Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.4, No.14, 2014
www.iiste.org

A Generalized Skew of type IV Logistic Distribution

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Abstract

In this paper, we derive, the probability density function (pdf) and cumulative distribution function (CDF) of the skew type IV generalized logistic distribution GSLD IV (α, β, λ). The general statistical properties of the GSLD IV (α, β, λ). such as: the moment generating function (mgf), characteristic function (ch.f), Laplace and fourier transformations are obtained in explicit form. Expressions for the nth moment, skewness and kurtosis coefficients are discussed. The mean deviation about the mean and about the median are also obtained. We consider the general case by inclusion of location and scale parameters. The results of Asgharzadeh (2013) are obtained as special cases. Graphically illustration of some results have been represented. Further we present a numerical example to illustrate some results of this paper.

Keywords: skew type IV generalized logistic distribution, moment generating function, skewness, kurtosis, mean deviation.

1. Introduction

A number of authors discussed important applications of the logistic distribution in many fields including survival analysis growth model public health and etc…. Several different forms of generalization of the logistic distribution have been proposed in the literature, and studied in Balakrishnan and leung (1988), Balakrishnan (1992) and Johnson et al (1995), i. e. types I,II,III and IV. The type IV generalized logistic distribution denoted by type IV GLD (α, β) has been discussed in Johnson et al (1995), and Nassar and Elmasry (2012). The probability density function (pdf) of type IV GLD (α, β) is given by

$$g(x) = \frac{1}{B(\alpha,\beta)} e^{-\beta x}(1+e^{-x})^{-(\alpha+\beta)}, \qquad -\infty < x < \infty \tag{1.1}$$

where $B(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\,dt$ is the complete beta function, β is the scale parameter and α is the shape parameter. This distribution is symmetric for every α . The type IV GLD (α, β) is the just the family of logistic distribution generated from the beta distribution, proposed by Jones (2004). It is well known, in general, that a generalized model is more flexible than the ordinary model and it is preferred by many data analysis in analyzing statistical data.
Let us introduce cumulative distribution function CDF of type IV GLD (α, β) as proposed by Jones (2004), as follows

$$G(x) = \frac{1}{B(\alpha,\beta)} \int_0^{\frac{1}{1+e^{-x}}} y^{\alpha-1}(1-y)^{\beta-1}\,dy = I_{\frac{1}{1+e^{-x}}}(\alpha,\beta), \qquad -\infty < x < \infty \tag{1.2}$$

where $I_y(\alpha,\beta) = \frac{B_y(\alpha,\beta)}{B(\alpha,\beta)}$ is the incomplete beta function ratio and the incomplete beta function is
$B_y(a,b) = \int_0^y t^{a-1}(1-t)^{b-1}\,dt$

The type IV GLD (α, β) generalizes the various forms of the logistic distribution. For example, if $\alpha = \beta = 1$, we obtain the standard logistic distribution. Also for the case $\alpha = \beta$, we have the type III generalized logistic distribution. Different properties about this distribution has been studied by Nassar and Elmasry (2012).

Azzalini (1985) showed that any symmetric distribution was viewed as member of more general class of skewed distribution. The pdf of the skew normal distribution defined by Azzalini (1985) is given by

$$f(x) = 2\phi(x)\,.\,\Phi(\lambda x)\,, \qquad -\infty < x < \infty \tag{1.3}$$

where $\lambda \in R$ is the skewness parameter, $\phi(x)$ and $\Phi(x)$ are respectively the pdf and CDF of $N(0,1)$.
This idea can be applied to any symmetric distribution. A long the same line, the skew logistic distribution with the skewness parameter λ has been studied by many others (see for example, Wahed and Ali (2001), Gupta et al (2002), Nadrajah and Kotz (2006)(2007), Nadrajah (2009), Gupta and Kundu (2010), and Chakraborty et al (2012)).
Koessler and Kumar (2010), illustrate an application with respect to an adoptive test for two-sample scale problem based on U-statistics. Asgharzadeh et al (2013) introduced a generalized skew type III logistic distribution and obtained several mathematical properties of this distribution such as CDF and moments. Furthermore estimation of unknown parameters by using the method of maximum likelihood and fisher information matrix are investigated. Abd-Elfattah et al (2014) studied the skew type I generalized logistic distribution with skewness parameter λ and obtained some important statistic properties of this distribution. Moreover skewness and kurtosis coefficients are illustrated numerically and graphically.
In this paper, we using type IV GLD (α, β) to study the skew of type IV GLD (α, β), with skewness parameter $\lambda \epsilon R$ and will be denoted as GSLDIV (α, β, λ). This paper is organized as follows: in next section we drive the pdf and CDF of

GSLDIV (α,β,λ) in explicit forms. In section 3, we obtained the moment generating function (mgf), characteristic function (ch. f), Laplace and Fourier transformations. Expressions for n<u>th</u> moment including the first four moments, skewness and kurtosis coefficients are given in section 4. The mean deviation about the mean and median are discussed in Section 5. Some important properties are discussed in Section 6. Numerical example is given in section 7. Finally, Conclusion remarks are provided in section 8. Graphical illustration of pdf, CDF of GSLDIV (α,β,λ), skewness and kurtosis have been represented. It should be noted that some known results of Asgharzadeh et al (2013), and Abd-Elfattah et al (2014) are obtained as special cases.

