assume that there exist constants $a, b, c \in [0,1)$ such that $a + b + c < 1$ for which,

$$d\big(F(x, y, z), F(u, v, w)\big) \leq a\, d(x, u) + b\, d(y, v)\ c\, d(z, w) \qquad 5.1.1$$

For all $x \geq u, y \leq v, z \geq w$. Assume either,

1. F is continuous,
2. X has the following properties:

- if non decreasing sequence $x_n \rightarrow x$, then $x_n \leq x$ for all n,
- if non increasing sequence $y_n \rightarrow y$, then $y_n \geq x$ for all n,

If there exist $x_0, y_0, z_0 \in X$ such that
$x_0 \leq F(x_0, y_0, z_0)$, $y_0 \geq F(y_0, x_0, y_0)$, and $z_0 \leq F(z_0, y_0, x_0)$
Then there exist $x, y, z \in X$ such that,

$$F(x,y,z) = x, F(y,x,y) = y, \text{and } F(z,y,x) = z$$

[Abbas, Aydi and Krapinar, Triplet fixed point in partially ordered metric spaces, submitted]. In this respect, let $(X, \leq)$ be a partially ordered set, $F: X^3 \to X$ and $g: X \to X$ two mappings. The mapping F is said to have the mixed g - monotone property if for any x, y, $z \to X$.

i. $x_1, x_2 \in X$, $gx_1 \leq gx_2 \Rightarrow F(x_1, y, z) \leq F(x_2, y, z)$ ,
ii. $y_1, y_2 \in X$, $gy_1 \geq gy_2 \Rightarrow F(x, y_1, z) \geq F(x, y_2, z)$ ,
iii. $z_1, z_2 \in X$, $gz_1 \leq gz_2 \Rightarrow F(x, y, z_1) \leq F(x, y, z_2)$

An element $(x, y, z) \in X^3$ is called a triplet coincidence point of F and g if

$$F(x,y,z) = gx, \ F(y,x,y) = gy, \text{ and } F(z,y,x) = gz.$$

while $(gx, gy, gz)$ is said a triplet point of coincidence of mappings F and g. Moreover $(x, y, z)$ is called a triplet common fixed point of F and g if

$$F(x,y,z) = gx, \ F(y,x,y) = gy, \text{ and } F(z,y,x) = gz.$$

At last mappings F and g are called commutative if

$$g\big(F(x,y,z)\big) = F(gx, gy, gz), \ \forall\, x, y, z \in X$$

In the same paper, they proved the following result.

**Theorem 5.1.2:-** Let $(X, \leq)$ be a partially ordered set and $(X, d)$ be a complete metric space. Assume there is a function $\varphi: [0, \infty) \to [0, \infty)$ such that $\varphi(t) < t$ for each $t > 0$. Also suppose that $F: X^3 \to X$ and $g: X \to X$ are such that F having the mixed g $-$ monotone property on X. Assume that there exist constants $a, b, c \in [0,1)$ such that $a + 2b + c < 1$ such that,

$$d\big(F(x,y,z), F(u,v,w)\big) \leq \varphi\big(a\, d(gx,gu) + b\, d(gy,gv) + c\, d(gz,gw)\big) \qquad 1.2$$

for all $gx \geq gu, gy \leq gv, gz \geq gw$.
Suppose $(X^3) \subset g(X)$ , g is continuous and commutes with F. Suppose either,

1. F is continuous,
2. X has the following properties:

- if non decreasing sequence $gx_n \to x$, then $gx_n \leq x$ for all n,
- if non increasing sequence $gy_n \to y$, then $gy_n \geq y$ for all n,

If there exist $x_0, y_0, z_0 \in X$ such that

$$gx_0 \leq F(x_0, y_0, z_0), \ gy_0 \geq F(y_0, x_0, y_0), \ and \ gz_0 \leq F(z_0, y_0, x_0) .$$

Then there exist $x, y, z \in X$ such that,

$$F(x,y,z) = gx, F(y,x,y) = gy, and\, F(z,y,x) = gz$$

that is, F and g have a triplet coincidence point.

In [23] Aydi et.al. Prove the following theorem

**Theorem5.1.3:-** Let $(X, \leq)$ be a partially ordered set and $(X, d)$ be a complete metric space. Assume there is a function $\varphi: [0, \infty) \to [0, \infty)$ such that $\varphi(t) < t$ for each $t > 0$. Also suppose that $F: X^3 \to X$ and $g: X \to X$ are such that F having the mixed g - monotone property on X. Assume that there exist constants $a, b, c \in [0,1)$ such that $a + 2b + c < 1$ such that,

$$d\big(F(x,y,z), F(u,v,w)\big) + d\big(F(y,x,y), F(v,u,v)\big) + d\big(F(z,y,x), F(w,v,u)\big) \leq 3\,\varphi\left(\frac{d(gx,gu) + d(gy,gv) + d(gz,gw)}{3}\right) \qquad 1.3$$

For all $gx \geq gu, gy \leq gv, gz \geq gw$.
Suppose $F(X^3) \subset g(X)$ , g is continuous and commutes with F. Suppose either,

1. F is continuous,
2. X has the following properties:

- if non decreasing sequence $gx_n \to x$, then $gx_n \leq x$ for all $n$,
- if non increasing sequence $gy_n \to y$, then $gy_n \geq y$ for all $n$,

If there exist $x_0, y_0, z_0 \in X$ such that

$$gx_0 \leq F(x_0, y_0, z_0), \ gy_0 \geq F(y_0, x_0, y_0), \ and \ gz_0 \leq F(z_0, y_0, x_0)$$

Then there exist $x, y, z \in X$ such that,

$$F(x,y,z) = gx, F(y,x,y) = gy, and\, F(z,y,x) = gz$$

that is, F and g have a triplet coincidence point.

Probabilistic functional analysis has emerged as one of the important mathematical disciplines in view of its role in analyzing probabilistic models in the applied sciences. The study of fixed points of random operators forms a central topic in this area. The Prague school of probabilistic initiated its study in the 1950. However, the research in this area flourished after the publication of the survey article of Bharucha-Reid [10]. Since then many interesting random fixed point results and several applications have appeared in the literature.

**5.2 Main Results**

First we give some definitions, which are use to prove of the main theorem.

**Definition 5.2.1 :-** Let $\Phi$ denote all functions $\varphi\colon[0,\infty) \to [0,\infty)$ which satisfy

i. $\varphi$ is continuous and non decreasing,

ii. $\varphi(t) = 0$ iff t = 0,

iii. $\varphi(r+s+t) \le \varphi(r)+\varphi(s)+\varphi(t)\ \forall\ r,s,t \in [0,\infty)$

For example, functions $\varphi_1(t) = kt$ where $k > 0, \varphi_2(t) = \frac{t}{t+1}, \varphi_3(t) = In(t+1)$, and $\varphi_4(t) = min\{t,1\}$ are in $\Phi$.

**Definition 5.2.2:-** Let $\Psi$ be the set of all functions $\psi : [0,\infty) \to [0,\infty)$ which satisfy $lim_{t\to q}\psi(t) > 0$ for all $q > 0$ and $lim_{t\to 0}\ \psi(t) = 0$

For example, functions $\psi_1(t) = kt$ where $k > 0$, $\psi_2(t) = \frac{ln(2t+1)}{2}$ are in $\Psi$.

Throughout this paper $(\Omega,\Sigma)$ denotes a measurable space, X be a partially ordered metric space.and C is non empty subset of X.

**Definition 5.2.2(a):** A function $f : \Omega \to$ C is said to be measurable if $f'(B \cap C) \in \Sigma$ for every Borel subset B of X.

**Definition 5.2.2 (b):** A function $f\colon\Omega \times C \to C$ is said to be random operator, if $f(.,X)\colon\Omega \to C$ is measurable for every$X \in C$.

**Definition5.2.2 (c):** A random operator $f\colon\Omega\times C \to C$ is said to be continuous if for fixed $t \in \Omega, f(t,.)\colon C \times C$ is continuous.

**Definition 5.2.2 (d):** A measurable function $g : \Omega \to C$ is said to be random fixed point of the random operator $f\colon\Omega\times C \to C$, if

$f(t,g(t)) = g(t), \forall\, t \in \Omega$.

In this paper, we present tripled coincidence point theorems for F : $X^3 \to$ X and g : X → X satisfying weak $(\mu,\varphi,\Psi)$contractions in partially ordered random metric spaces. Our results motivatedby Berinde and Borcut [22], Abbas et. al., Aydi et al. [23] Choudhary [30-32]. Gupta et. al.[33]

Now we prove our main results.

**Theorem 5.2.3:-** Let $(X,\le)$ be a partially ordered set and (X,d) be a complete random metric space. Let $F : X^3 \to X$ be a continuous mapping having the mixed g - monotone property on X and $F(X^3) \subset g(X)$ . $\xi\ \varepsilon\ \Omega, be\ a\ measerable\ sellector$ .Suppose there exist $\mu,\varphi \in \Phi$ , $\psi \in \Psi$ for which,

$$\mu\left(d\left(F(\xi,(x(\xi),y(\xi),z(\xi))),F(\xi,(u(\xi),v(\xi),w(\xi)))\right)\right)$$
$$\le \frac{1}{3}\varphi\big(d(gx(\xi),gu(\xi)) + d(gy(\xi),gv(\xi)) + d(gz(\xi),gw(\xi))\big)$$
$$-\frac{1}{3}\psi\begin{pmatrix} d(gx(\xi),gu(\xi)) + d(gy(\xi),gv(\xi)) \\ + d(gz(\xi),gw(\xi))\end{pmatrix} \qquad (5.2.3.1)$$

For all $gx(\xi) \ge gu(\xi),\ gy(\xi) \le gv(\xi)\ and\ gz(\xi) \ge gw(\xi)$.

Assume that F is continuous; g is continuous and commutes with F.

If there exist $x_0(\xi),y_0(\xi),z_0(\xi) \in X$ such that

$$gx_0(\xi) \le F(\xi,(x_0(\xi),y_0(\xi),z_0(\xi))),\ gy_0(\xi) \ge F(\xi,(y_0(\xi),x_0(\xi),y_0(\xi)),$$
$$and\ gz_0(\xi) \le F(\xi,(z_0(\xi),y_0(\xi),x_0(\xi))$$

Then there exist $x(\xi),y(\xi),z(\xi) \in X$ such that,

$$F\big(\xi,(x(\xi),y(\xi),z(\xi))\big) = gx(\xi),$$
$$F(\xi,(y(\xi),x(\xi),y(\xi))) = gy(\xi), and\ F(\xi,(z(\xi),y(\xi),x(\xi))) = gz(\xi).$$

That is, F and g have a triplet coincidence point.

**Proof:** Let $x_0(\xi)\,,y_0(\xi),z_0(\xi) \in X$ such that

$$gx_0(\xi) \le F\big(\xi,(x_0(\xi),y_0(\xi),z_0(\xi))\big),$$
$$gy_0(\xi) \ge F(\xi,(y_{0(\xi)},x_0(\xi),y_0(\xi)),$$
$$and\ gz_0(\xi) \le F(\xi,(z_0(\xi),y_0(\xi),x_0(\xi))$$

We can choose $x_1(\xi)\,,y_1(\xi),z_1(\xi) \in X$ such that

$$gx_1(\xi) = F\big(\xi,(x_0(\xi),y_0(\xi),z_0(\xi))\big),$$
$$gy_1(\xi) = F(\xi,(y_0(\xi),x_0(\xi),y_0(\xi))),$$
$$and\ gz_1(\xi) = F(\xi,(z_0(\xi),y_0(\xi),x_0(\xi))) \qquad (5.2.3.2)$$

This can be done because $F(X^3) \subset g(X)$. Continuing this process, we construct a sequence $\{x_n(\xi)\}$, $\{y_n(\xi)\}$ $and$ $\{z_n(\xi)\}$ in X such that

$$gx_{n+1}(\xi) = F(\xi,(x_n(\xi),y_n(\xi),z_n(\xi)),$$
$$gy_{n+1}(\xi) = F(\xi,(y_n(\xi),x_n(\xi),y_n(\xi))),$$

$$and\ gz_{n+1}(\xi) = F(\xi,(z_n(\xi),y_n(\xi),x_n(\xi))) \quad (5.2.3.3)$$

By induction, we will prove that

$$gx_n(\xi) \le gx_{n+1}(\xi),\ gy_n(\xi) \ge gy_{n+1}(\xi)$$
$$and\ gz_n(\xi) \le gz_{n+1}(\xi) \quad (5.2.3.4)$$

Since,

$$gx_0(\xi) \le F(\xi,(x_0(\xi),y_0(\xi),z_0(\xi))),\ gy_0(\xi) \ge F(\xi,(y_0(\xi),x_0(\xi),y_0(\xi))),$$
$$and\ gz_0(\xi) \le F(z_0(\xi),y_0(\xi),x_0(\xi))) \quad (5.2.3.5)$$

Therefore by ((5.2.3.2)) we have

$$gx_0(\xi) \le gx_1(\xi),\ gy_0(\xi) \ge gy_1(\xi)\ and\ gz_0(\xi) \le gz_1(\xi)$$

Thus (5.2.3.4) is true for n = 0. We suppose that (5.2.3.4) is true for some $n > 0$.
Since F has the mixed g - monotone property, by (5.2.3.4)we have that

$$gx_{n+1}(\xi) = F(\xi,(x_n(\xi),y_n(\xi),z_n(\xi))) \le F(\xi,(x_{n+1}(\xi),y_n(\xi),z_n(\xi))$$
$$\le F(\xi,(x_{n+1}(\xi),y(\xi)_n,z_{n+1}(\xi))$$
$$\le F(\xi,(x_{n+1}(\xi),y_{n+1}(\xi),z_{n+1}(\xi))) = gx_{n+2}(\xi)$$
$$gy_{n+2}(\xi) = F(\xi,(y_{n+1}(\xi),x_{n+1}(\xi),y_{n+1}(\xi)))$$
$$\ge F(\xi,(y_{n+1}(\xi),x_n(\xi),y_{n+1}(\xi))$$
$$\ge F(\xi,(y_n(\xi),x_n(\xi),y_{n+1}(\xi))$$
$$\ge F(\xi,(y_n(\xi),x_n(\xi),y_n(\xi)) = gy_{n+1}(\xi)$$

and

$$gz_{n+1}(\xi) = F(\xi,(z_n(\xi),y_n(\xi),x_n(\xi))) \le F(\xi,(z_{n+1}(\xi),y_n(\xi),x_n(\xi)))$$
$$\le F(\xi,(z_{n+1},y_{n+1},x_n)$$
$$\le F(\xi,(z_{n+1}(\xi),y_{n+1}(\xi),x_{n+1}(\xi))) = gz_{n+2}(\xi)$$