2. Probability density function and Cumulative distribution function of GSLD IV (α,β,λ)

In this section we derive a form of pdf and CDF of GSLDIV (α,β,λ). Using (1.1),(1.2), and (1.3), we define the pdf of GSLDIV (α,β,λ) as follows:

Definition (2.1): we say that the random variable X is distributed according to GSLDIV (α,β,λ), if its pdf is given by

$$f(x,\alpha,\beta,\lambda) = 2g(x)\,.\,G(\lambda x)\,, \qquad -\infty < x < \infty \tag{2.1}$$

where $\lambda \in R$ is the skewness parameter, $g(x)$ and $G(x)$ are the pdf and CDF of type IV GLD (α,β) given in (1.1) and (1.2).

Throughout the rest of this paper (unless otherwise stated), we shall assume that $\lambda > 0$ since the corresponding results for $\lambda < 0$ can be obtained using the fact that -X has GSLDIV $(\alpha,\beta,-\lambda)$.

The pdf of GSLDIV (α,β,λ) given by (2.1) can be expressing in other forms (a) a double series representation (b) a triple series representation as given in the following lemma.

Lemma (2.1): if X be a random variable having GSLDIV (α,β,λ), then its pdf is given by

$$\text{(a)} f(x,\alpha,\beta,\lambda) = \begin{cases} \frac{2}{B^2(\alpha,\beta)(1+e^{-x})^{(\alpha+\beta)}}\,.\,A(i,j)\,.\,e^{-(\beta+\lambda j)x}\,, & x>0 \\ \\ \frac{2}{B^2(\alpha,\beta)(1+e^{x})^{(\alpha+\beta)}}\,.\,A(i,j)\,.\,e^{[(i+j+\alpha)\lambda+\alpha]x}\,, & x<0 \end{cases} \tag{2.2}$$

$$\text{(b)} f(x,\alpha,\beta,\lambda) = \begin{cases} \frac{2}{B^2(\alpha,\beta)}\,.\sum_{k=0}^{\infty}\binom{-\alpha-\beta}{k}\,.\,A(i,j)\,.\,e^{-B_2 x}\,, & x>0 \\ \\ \frac{2}{B^2(\alpha,\beta)}\,.\sum_{k=0}^{\infty}\binom{-\alpha-\beta}{k}\,.\,A(i,j)\,.\,e^{B_1 x}\,, & x<0 \end{cases} \tag{2.3}$$

where, $A(i,j) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{(-1)^i}{i+\alpha}\binom{\beta-1}{i}\binom{-i-\alpha}{j}$,

$$B_1 = (i+j+\alpha)\lambda + \alpha + k, \quad \text{and} \quad B_2 = \beta + k + \lambda j \tag{2.4}$$

Proof:

(a) The pdf $g(x)$ given in (1.1) can be written in the form

$$g(x) = \begin{cases} \frac{1}{B(\alpha,\beta)}\,.\,e^{-\beta x}\,.\,(1+e^{-x})^{-(\alpha+\beta)}, & x>0 \\ \\ \frac{1}{B(\alpha,\beta)}\,.\,e^{\alpha x}\,.\,(1+e^{x})^{-(\alpha+\beta)}\,, & x<0 \end{cases} \tag{2.5}$$

Using the binomial expansion for $(1+e^{-x})^{-(\alpha+\beta)}$ in (2.5), we get

$$g(x) = \begin{cases} \frac{1}{B(\alpha,\beta)}\,.\sum_{k=0}^{\infty}\binom{-\alpha-\beta}{k}\,.\,e^{-(\beta+k)x}, & x>0 \\ \\ \frac{1}{B(\alpha,\beta)}\,.\sum_{k=0}^{\infty}\binom{-\alpha-\beta}{k}\,.\,e^{(\alpha+k)x}\,, & x<0 \end{cases} \tag{2.6}$$

Now, utilizing the binomial expansion for $(1-y)^{\beta-1}$and $(1+e^{-x})^{-(i+\alpha)}$ in (1.2) with elementary calculations, we can obtain

$$G(\lambda x)=\begin{cases}\frac{1}{B(\alpha,\beta)}\,.A(i,j).\,e^{-\lambda jx}, & x>0\\ \\ \frac{1}{B(\alpha,\beta)}\,.A(i,j).\,e^{(i+j+\alpha)\lambda x}\;, & x<0\end{cases} \tag{2.7}$$

Substituting from (2.5) and (2.7) into (2.1) the result of part (a) of double series representation for the pdf $f(x,\alpha,\beta,\lambda)$ is obtained.

(b) Employing (2.1), (2.6), and (2.7), one can obtain the triple series representation given in (2.3).

It is clear that if we take $\alpha=\beta$, the result of Asgharzadeh (2013) is obtained as special case.