That is (5.2.3.4)is true for any $n \in N$. If for some $k \in N$,

$$gx_k(\xi) = gx_{k+1}(\xi),\ gy_k(\xi) = gy_{k+1}(\xi)\ and\ gz_k(\xi) = gz_{k+1}(\xi)$$

then, by(5.2.3.4) $(x_k(\xi),y_k(\xi),z_k(\xi))$ is a triplet coincidence point of F and g. Now we assume that at least

$$gx_n(\xi) \ne gx_{n+1}(\xi),\ gy_n(\xi) \ne gy_{n+1}(\xi) and$$
$$gz_n(\xi) \ne gz_{n+1}(\xi)$$

for any $n \in N$. From (5.2.3.4)and the inequality (5.2.3.1)we have

$$d(gx_{n+1}(\xi),gx_n(\xi))$$
$$= d(F(\xi,(x_n(\xi),y_n(\xi),z_n(\xi))),F(\xi,(x_{n-1}(\xi),y_{n-1}(\xi),z_{n-1}(\xi)))$$
$$\mu\left(d(F(\xi,(x_n(\xi),y_n(\xi),z_n(\xi)),F(\xi,(x_{n-1}(\xi),y_{n-1}(\xi),z_{n-1})(\xi)))\right)$$
$$\le \frac{1}{3}\varphi\left(d(gx_n(\xi),gx_{n-1}(\xi)) + d(gy_n(\xi),gy_{n-1}(\xi)) + d(gz_n(\xi),gz_{n-1}(\xi))\right)$$
$$-\frac{1}{3}\psi\left(d(gx_n(\xi),gx_{n-1}(\xi)) + d(gy_n(\xi),gy_{n-1}(\xi)) + d(gz_n(\xi),gz_{n-1}(\xi))\right)$$
$$\mu\left(d(gx_{n+1}(\xi),gx_n(\xi))\right)$$
$$\le \frac{1}{3}\varphi\left(d(gx_n(\xi),gx_{n-1}(\xi)) + d(gy_n(\xi),gy_{n-1}(\xi)) + d(gz_n(\xi),gz_{n-1}(\xi))\right)$$
$$-\frac{1}{3}\psi\left(d(gx_n(\xi),gx_{n-1}(\xi)) + d(gy_n(\xi),gy_{n-1}(\xi)) + d(gz_n(\xi),gz_{n-1}(\xi))\right) \quad (5.2.3.6)$$

Similarly we get

$$\mu\left(d(gy_{n+1}(\xi),gy_n(\xi))\right)$$
$$\le \frac{1}{3}\varphi\left(d(gx_n(\xi),gx_{n-1}(\xi)) + d(gy_n(\xi),gy_{n-1}(\xi)) + d(gy_n(\xi),gy_{n-1}(\xi))\right)$$
$$-\frac{1}{3}\psi\left(d(gx_n(\xi),gx_{n-1}(\xi)) + d(gy_n(\xi),gy_{n-1}(\xi)) + d(gy_n(\xi),gy_{n-1}(\xi))\right) \quad (5.2.3.7)$$
$$\mu\left(d(gz_{n+1},gz_n)\right)$$
$$\le \frac{1}{3}\varphi\left(d(gx_n(\xi),gx_{n-1}(\xi)) + d(gy_n(\xi),gy_{n-1}(\xi)) + d(gz_n(\xi),gz_{n-1}(\xi))\right)$$
$$-\frac{1}{3}\psi\left(d(gx_n(\xi),gx_{n-1}(\xi)) + d(gy_n(\xi),gy_{n-1}(\xi)) + d(gz_n(\xi),gz_{n-1}(\xi))\right) \quad (5.2.3.8)$$

For each $n \ge 1$.
By adding (2.6), (2.7) and (2.8) and from the property of $\mu$ we get

$$\mu\left(H(\xi,(x_n(\xi),y_n(\xi),z_n(\xi))\right)$$
$$\le \varphi\left(H(\xi,(x_n(\xi),y_n(\xi),z_n(\xi))\right) - \psi\left(H(\xi,(x_{n-1}(\xi),y_{n-1}(\xi),z_{n-1}(\xi))\right) \quad 2.9$$

where

$$H(\xi,(x_n(\xi),y_n(\xi),z_n(\xi)) = d\left(g(x_n),g(x_{n+1}(\xi))\right) + d\left(g(y_n(\xi)),g(y_{n+1}(\xi))\right) +$$
$$d\left(g(z_n(\xi)),g(z_{n+1})\right)$$

or

$$H(\xi,(x_n(\xi),y_n(\xi),z_n(\xi))$$
$$= d\left((\xi,F(x_n(\xi),y_n(\xi),z_n(\xi)),F(\xi,(x_{n-1}(\xi),y_{n-1}(\xi),z_{n-1}(\xi))\right)$$

$$+ d\big(F(\xi,(y_n(\xi),x_n(\xi),y_n(\xi))),F(\xi,\big(y_{n-1}(\xi),x_{n-1}(\xi),y_{n-1}(\xi)\big))\big)$$
$$+ d(F(\xi,(z_n(\xi),y_n(\xi),x_n(\xi))),F(\xi,(z_{n-1}(\xi),y_{n-1}(\xi),x_{n-1}(\xi)))$$

Using the fact of $\mu,\varphi$ are non decreasing, we get

$$H(\xi,(x_n(\xi),y_n(\xi),z_n(\xi)) \le H(\xi,(x_{n-1}(\xi),y_{n-1}(\xi),z_{n-1}(\xi))$$

We set,

$$\delta_n = H(\xi,(x_{n-1}(\xi),y_{n-1}(\xi),z_{n-1}(\xi)) = d\big(gx_n(\xi),gx_{n-1}(\xi)\big) + d\big(gy_n(\xi),gy_{n-1}(\xi)\big) + d(gz_n(\xi),gz_{n-1}(\xi)) \quad (5.2.3.10)$$

Then the sequence $\{\delta_n\}$ is decreasing. Therefore, there is some $\delta \ge 0$ such that

$$lim_{n\to\infty}\, \delta_n = lim_{n\to\infty}\, \big( d(gx_n(\xi),gx_{n-1}(\xi)) + d(gy_n(\xi),gy_{n-1}(\xi)) + d(gz_n(\xi),gz_{n-1}(\xi))\big) = \delta \quad (5.2.3.11)$$

We shall show that $\delta = 0$. Suppose, to the contrary, that $\delta > 0$. Then taking the limit as $n \to \infty$ of both sides and have in mind that we suppose $lim_{n\to q}\, \psi(t) > 0$ for all $q > 0$ and $\mu,\varphi$ are continuous, we have

$$\mu(\delta) = lim_{n\to\infty}\, \varphi(\delta_n)$$
$$lim_{n\to\infty}\, \mu(\delta_n) \le lim_{n\to\infty}\, \big(\varphi(\delta_{n-1}) - \psi(\delta_{n-1})\big) \le \mu(\delta)$$

a contradiction. Thus $\delta = 0$, that is

$$lim_{n\to\infty}\, \delta_n = lim_{n\to\infty}\, \Big( d\big(gx_n(\xi),gx_{n-1}(\xi)\big) + d\big(gy_n(\xi),gy_{n-1}(\xi)\big) + d\big(gz_n(\xi),gz_{n-1}(\xi)\big)\Big) = 0 \quad (5.2.3.12)$$