The calculations through this paper are based on the generalized hypergeometric function mF_n defined by $mF_n(\tau_1,\tau_2,\ldots,\tau_m;\mathcal{V}_1,\mathcal{V}_2,\ldots,\mathcal{V}_n;x)=\sum_{k=0}^{\infty}\frac{(\tau_1)_k(\tau_2)_k\cdots(\tau_m)_k}{(\mathcal{V}_1)_k(\mathcal{V}_2)_k\cdots(\mathcal{V}_n)_k}\,\frac{x^k}{k!}$

and $(\mathrm{c})_\mathrm{k}=\mathrm{c}(\mathrm{c}+1)\ldots\ldots(\mathrm{c}+\mathrm{k}-1)$ denotes the ascending factorial. The properties of this special functions being used can be found in Gradshteyn and Ryzhik (2000).

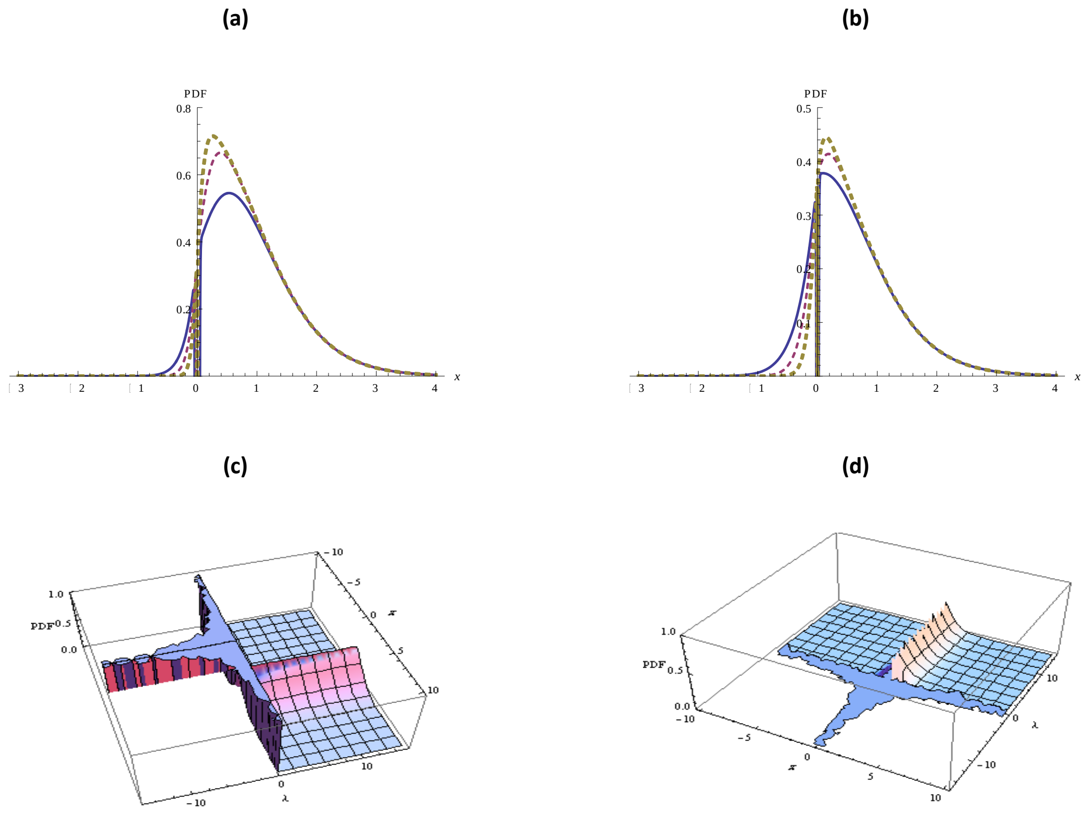

Figure 1: The shapes of pdf of GSLDIV (α, β, λ) (a) when (α=2,β=2, λ=2 Thick line), (α=2,β=2, λ=5 Dashed line), and (α=2,β=2, λ=10 Dashed &Thick line). (b) when (α=1,β=2, λ=2 Thick line), (α=2,β=2, λ=5 Dashed line), and (α=2,β=2, λ=10 Dashed &Thick line). (c) when [α=1, β=1 and λ ∊ (-15, 15)]. (d) when [α=1, β=2 and λ ∊ (−15, 15)].

Theorem (2.1): if X is a GSLDIV (α, β, λ) random variable, then its CDF is given by

$$(a)F(x,\alpha,\beta,\lambda)=\begin{cases}\frac{2}{B^2(\alpha,\beta)}.A(i,j).\left[\frac{A_1-A_2e^{-(\beta+\lambda j)x}}{\beta+\lambda j}+\frac{A_3}{(i+j+\alpha)\lambda+\alpha}\right], & x>0\\ \\ \frac{2}{B^2(\alpha,\beta)}.A(i,j).\frac{A_4e^{[(i+j+\alpha)\lambda+\alpha]x}}{(i+j+\alpha)\lambda+\alpha}\;, & x<0\end{cases} \tag{2.8}$$

$$(b)F(x,\alpha,\beta,\lambda) = \begin{cases} 1 - \frac{2}{B^2(\alpha,\beta)} \cdot \sum_{k=0}^{\infty} \binom{-\alpha-\beta}{k} \cdot A(i,j) . \frac{e^{-B_2 x}}{B_2} \ , & x > 0 \\ \\ \frac{2}{B^2(\alpha,\beta)} \cdot \sum_{k=0}^{\infty} \binom{-\alpha-\beta}{k} \cdot A(i,j) . \frac{e^{B_1 x}}{B_2} \ , & x < 0 \end{cases} \tag{2.9}$$

where $A(i,j)$, B_1, B_2 are given by (2.4),
$A_1 = 2F_1(\alpha+\beta, \beta+\lambda j, \beta+\lambda j+1, -1)$,
$A_2 = 2F_1(\alpha+\beta, \beta+\lambda j, \beta+\lambda j+1, -e^{-x})$,
$A_3 = 2F_1(\alpha+\beta, \alpha+(i+j+\alpha)\lambda, \alpha+(i+j+\alpha)\lambda+1, -1)$, and
$A_4 = 2F_1(\alpha+\beta, \alpha+(i+j+\alpha)\lambda, \alpha+(i+j+\alpha)\lambda+1, -e^{x})$

Proof:

The proof of this theorem has two cases separately, when $x \geq 0$ and $x < 0$, as follows:

(a) Firstly, when $x \geq 0$, employing the double series representation of the pdf given in (2.2), the CDF of GSLDIV (α, β, λ) can be written as

$$F(x,\alpha,\beta,\lambda) = P(X \leq x) = \int_{-\infty}^{x} f(x,\alpha,\beta,\lambda)\, dx = F(0) + \frac{2}{B^2(\alpha,\beta)} \cdot A(i,j) \cdot I_{(x)}$$

Where $I_{(x)}$ is the integral given by

$$I_{(x)} = \int_0^x \frac{e^{-(\beta+\lambda j)x}}{(1+e^{-x})^{(\alpha+\beta)}}\, dx \tag{2.10}$$

Substituting $Z = e^{-x}$ the integral $I_{(x)}$ reduce to

$$I_{(x)} = \int_{e^{-x}}^{1} \frac{Z^{\beta+\lambda j-1}}{(1+Z)^{(\alpha+\beta)}}\, dz$$

$$= \int_0^1 \frac{Z^{\beta+\lambda j-1}}{(1+Z)^{(\alpha+\beta)}}\, dz - \int_0^{e^{-x}} \frac{Z^{\beta+\lambda j-1}}{(1+Z)^{(\alpha+\beta)}}\, dz = I_1 - I_2 \tag{2.11}$$

To evaluate the integrals I_1 and I_2, we use the formula (3.194.1) in Gradshteyn and Ryzhik (2000) , which stated as follows

$$\int_0^u \frac{x^{\mu-1}}{(1+bx)^{\mathcal{V}}} dx = \frac{u^\mu}{\mu} 2F_1(\mathcal{V}, \mu, 1+\mu, -bu) \tag{2.12}$$

Then, the integrals I_1 and I_2 can be calculated as

$$I_1 = \frac{A_1}{\beta+\lambda j}, \qquad I_2 = \frac{A_2 e^{-(\beta+\lambda j)x}}{\beta+\lambda j} \tag{2.13}$$

From (2.10),(2.11), and (2.13), we get

$$F(x) = F(0) + \frac{2}{B^2(\alpha,\beta)} \cdot A(i,j) \cdot \left[\frac{A_1}{\beta+\lambda j} - \frac{A_2 e^{-(\beta+\lambda j)x}}{\beta+\lambda j}\right] \tag{2.14}$$

Similarly, $F(0)$ can be calculated as above and we can evaluate it as follows

$$F(0) = \int_{-\infty}^{0} f(x,\alpha,\beta,\lambda)\, dx = \frac{2}{B^2(\alpha,\beta)} \cdot A(i,j) \cdot I_3 \tag{2.15}$$

Where I_3 is the integral given by

$$I_3 = \int_{-\infty}^{0} \frac{e^{[(i+j+\alpha)\lambda+\alpha]x}}{(1+e^x)^{(\alpha+\beta)}}\, dx = \int_0^1 \frac{Z^{(i+j+\alpha)\lambda+\alpha-1}}{(1+Z)^{(\alpha+\beta)}}\, dz = \frac{A_3}{(i+j+\alpha)\lambda+\alpha} \tag{2.16}$$

Substituting from (2.16) into (2.15) we have

$$F(0) = \frac{2}{B^2(\alpha,\beta)} \cdot A(i,j) \cdot \frac{A_3}{(i+j+\alpha)\lambda+\alpha} \tag{2.17}$$