In what follows, we shall prove that $\{gx_n(\xi)\},\{gy_n(\xi)\}$ *and* $\{gz_n(\xi)\}$ are Cauchy sequences. Suppose, to the contrary, that atleast one of $\{gx_n(\xi)\}\,\{gy_n(\xi)\},\{gz_n(\xi)\}$ in not Cauchy sequence. Then there exists an $\epsilon > 0$ for which we can find subsequence $\{gx_{n(k)}(\xi)\}$ ,$\{gx_{m(k)}(\xi)\}$ of $\{gx_n(\xi)\}$ and $\{gy_{n(k)}(\xi)\},\{gy_{m(k)}(\xi)\}$ of $\{gy_n(\xi)\}$
and $\{gz_{n(k)}(\xi)\},\{gz_{m(k)}(\xi)\}$of $\{gz_n(\xi)\}$ *with* $n(k) > m(k) \ge k$ such that

$$d\big(gx_{n(k)}(\xi),gx_{m(k)}(\xi)\big) + d\big(gy_{n(k)}(\xi),gy_{m(k)}(\xi)\big) + d\big(gz_{n(k)}(\xi),gz_{m(k)}(\xi)\big) \ge \epsilon \quad (5.2.3.13)$$

Additionally correspondence to $m(k)$. we may choose $n(k)$ such that it is the smallest integer satisfying (5.2.3.13) and $n(k) > m(k) \ge k$. Thus

$$d\big(gx_{n(k)-1}(\xi),gx_{m(k)}(\xi)\big) + d\big(gy_{n(k)-1}(\xi),gy_{m(k)}(\xi)\big) + d\big(gz_{n(k)-1}(\xi),gz_{m(k)}(\xi)\big) < \epsilon \quad (5.2.3.14)$$

By using triangle inequality and (2.13) and (2.14)

$$\begin{aligned}\epsilon \le p_k(\xi) &= d\Big(gx_{n(k)}(\xi),gx_{m(k)}(\xi)\Big) + d\big(gy_{n(k)}(\xi),gy_{m(k)}(\xi)\big) + d\big(gz_{n(k)}(\xi),gz_{m(k)}(\xi)\big)\\ &\le d\big(gx_{n(k)}(\xi),gx_{n(k)-1}(\xi)\big) + d\big(gy_{n(k)-1}(\xi),gy_{m(k)}(\xi)\big) + d\big(gz_{n(k)}(\xi),gz_{n(k)-1}(\xi)\big)\\ &\quad + d\big(gy_{n(k)-1(\xi)},gy_{m(k)}(\xi)\big) + d\big(gz_{n(k)}(\xi),gz_{n(k)-1}(\xi)\big) + d\big(gz_{n(k)-1}(\xi),gz_{m(k)}(\xi)\big)\\ &< d\big(gx_{n(k)},gx_{n(k)-1}(\xi)\big) + d\big(gy_{n(k)}(\xi),gy_{n(k)-1}(\xi)\big) + d\big(gz_{n(k)}(\xi),gz_{n(k)-1}(\xi)\big) + \epsilon \quad (5.2.3.15)\end{aligned}$$

letting $k \to \infty$ in 5.2.3.15) and using 5.2.3.12)

$$lim_{k\to\infty}\, p_k(\xi) = lim_{k\to\infty} \left\{\begin{matrix} d\Big(gx_{n(k)}(\xi),gx_{m(k)}(\xi)\Big) \\ + d\Big(gy_{n(k)}(\xi),gy_{m(k)}(\xi)\Big) \\ + d\Big(gz_{n(k)}(\xi),gz_{m(k)}(\xi)\Big)\end{matrix}\right\}$$

$$lim_{k\to\infty} p_k(\xi) = \epsilon \quad 5.2.3.16)$$

Again by triangular inequality,

$$\begin{aligned}p_k(\xi) = {} & d\Big(gx_{n(k)}(\xi),gx_{m(k)}(\xi)\Big) + d\Big(gy_{n(k)}(\xi),gy_{m(k)}(\xi)\Big) + d\Big(gz_{n(k)}(\xi),gz_{m(k)}(\xi)\Big)\\ \le {} & d\Big(gx_{n(k)}(\xi),gx_{n(k)+1}(\xi)\Big) + d\Big(gx_{n(k)+1}(\xi),gx_{m(k)+1}(\xi)\Big) + d\Big(gx_{m(k)+1}(\xi),gx_{m(k)}(\xi)\Big)\\ & d\Big(gy_{n(k)}(\xi),gy_{n(k)+1}(\xi)\Big) + d\Big(gy_{n(k)+1}(\xi),gy_{m(k)+1}(\xi)\Big) + d\Big(gy_{m(k)+1}(\xi),gy_{m(k)}(\xi)\Big)\end{aligned}$$

$$d\left(gz_{n(k)}(\xi), gz_{n(k)+1}(\xi)\right) + d\left(gz_{n(k)+1}, gz_{m(k)+1}\right) + d\left(gz_{m(k)+1}(\xi), gz_{m(k)}(\xi)\right) \le \delta_{n(k)+1} + \delta_{m(k)+1} + d\left(gx_{n(k)+1}(\xi), gx_{m(k)+1}(\xi)\right) + d\left(gy_{n(k)+1}, gy_{m(k)+1}\right) + d\left(gz_{n(k)+1}, gz_{m(k)+1}(\xi)\right) \quad 2.17$$

Since $n(k) > m(k)$, then

$$gx_{n(k)}(\xi) \ge gx_{m(k)}(\xi), \quad gy_{n(k)}(\xi) \le gy_{m(k)}(\xi),$$
$$gz_{n(k)}(\xi) \ge gz_{m(k)}(\xi)$$

(5.2.3.17)Take (5.2.3.17)in (5.2.3.1)to get,

$$\left\{\begin{array}{l} d\left(gx_{n(k)+1}(\xi), gx_{m(k)+1}(\xi)\right) \\ + d\left(gy_{n(k)+1}(\xi), gy_{m(k)+1}(\xi)\right) \\ + d\left(gz_{n(k)+1}(\xi), gz_{m(k)+1}(\xi)\right) \end{array}\right\}$$

$$= \left\{\begin{array}{c} d\left(F(\xi,(x_{n(k)(\xi)}, y_{n(k)}(\xi), z_{n(k)}(\xi))), F(\xi,\left(x_{m(k)(\xi)}, y_{m(k(\xi))}, z_{m(k)}(\xi)\right))\right) \\ + d\left(F(\xi,(y_{n(k)}(\xi), x_{n(k)}(\xi), y_{n(k)}(\xi)))), F(\xi,(y_{m(k)}(\xi), x_{m(k)}(\xi), y_{m(k)}(\xi)))\right) \\ + d\left(F(\xi,(z_{n(k)}, y_{n(k)}(\xi), x_{n(k)}(\xi))), F(\xi,(z_{m(k)}(\xi), y_{m(k)}(\xi), x_{m(k)}(\xi)))\right) \end{array}\right\}$$

Using the property of φ we get,

$$\mu\left(p_k(\xi)\right) \le \varphi\left(\delta_{n(k)+1}(\xi)\right) + \varphi\left(\delta_{m(k)+1}(\xi)\right) + \varphi(p_k(\xi)) - \psi(p_k(\xi))$$