Combining (2.14), and (2.17) the result for $x \geq 0$ is yielded

(b) Second case: when $x < 0$, by similar calculation and using (2.12), we get

$$F(x,\alpha,\beta,\lambda) = \int_{-\infty}^{x} f(x,\alpha,\beta,\lambda)\, dx = \frac{2}{B^2(\alpha,\beta)} \cdot A(i,j) \cdot I_4$$

where $I_4 = \int_{-\infty}^{x} \frac{e^{[(i+j+\alpha)\lambda+\alpha]x}}{(1+e^x)^{(\alpha+\beta)}}\, dx$

$$= \int_0^{e^x} \frac{Z^{(i+j+\alpha)\lambda+\alpha-1}}{(1+Z)^{(\alpha+\beta)}}\, dz = \frac{A_4 e^{[(i+j+\alpha)\lambda+\alpha]x}}{(i+j+\alpha)\lambda+\alpha}$$

Consequentially for $x < 0$, the CDF, $F(x,\alpha,\beta,\lambda)$ is given by

$$F(x,\alpha,\beta,\lambda) = \frac{2}{B^2(\alpha,\beta)} \cdot A(i,j) \cdot \frac{A_4 e^{[(i+j+\alpha)\lambda+\alpha]x}}{(i+j+\alpha)\lambda+\alpha}$$

which complete the proof of part (a) of this theorem. In the same manner part (b) of this theorem can be also obtained by using the triple series form of the pdf, $f(x,\alpha,\beta,\lambda)$ given in (2.3). If we put $\alpha=\beta$, the results of Asgharzadeh (2013) are obtained as special cases.

Properties of GSLD IV (α,β,λ)

Some properties of GSLD IV (α,β,λ) can be obtained from (2.1) as follows:

(i) When $\alpha=\beta=1$, $f(x,\alpha,\beta,\lambda)$ reduces to the standard skew logistic pdf.

(ii) When $\lambda=0$, $f(x,\alpha,\beta,\lambda)$ reduces to the type IV generalized logistic pdf.

(iii)When $\alpha=\beta$, $f(x,\alpha,\beta,\lambda)$ reduces to the type III generalized skew logistic distribution (α,λ) [see Asgharzadeh et al (2013)].

(iv) If X has GSLDIV (α,β,λ), then -X has GSLDIV $(\alpha,\beta,-\lambda)$.

(v) $f(x,\alpha,\beta,\lambda)+f(x,\alpha,\beta,-\lambda)=2g(x)$, for all $x\in R$.

(vi) $f(x,\alpha,\beta,\lambda)\longrightarrow 2g(x)I\{x\geq 0\}$ as $\lambda\longrightarrow\infty$, and
$f(x,\alpha,\beta,\lambda)\longrightarrow 2g(x)I\{x\leq 0\}$ as $\lambda\longrightarrow-\infty$ for all $\alpha>0,\beta>0$.

(vii) $f(x,\alpha,\beta,\lambda)\longrightarrow 0$ as $x\longrightarrow\mp\infty$ for all $\alpha>0,\beta>0$ and $\lambda\in R$.