Letting $k \to \infty$ and using (5.2.3.10)and (5.2.3.14)we get

$$\mu(\epsilon) \le \varphi(0) + \varphi(\epsilon) - lim_{k\to\infty} \psi(p_k(\xi)) < \varphi(\epsilon)$$

This is contradiction. This shows that $\{gx_n(\xi)\}, \{gy_n(\xi)\} and \{gz_n(\xi)\}$are Cauchy sequences. Since X is a complete metric space, there exist $x, y, z \in X$ such that

$$lim_{n\to\infty}\{gx_n(\xi)\} = x(\xi), \quad lim_{n\to\infty}\{gy_n(\xi)\} = y(\xi),$$
$$lim_{n\to\infty}\{gz_n(\xi)\} = z(\xi) \quad (5.2.3.18)$$

From (5.2.3.18)and the continuity of g,

$$lim_{n\to\infty}\{g(gx_n(\xi))\} = gx(\xi)$$
$$, im_{n\to\infty}\{g(gy_n(\xi))\} = gy,$$
$$lim_{n\to\infty}\{g(gz_n(\xi))\} = gz(\xi) \quad (5.2.3.19)$$

From the commutativity of F and g, we have

$$g(gx_{n+1}(\xi)) = g\left(F(\xi,(x_n(\xi), y_n(\xi), z_n(\xi))\right)$$
$$= F(\xi,(gx_n(\xi), gy_n(\xi), gz_n(\xi))$$
$$g(gy_{n+1}) = g\left(F\left(\xi,(y_n(\xi), x_n(\xi), y_n(\xi))\right)\right)$$
$$= F(\xi,(gy_n(\xi), gx_n(\xi), gy_n(\xi))$$
$$g(gz_{n+1}(\xi)) = g\left(F\left(\xi,(z_n(\xi), y_n(\xi), x_n(\xi))\right)\right)$$
$$= F(\xi,(gz_n(\xi), gy_n(\xi), gx_n(\xi))) \quad (5.2.3.20)$$

Now we shall show that

$$gx(\xi) = F(\xi,(x(\xi), y(\xi), z(\xi)), \quad gy(\xi) = F(\xi,(y(\xi), x(\xi), y(\xi)),$$
$$and \quad gz = F(\xi,(z(\xi), y(\xi), x(\xi)))$$

Suppose that F is continuous. Letting $n \to \infty$ in (5.2.3.20), therefore we obtain

$$gx(\xi) = lim_{n\to\infty}\{g(gx_n(\xi))\}$$
$$= lim_{n\to\infty}F(\xi,(gx_n(\xi), gy_n(\xi), gz_n(\xi))) = F(\xi,(x(\xi), y(\xi), z(\xi)))$$
$$gy(\xi) = lim_{n\to\infty}\{g(gy_n(\xi))\}$$
$$= lim_{n\to\infty}F(\xi,(gy_n(\xi), gx_n(\xi), gy_n(\xi)) = F(\xi,(y(\xi), x(\xi), y(\xi)))$$
$$gz(\xi) = lim_{n\to\infty}\{g(gz_n(\xi))\}$$
$$= lim_{n\to\infty}F(\xi,(gz_n(\xi), gy_n(\xi), gy_n(\xi)))$$
$$= F(\xi,(z(\xi), y(\xi), x(\xi))) \quad (5.2.3.21)$$

We have proved that F and g have a tripled coincidence point.

**Corollary5.2.4:-** Let $(X, \le)$ be a partially ordered set and (X, d) be a complete metric space. Let $F : X^3 \to X$ be a continuous mapping having the mixed g - monotone property on X and $(X^3) \subset g(X)$ . Suppose there exist $\alpha \in [0,1)$ $\xi \, \varepsilon \, \Omega, be\ a\ measerable\ sellector,$ for which,

 

$$d(F(\xi,(x(\xi),y(\xi),z(\xi))),F(\xi,(u(\xi),v(\xi),w(\xi))))$$
$$\leq \alpha\,(d(gx(\xi),gu(\xi)) + d(gy(\xi),gv(\xi)) + d(gz(\xi),gw(\xi))) \quad (5.2.4.1)$$

For all $gx(\xi) \geq gu(\xi),\ gy(\xi) \leq gv(\xi)\ and\ gz(\xi) \geq gw(\xi)$.
Assume that F is continuous; g is continuous and commutes with F.
If there exist $x_0(\xi), y_0(\xi), z_0(\xi) \in X$ such that

$$gx_0(\xi) \leq F(\xi,(x_0(\xi),y_0(\xi),z_0(\xi))),$$
$$gy_0(\xi) \geq F(\xi,(y_0(\xi),x_0(\xi),y_0(\xi))),\ and\ gz_0(\xi) \leq F(\xi,(z_0(\xi),y_0(\xi),x_0(\xi))$$

Then there exist $x(\xi), y(\xi), z(\xi) \in X$ such that,

$$F(\xi,(x(\xi),y(\xi),z(\xi)) = gx(\xi),$$
$$F(\xi,(y(\xi),x(\xi),y(\xi)) = gy(\xi), and\ F(\xi,(z(\xi),y(\xi),x(\xi))) = gz(\xi).$$

That is, F and g have a triplet coincidence point.
**Proof:-** It can be proved easily using above Theorem.
**Corollary 5.2.5:-** Let $(X,\leq)$ be a partially ordered set and $(X,d)$ be a complete metric space. Let $F: X^3 \to X$ be a continuous mapping having the mixed g - monotone property on X and $F(X^3) \subset g(X)$. Suppose there exist $\varphi \in \Phi$, $\psi \in \Psi$ $\xi\ \varepsilon\ \Omega, be\ a\ measerable\ sellector$ ,for which,

$$d(F(\xi,(x(\xi),y(\xi),z(\xi))),F(\xi,(u(\xi),v(\xi),w(\xi))))$$
$$\leq (d(gx(\xi),gu(\xi)) + d(gy(\xi),gv(\xi)) + d(gz(\xi),gw(\xi)))$$
$$-\psi(d(gx(\xi),gu(\xi)) + d(gy(\xi),gv(\xi)) + d(gz(\xi),gw(\xi)))$$

For all $gx(\xi) \geq gu(\xi),\ gy(\xi) \leq gv(\xi)\ and\ gz(\xi) \geq gw(\xi)$.
Assume that F is continuous; g is continuous and commutes with F. If there exist $x_0(\xi), y_0(\xi), z_0(\xi) \in X$ such that

$$gx_0(\xi) \leq F(\xi,(x_0(\xi),y_0(\xi),z_0(\xi))),\ gy_0 \geq F(\xi,(y_0(\xi),x_0(\xi),y_0(\xi)),$$
$$and\ gz_0(\xi) \leq F(\xi,(z_0(\xi),y_0(\xi),x_0(\xi))$$

Then there exist $x(\xi), y(\xi), z(\xi) \in X$ such that,

$$F(\xi,(x,y(\xi),z(\xi)) = gx(\xi), F(\xi,(y(\xi),x(\xi),y(\xi))$$
$$= gy(\xi), and\ F(\xi,(z(\xi),y(\xi),x(\xi)) = gz(\xi).$$

That is, F and g have a triplet coincidence point.
**Proof:-** Using above Theorem , taking $\mu(t), = \varphi(t) = t$ corollary 52.5.can be proved easily.
**Theorem 5.2.6:-** Let $(X,\leq)$ be a partially ordered set and $(X,d)$ be a complete metric space. Let $F: X^3 \to X$ be a continuous mapping having the mixed g - monotone property on X and $(X^3) \subset g(X)$ . $\xi\ \varepsilon\ \Omega, be\ a\ measerable\ sellector$ .Suppose there exist $\mu,\ \varphi \in \Phi$, $\psi \in \Psi$ for which,

$$\mu\left\{\begin{array}{c} d(F(\xi,(x(\xi),y(\xi),z(\xi))),F(\xi,(u(\xi),v(\xi),w(\xi)))) \\ + d(F(\xi,(y(\xi),x(\xi),y(\xi))),F(\xi,(v(\xi),u(\xi),v(\xi)))) \\ + d(F(\xi,(z(\xi),y(\xi),x(\xi))),F(\xi,(w(\xi),v(\xi),u(\xi)))) \end{array}\right\}$$
$$\leq \varphi\left\{\begin{array}{c} d(gx(\xi),gu(\xi)) \\ + d(gy(\xi),gv(\xi)) \\ + d(gz,gw) \end{array}\right\} - \psi\left\{\begin{array}{c} d(gx(\xi),gu(\xi)) \\ + d(gy(\xi),gv(\xi)) \\ + d(gz(\xi),gw(\xi)) \end{array}\right\} \quad (5.2.6.1)$$

For all $gx(\xi) \geq gu(\xi),\ gy(\xi) \leq gv(\xi)\ and\ gz(\xi) \geq gw(\xi)$.
Assume that F is continuous, g is continuous and commutes with F. If there exist $x_0(\xi), y_0(\xi), z_0(\xi) \in X$ such that

$$gx_0(\xi) \leq F(\xi,(x_0(\xi),y_0(\xi),z_0(\xi))),$$
$$gy_0(\xi) \geq F(\xi,(y_0(\xi),x_0(\xi),y_0(\xi)),\ and\ gz_0 \leq F(\xi,(z_0(\xi),y_0(\xi),x_0(\xi))$$

Then there exist $x, y, z \in X$ such that,

$$F(\xi,(x(\xi),y(\xi),z(\xi))) = gx(\xi), F(\xi,(y,x,y) = gy(\xi),$$
$$and\ F(\xi,(z(\xi),y(\xi),x(\xi)) = gz(\xi).$$

That is, F and g have a triplet random fixed point.
**Proof:-** From the Theorem we can

$$\mu\left(d(F(\xi,(x(\xi),y(\xi),z(\xi)),F(\xi,(u(\xi),v(\xi),w(\xi))))\right)$$
$$\leq \frac{1}{3}\varphi(d(gx(\xi),gu(\xi)) + d(gy(\xi),gv(\xi)) + d(gz(\xi),gw(\xi)))$$
$$-\frac{1}{3}\psi(d(gx(\xi),gu(\xi)) + d(gy(\xi),gv(\xi)) + d(gz(\xi),gw(\xi)))$$

------------------------------------------------------------ (5.2.6.2)

Similarly we get,

$$\mu(d(F(\xi,(y(\xi),x(\xi),y(\xi))),F(\xi,(v(\xi),u(\xi),v(\xi)))))$$
$$\leq \frac{1}{3}\varphi(d(gx,gu) + d(gy,gv) + d(gy,gv))$$

$-\frac{1}{3}\psi\big( d(gx(\xi), gu(\xi)) + d(gy(\xi), gv(\xi)) + d(gy(\xi), gv(\xi))\big)$

(5.2.6.3)

and

$$\mu\left(d\big(F(\xi,(z(\xi),y(\xi),x(\xi))),F(\xi,(w(\xi),v(\xi),u(\xi)))\big)\right)$$
$$\leq \frac{1}{3}\varphi\big( d(gx(\xi),gu(\xi)) + d(gy(\xi),gv(\xi)) + d(gz(\xi),gw(\xi))\big)$$
$$-\frac{1}{3}\psi\big( d(gx,gu(\xi)) + d(gy(\xi),gv(\xi)) + d(gz(\xi),gw(\xi))\big)$$

(5.2.6.4)

by adding (5.2.6.2), (5.2.6.3)and (5.2.6.4)and property of $\mu$ then the result is follows similarly to the prove of Theorem and nothing to remain prove in Theorem .

**Remark 5.2.7:-** It is clear that Theorem 5.2.6 is the special case of Theorem 5.1.3.

**Remark 5.2.8:-** It is clear that Theorem 2.3 is the special case of Theorem 5.1.1 for perticular valuies of a,b and c

**Theorem 5.2.9:-** In addition to hypothesis of Theorem 5.2.3 suppose that for all $(x(\xi),y(\xi),z(\xi))$ and $(u(\xi),v(\xi),w(\xi))$ in $X^3$, there exists $(a(\xi),b(\xi),c(\xi))$ in $X^3$ $\xi\ \varepsilon\ \Omega, be\ a\ measerable\ sellector$ , such that $(F(\xi,(a(\xi),b(\xi),c(\xi))),F(\xi(b(\xi),a(\xi),b(\xi))),F(\xi(c(\xi),b(\xi),a(\xi))))$

are comparable $to\begin{Bmatrix}(F(\xi,(x(\xi),y(\xi),z(\xi))\ ),\\ F(\xi,(y(\xi),x(\xi),y(\xi)),\\ F(\xi,(z(\xi),y(\xi),x(\xi))\ ))\end{Bmatrix}$ and $\left\{\begin{pmatrix}F(\xi,(u(\xi),v(\xi),w(\xi))\ ),\\ F(\xi,(v(\xi),u(\xi),v(\xi))\ ),\\ F(\xi,(w(\xi),v(\xi),u(\xi))\ )\end{pmatrix}\right\}$

Also assume that $\mu,\varphi$ are non decreasing. Then F and g have unique tripled common random fixed point $(x(\xi),y(\xi),z(\xi))$ that is

$$x(\xi) = gx(\xi) = F\Big(\xi,\big(x(\xi),y(\xi),z(\xi)\big)\Big),$$
$$y(\xi) = gy(\xi) = F(\xi,(y(\xi),x(\xi),y(\xi)))$$
$$and\ z(\xi) = gz(\xi) = F(\xi,(z(\xi),y(\xi),x(\xi))).$$

**Proof:-** Assume now, that $(x,y,z)$ and $(u,v,z)$ are two tripled coincidence points of F and g that is

$$F(\xi,(x(\xi),y(\xi),z(\xi))) = gx(\xi), F(\xi,(y(\xi),x(\xi),y(\xi))) = gy(\xi)$$
$$and\ F(\xi,(z(\xi),y(\xi),x(\xi))) = gz(\xi),$$
$$F\Big(\xi,\big(u(\xi),v(\xi),w(\xi)\big)\Big) = gu(\xi),$$
$$F(\xi,\big(v(\xi),u(\xi),v(\xi)\big)) = gv(\xi)\ and\ F(\xi,\big(w(\xi),v(\xi),u(\xi)\big)))$$
$$= gw(\xi)$$

We will show that $(gx(\xi),gy(\xi),gz(\xi))\ and\ (gu(\xi),gv(\xi),gw(\xi))$ are equal.