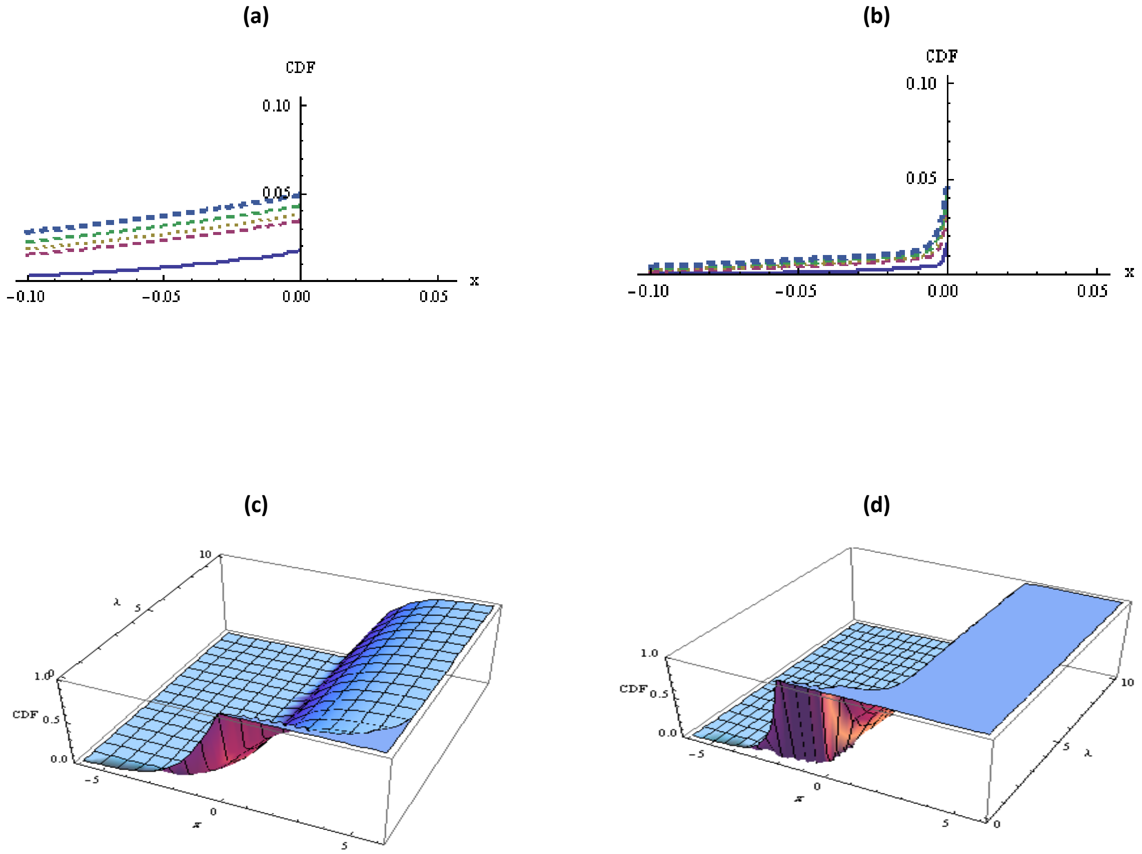

Figure 2: The shapes of CDF of GSLDIV (α, β, λ) (a) when (α=1,β=1,λ=20 Thick line), (α=1,β=1,λ=10 Dashed line), (α=1,β=1,λ=9 Dotted line), (α=1,β=1,λ=9 Dotted &Dashed line), and (α=1,β=1, λ=8 Dashed &Thick line). (b) when (α=2,β=1,λ=20 Thick line), (α=2,β=1,λ=10 Dashed line), (α=2,β=1,λ=9 Dotted line), (α=2,β=1,λ=9 Dotted &Dashed line), and (α=2,β=1, λ=8 Dashed &Thick line). (c) when [α=1, β=1 and λ ∊ (0, 10)]. (d) when [α=2, β=1 and λ ∊ (0, 10)].

3. Moment generating function and characteristic function

In this section, we derive the moment generating function (mgf) and the characteristic function (ch.f) of the random variable X with GSLDIV (α, β, λ). We consider the triple series representation of the pdf, f(x, α, β, λ) given by (2.3).

Theorem (3.1): The mgf of the random variable X with GSLDIV (α, β, λ) is given by

$$M_X(t)=\frac{2}{B^2(\alpha,\beta)}\cdot\sum_{k=0}^{\infty}\binom{-\alpha-\beta}{k}.A(i,j).\left[\frac{1}{B_1+t}+\frac{1}{B_2-t}\right] \tag{3.1}$$

Proof: Let $f(x,\alpha,\beta,\lambda)$ be the pdf of GSLDIV (α, β, λ) given by (2.3), then

$$M_X(t)=E(e^{tX})=\int_{-\infty}^{\infty}e^{tx}\,f(x,\alpha,\beta,\lambda)dx=\frac{2}{B^2(\alpha,\beta)}\cdot\sum_{k=0}^{\infty}\binom{-\alpha-\beta}{k}.A(i,j).[I_5+I_6]$$

where

$I_5=\int_{-\infty}^{0}e^{[B_1+t]x}\,dx=\frac{1}{B_1+t}$, and

$I_6=\int_{0}^{\infty}e^{-(B_2-t)x}\,dx=\frac{1}{B_2-t}$

which prove the theorem.

Remark(3.1)

if the random variable X has GSLDIV (α, β, λ), then its characteristic function $\emptyset_x(t)$ is given by

$$\emptyset_x(t) = \frac{2}{B^2(\alpha,\beta)} \cdot \sum_{k=0}^{\infty} \binom{-\alpha-\beta}{k} . A(i,j). \left[\frac{1}{B_1+ut} + \frac{1}{B_2-ut}\right]$$

Where, u = $\sqrt{-1}$ is the complex imaginary part.

Other useful properties of GSLDIV $(\alpha, \beta, \lambda)$are the Laplace and Fourier Transforms, which are given by the following remark (3.2) and remark (3.3), for the triple series representation of pdf given in (2.3).

Remark (3.2)

The Laplace Transform of the random variable X having GSLDIV $(\alpha, \beta, \lambda)$is:

$$L(t) = E(e^{-tx}) = \frac{2}{B^2(\alpha,\beta)} \cdot \sum_{k=0}^{\infty} \binom{-\alpha-\beta}{k} . A(i,j). \left[\frac{1}{B_1-t} + \frac{1}{B_2+t}\right]$$

Remark (3.3)

The Fourier Transform of the random variable X having GSLDIV $(\alpha, \beta, \lambda)$is:

$$Fo(t) = E(e^{-utx}) = \frac{2}{B^2(\alpha,\beta)} \cdot \sum_{k=0}^{\infty} \binom{-\alpha-\beta}{k} . A(i,j). \left[\frac{1}{B_1-ut} + \frac{1}{B_2+ut}\right]$$

where $A(i,j)$, B_1, and B_2 are given by (2.4).

4. Skewness and Kurtosis coefficients

The skewness coefficient is measured by $\gamma_1 = \frac{M_3}{\sigma^3}$, and the kurtosis coefficient is measured by $\gamma_2 = \frac{M_4}{\sigma^4}$, where M_3 and M_4 are the third and fourth moments about the mean and, σ is the standard deviation of the random variable X having GSLDIV (α, β, λ). We firstly derive the n<u>th</u> moment of the random variable X having pdf given in (2.3) by the following theorem.