By assumption, there is $(a(\xi),b(\xi),c(\xi)\ )$ (in $X^3$ such that $\big(F(\xi,(a(\xi),b(\xi),c(\xi))),F(\xi,(b(\xi),a(\xi),b(\xi))),F(\xi,(c(\xi),b(\xi),a(\xi)))\big)$ is comparable to $(F(\xi,(x(\xi),y(\xi),z(\xi))),F(y(\xi),x(\xi),y(\xi)),\ F(\xi,(z(\xi),y(\xi),x(\xi))))$

$$and\ (F(\xi,(u(\xi),v(\xi),w(\xi))),F(\xi,(v(\xi),u(\xi),v(\xi))),F(\xi,(w(\xi),v(\xi),u(\xi)))).$$

Define the sequence $\{ga_n(\xi)\},\{gb_n(\xi)\}and\ \{gc_n(\xi)\}$such that $a(\xi) = a_0(\xi)$,
$b(\xi) = b_0(\xi), c(\xi) = c_0(\xi)$ and

$$ga_n(\xi) = F(\xi,(a_{n-1}(\xi),b_{n-1}(\xi),c_{n-1}(\xi))$$
$$gb_n(\xi) = F(\xi,(b_{n-1}(\xi),a_{n-1}(\xi),b_{n-1}(\xi))$$
$$gc_n(\xi) = F(\xi,(c_{n-1}(\xi),b_{n-1}(\xi),a_{n-1}(\xi))$$

for all n. Further, set $x(\xi) = x_0(\xi), y(\xi) = y_0(\xi)$,
$z(\xi) = z_0(\xi)\ and\ u(\xi) = u_0(\xi), v(\xi) = v_0(\xi), w(\xi) = w_0(\xi)$ and similarly define the sequences $\{gx_n(\xi)\},\{gy_n(\xi)\},\{gz_n(\xi)\}$and $\{gu_n\},\{g_n\ (\xi)\},\{gw_n(\xi)\}$. Then,

$$gx_n(\xi) = F(\xi,(x(\xi),y(\xi),z(\xi)))\quad gu_n(\xi) = F(\xi,(u(\xi),v(\xi),w(\xi)))$$
$$gy_n(\xi) = F(\xi,(y(\xi),x(\xi),y(\xi)))\quad gv_n(\xi) = F(\xi,(v(\xi),u(\xi),v(\xi)))\qquad (5.2.9.1)$$
$$gz_n(\xi) = F\Big(\xi,\big(z(\xi),y(\xi),x(\xi)\big)\Big)$$
$$gw_n(\xi) = F(\xi,(w(\xi),v(\xi),u(\xi))$$

for all n ≥ 1. Since

$$(F(\xi,(x(\xi),y(\xi),z(\xi)),F(\xi,(y(\xi),x(\xi),y(\xi)),F(\xi,(z(\xi),y(\xi),x(\xi)))$$
$$= (gx_1(\xi),gy_1(\xi),gz_1(\xi)) = (gx(\xi),gy(\xi),gz(\xi))$$ is comparable to
$$\Big(F(\xi,(a(\xi),b(\xi),c(\xi))),F(\xi,(b(\xi),a(\xi),b(\xi))),F(\xi,\big(c(\xi),b(\xi),a(\xi)\big)\Big)$$
$= (ga_1(\xi),gb_1(\xi),gc_1(\xi))$, then it is easy to see that
$(gx(\xi),gy(\xi),gz(\xi)) \geq (ga_1(\xi),gb_1(\xi),gc_1(\xi))$. Recursively, we get that

$$(gx(\xi),gy(\xi),gz(\xi)) \geq (ga_n(\xi),gb_n(\xi),gc_n(\xi))\quad \forall\, n \geq 0. \qquad 2.29$$

Now we have

$$\mu\left(d\big(F(\xi,(x(\xi),y(\xi),z(\xi))),F(\xi,\big(a_n(\xi),b_n(\xi),c_n(\xi)\big))\big)\right)$$

$\leq \frac{1}{3}\varphi\big(d(gx(\xi),ga_n(\xi)) + d(gy(\xi),gb_n(\xi)) + d(gz(\xi),gc_n(\xi))\big)$
$-\frac{1}{3}\psi\big(d(gx(\xi),ga_n(\xi)) + d(gy(\xi),gb_n(\xi)) + d(gz(\xi),gc_n(\xi))\big)$
(5.2.9.2)

From (5.2.9.2)
We deduce that $\gamma_{n+1}(\xi) \leq \varphi(\gamma_n(\xi))$,
Where
$\gamma_n(\xi) = d\big(F(\xi,(x(\xi),y(\xi),z(\xi)),F(\xi,(a_{n-1}(\xi),b_{n-1}(\xi),c_{n-1}(\xi))\big).$

$$\gamma_n(\xi) \leq \varphi^n(\gamma_0(\xi))$$

That is the sequence $d\big(F(\xi,(x(\xi),y(\xi),z(\xi))),F(\xi,(a_{n-1}(\xi),b_{n-1}(\xi),c_{n-1}(\xi)))\big)$ is decreasing. Therefore, there exists $\alpha \geq 0$ such that

$$lim_{n\to\infty}\left\{\begin{matrix} d(gx(\xi),ga_n(\xi)) \\ + d(gy(\xi),gb_n(\xi)) \\ + d(gz(\xi),gc_n(\xi))\end{matrix}\right\} = \alpha(\xi). \quad (5.2.9.3)$$

We shall show that $\alpha(\xi) = 0$. Suppose, to the contrary, that $\alpha(\xi) > 0$. Taking the limit as $n \to \infty$ in (5.2.9.2)we have

$$\mu(\alpha(\xi)) \leq \varphi(\alpha(\xi)) - lim_{n\to\infty}\,\psi\left\{\left(\begin{matrix} d(gx(\xi),ga_n(\xi)) \\ + d(gy(\xi),gb_n(\xi)) \\ + d(gz(\xi),gc_n(\xi))\end{matrix}\right)\right\} < \varphi(\alpha)$$

a contradiction. Thus, $\alpha = 0$, that is

$$lim_{n\to\infty}\left[\begin{matrix} d(gx(\xi),ga_n(\xi)) \\ + d(gy(\xi),gb_n(\xi)) \\ + d(gz(\xi),gc_n(\xi))\end{matrix}\right] = 0. \quad (5.2.9.4)$$

$$lim_{n\to\infty}\big[d(gx(\xi),ga_n(\xi))\big] = lim_{n\to\infty}\big[d(gy(\xi),gb_n(\xi))\big] = lim_{n\to\infty}[d(gz(\xi),gc_n(\xi))] = 0.$$

Similarly we show that

$$lim_{n\to\infty}\big[d(gu(\xi),ga_n(\xi))\big] = lim_{n\to\infty}\big[d(gv(\xi),gb_n(\xi))\big] = lim_{n\to\infty}[d(gw(\xi),gc_n(\xi))] = 0$$

Combining yields that $(gx(\xi),gy(\xi),gz(\xi))$
$and\ (gu(\xi),gv(\xi),gw(\xi))$ are equal.
Since $F(\xi,(x(\xi),y(\xi),z(\xi))) = gx(\xi), F(\xi,(y(\xi),x(\xi),y(\xi))$
$= gy(\xi)\ and\ F(\xi,(z(\xi),y(\xi),x(\xi))) = gz(\xi)$ by commutativity of F and g, we have

$$g(F(\xi,(x(\xi),y(\xi),z(\xi))) = g(gx(\xi)) = F(\xi,(gx(\xi),gy(\xi),gz(\xi))$$
$$g(F(\xi,((\xi)y,x(\xi),y(\xi)))) = g(gy(\xi)) = F(\xi,(gy(\xi),gx(\xi),gy(\xi)))$$
$$g\Big(F\Big(\xi,(z(\xi),y(\xi),x(\xi))\Big)\Big) = g(gz(\xi)) = F(\xi,(gz(\xi),gy(\xi),gx(\xi))),$$

Denote $gx(\xi) = x'(\xi), gy(\xi) = y'(\xi)\ and\ gz(\xi) = z'(\xi)$.
It is clear that,

$$F(\xi,(x'(\xi),y'(\xi),z'(\xi)) = gx'(\xi), F(\xi,(y(\xi)',x'(\xi),y'(\xi)) = gy'(\xi)$$
$$and\ F(\xi,(z(\xi)',y'(\xi),x'(\xi)) = gz'(\xi)$$

That is, $(x'(\xi),y'(\xi),z'(\xi))$ is a tripled coincidence point of F and g. Consequently, $(gx'(\xi),gy'(\xi),gz'(\xi))$ and $(gx(\xi),gy(\xi),gz(\xi))$ are equal, that is
$gx(\xi) = gx'(\xi), gy(\xi) = gy'(\xi)$ and $gz(\xi) = gz(\xi)'$.
We deduce $gx(\xi) = gx'(\xi) = x(\xi), gy(\xi) = gy'(\xi) = y(\xi)$
and $gz(\xi) = gz'(\xi) = z(\xi)$. Therefore, $(x'(\xi),y(\xi)',z(\xi)')$ is a tripled common random fixed point of F and g. Uniqueness can be proved easily.

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## 7. References

1. Bhaskar T.G. and Lakshmikantham V., Fixed point theory in partially ordered metric spaces and applications. Nonlinear Anal. 65, 1379 - 1393 (2006).
2. Lakshmikantham V., Ciric Lj. B., "Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces". Nonlinear Anal. 70,/ 4341-4349 (2009).

3. Karapinar E. Couple fixed point on cone metric spaces. Gazi Univ. J. Sci. 241/, 51-58 (2011).
4. Karapinar E., Coupled fixed point theorems for nonlinear contractions in cone metric spaces. Comput. Math. Appl. 5912 3656-3668 (2010).
5. Aydi H., Mujahid A. and Postolache M., Coupled coincidence points for hybrid pair of mappings via mixed monotone property. J. Adv. Math. Studies 51, 118-126 (2012).
6. Aydi H., Some coupled fixed point results on partial metric spaces. Int. J. Math. Math. Sci., 2011/, Article ID 647091, 11 (2011).
7. Aydi H., Samet B. and Vetro C., Coupled fixed point results in cone metric spaces for ~ w-compatible mappings. Fixed Point Theory Appl. 2011/, 27 (2011).
8. Aydi H., Damjanovic B., Samet B. and Shatanawi W., Coupled fixed point theorems for nonlinear contractions in partially ordered G-metric spaces. Math. Comput. Model. 54, 2443-2450 (2011).
9. Aydi H., Shatanawi W. and Postolache M. Coupled fixed point results for φ, \■ −weakly contractive mappings in ordered G-metric spaces. Comput. Math. Appl. 63/, 298-309 (2012).
10. Choudhury B.S., Metiya N. and Kundu A., Coupled coincidence point theorems in ordered metric spaces. Ann. Univ. Ferrara 57,/ 1-16 2011.
11. Choudhury, B.S. and Kundu A., A coupled coincidence point result in partially ordered metric spaces for compatible mappings. Nonlinear Anal. 73,/ 2524-2531 2010.
12. Luong N.V. and Thuan N.X., Coupled fixed points in partially ordered metric spaces and application. Nonlinear Anal. 74,/ 983-992 (2011).
13. Samet B., Coupled fixed point theorems for a generalized Meir- Keeler contraction in partially ordered metric spaces. Nonlinear Anal. 7412/, 4508-4517 (2010).
14. Samet B. and Vetro C., Coupled fixed point, f-invariant set and fixed point of N-order. Ann. Funct. Anal. 12, 46-56 (2010).
15. Alber Yal, and Guerre - Delabriere S., 'Principles of weakly contractive maps in Hilbert spaces. In Gohberg I, Lyubich Yu, editors. New results in operator theory', Advances and applications, 98 (1997 )7-22.
16. Rhoades B.E., Some theorems on weakly contractive maps., Nonlinear Analysis, 47 (2001) 2683-2693.
17. Khan M.S., Swaleh M.and Sessa S., `Fixed point theorems for altering distances between the points', Bull. Aust. Math. Soc. 30 (1984 )1-9.
18. Naidu SVR, `Some fixed point theorems in metric spaces by altering distances', Czechoslovak Math. J. 53/1 (2003) 205-12.
19. Sastry KPR and Babu GVR, `Some fixed point theorems by altering distances between the points.', Ind. J Pure Appl. Math. 30/6 (1999) 641-7.
20. Choudhury B.S., `A common unique fixed point result in metric space involving generalized altering distance.', Math. Commun. 10 (2005) 105-10.
21. Doric D., `Common fixed point for generalized $\psi, \phi$ − weak contractions.',Appl.Math. Lett. 22 2009 1896-900.
22. Berinde V. and Borcut M., Tripled fixed point theorems for contractive type mappings in partially ordered metric spaces. Nonlinear Anal. 7415, 4889-4897 (2011).
23. Aydi H., Karapinar E., and Postolache M., Tripled coincidence point theorems for weak φ- contractions in partially ordered metric spaces., Fixed Point Theory and Applications 2012, 2012:44 doi: 10.1186/1687-1812-(2012)-44.
24. Beg I. and Butt A.R. ,'Common fixed point for generalized set valued contractions satisfying an implicit relation in partially ordered metric spaces. Math. Commun. 15 (2010) 65-76.
25. Datta P.N. and Choudhury B.S., `A generalisation of contaction principle in metric spaces', Fixed Point Theory and Appl. (2008 )article ID 406368.
26. Choudhury B.S. ,`Certain common fixed point theorems in complete metric spaces', Soochow J Math. 22/3/(1996) 427-34.
27. Fisher B., `Common fixed point of mappings and setvalued mappings', Rostock Math Colloq 18/(1981) 69-77.
28. Fisher B. and Iseki K., `Fixed points for set valued mappings on complete and compact metric spaces', Math. Japonica 28/(1983) 639-46.
29. Bharucha –Reid, A.T. Fixed point theorems in probabilistic analysis, Bull. Amer. Math. Soc.82 (1976) 641-657.
30. Choudhary, B.S. and Ray, M. "Convergence of an iteration leading to a solution of a random operator equation" J. Appl. Math. Stochastic Anal. 12 (1999). No 2,161-168.
31. Choudhary, B.S. and Upadhyay, A. "An iteration leading to random solutions and fixed points of operators" Soochow J. Math.25 (1999). No 4,395-400.
32. Choudhary, B.S. "A common unique fixed point theorem for two random operators in Hilbert spaces" I. J. M.M. S. 32 (2002) 177-182.
33. Gupta, Animesh, Kushwaha Renu, "Tripled Common Fixed Point For Weak $(\mu, \varphi, \psi)$ − Contractions In Partially Ordered Metric Spaces," Mathematical Theory And Modeling, Vol.3 No. 6 (2013) 46-53.