Theorem (4.1)

The n<u>th</u> moment of the SGLDIV(α, β, λ)random variable X is

(a) If n is even order, we have

$$E(X^n) = \frac{4\,(n!)}{B^2(\alpha,\beta)} \cdot \sum_{k=0}^{\infty} \binom{-\alpha-\beta}{k} \cdot \frac{A(i,j)}{(B_2)^{n+1}}$$

(b) If n is odd order, we have

$$E(X^n) = \frac{2\,(n!)}{\beta^2(\alpha,\beta)} \sum_{k=0}^{\infty} \binom{-\alpha-\beta}{k} . A(i,j). \left[\frac{1}{(B_2)^{n+1}} - \frac{1}{(B_1)^{n+1}}\right]$$

Proof:

Since, $E(X^n) = \int_{-\infty}^{\infty} x^n \, f(x, \alpha, \beta, \lambda)\, dx$

(a) If n is even order, we have

$$E(X^n) = 2\int_0^{\infty} x^n \, f(x, \alpha, \beta, \lambda)\, dx$$

using the triple series representation of $f(x, \alpha, \beta, \lambda)$ given in (2.3), and by elementary calculation, we get the result

(b) If n is odd order, we have

$$E(X^n) = \int_{-\infty}^{0} x^n \, f(x, \alpha, \beta, \lambda)\, dx + \int_0^{\infty} x^n \, f(x, \alpha, \beta, \lambda)\, dx$$

Employing (2.3), and evaluating the integrals, we obtain the result.

The first four moments of X can be obtained as follows

$$E(X) = \frac{2}{\beta^2(\alpha,\beta)} \sum_{k=0}^{\infty} \binom{-\alpha-\beta}{k} . A(i,j). \left[\frac{1}{(B_2)^2} - \frac{1}{(B_1)^2}\right]$$

$$E(X^2) = \frac{8}{B^2(\alpha,\beta)} \cdot \sum_{k=0}^{\infty} \binom{-\alpha-\beta}{k} \cdot \frac{A(i,j)}{(B_2)^3}$$

$$E(X^3) = \frac{12}{\beta^2(\alpha,\beta)} \sum_{k=0}^{\infty} \binom{-\alpha-\beta}{k} . A(i,j). \left[\frac{1}{(B_2)^4} - \frac{1}{(B_1)^4}\right]$$

$$E(X^4) = \frac{96}{B^2(\alpha,\beta)} \cdot \sum_{k=0}^{\infty} \binom{-\alpha-\beta}{k} \cdot \frac{A(i,j)}{(B_2)^5}$$

Therefore, using this four moments, we can easily obtain the variance σ^2, the skewness coefficient γ_1 and kurtosis coefficient γ_2 , which will be illustrated here numerically in section(7). If $(\alpha = \beta)$ the results of Asgharzadeh et al (2013) are obtained as special cases.

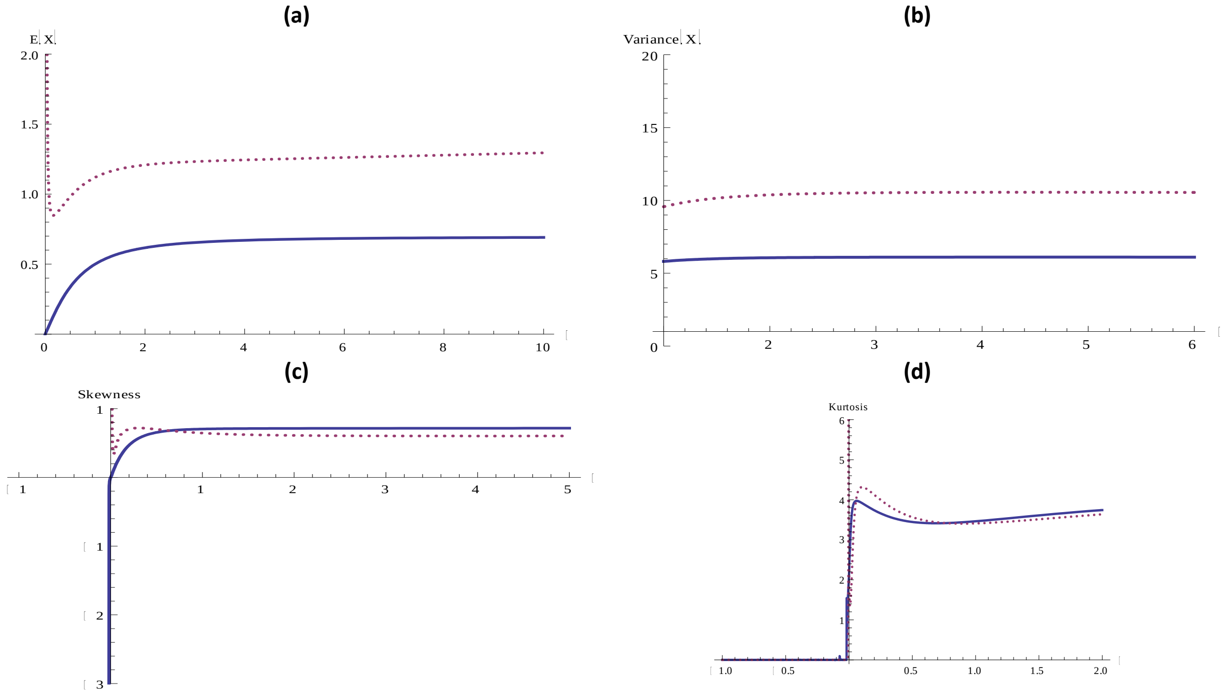

Figure 3: Illustrates the shapes of (a) the expectation value E(X) of GSLDIV (α, β, λ), when λ ϵ (0, 10), β = 1 and α =1 (Thick line) and α =2 (dotted line). (b) the variance var(X) , when λ ϵ (1, 6), β = 1 and α =1 (Thick line) and α =2 (dotted line). (c) the skewness coefficient, when λ ϵ (-1, 5), β = 1 and α =1 (Thick line) and α =2 (dotted line). (d) the kurtosis coefficient, when λ ϵ (-1, 10), β = 1 and α =1 (Thick line) and α =1.5 (dashed line).

5. Mean Deviation

The mean deviation about the mean μ denoted by $\Delta_1(X)$ and about the median M denoted by $\Delta_2(X)$ are defined by

$\Delta_1(X) = \int_{-\infty}^{\infty} |x-\mu| \; f(x,\alpha,\beta,\lambda)\, dx,$ and

$\Delta_2(X) = \int_{-\infty}^{\infty} |x-M| \; f(x,\alpha,\beta,\lambda)\, dx$

Receptivity, where $\mu = E(X)$ is the expectation of a random variable X having GSLDIV (α,β,λ)

Theorem (5.1):

The mean deviation about the mean μ of a random variable X having GSLD IV (α,β,λ) is giving as:

$$\Delta_1(X) = \begin{cases} \Delta_{11}(\mu)\,, & if\ \mu \le 0 \\ \Delta_{12}(\mu)\,, & if\ \mu \ge 0 \end{cases}$$

where,

$$\Delta_{11}(\mu) = \frac{2}{B^2(\alpha,\beta)} \cdot \sum_{k=0}^{\infty} \binom{-\alpha-\beta}{k} \cdot A(i,j). \left[\frac{1}{{B_1}^2}\left(2e^{\mu B_1} - 1\right) + \frac{1}{{B_2}^2}\right] - \mu,$$

$$\Delta_{12}(\mu) = \frac{2}{B^2(\alpha,\beta)} \cdot \sum_{k=0}^{\infty} \binom{-\alpha-\beta}{k} \cdot A(i,j). \left[\frac{1}{{B_2}^2}\left(2e^{\mu B_2} - 1\right) + \frac{1}{{B_1}^2}\right] + \mu,$$

Proof:

$\Delta_1(X)$ can be written in following form given by Nadarajah (2009) as:

$$\Delta_1(X) = 2\mu F(\mu) - \mu - \int_{-\infty}^{\mu} x \; f(x,\alpha,\beta,\lambda)\, dx + \int_{\mu}^{\infty} x \; f(x,\alpha,\beta,\lambda)\, dx \tag{5.1}$$

The proof is depending on two separately cases, when $\mu \le 0$ and when $\mu > 0$, the calculations are employing the triple series form of the pdf given in (2.3). Now, if $\mu \le 0$, we have

$$F(\mu) = \frac{2}{B^2(\alpha,\beta)} \cdot \sum_{k=0}^{\infty} \binom{-\alpha-\beta}{k} \cdot A(i,j). \frac{1}{B_1} . e^{\mu B_1} \tag{5.2}$$

The integrals in (5.1) can be obtained as following

$$\int_{-\infty}^{\mu} x \; f(x,\alpha,\beta,\lambda)\, dx = \frac{2}{B^2(\alpha,\beta)} \cdot \sum_{k=0}^{\infty} \binom{-\alpha-\beta}{k} \cdot A(i,j).\ I_6 \tag{5.3}$$

where,

$$I_6 = \int_{-\infty}^{\mu} x\, e^{xB_1}\, dx = \frac{1}{B_1}\left(\mu - \frac{1}{B_1}\right) e^{\mu B_1} \tag{5.4}$$

From equations (5.3) and (5.4) we get

$$\int_{-\infty}^{\mu} x\ f(x,\alpha,\beta,\lambda)\ dx = \frac{2}{B^2(\alpha,\beta)}\ .\sum_{k=0}^{\infty}\binom{-\alpha-\beta}{k}\ .A(i,j).\frac{1}{B_1}(\mu-\frac{1}{B_1})e^{\mu B_1} \tag{5.5}$$

Similarly, we have

$$\int_{\mu}^{\infty} x\ f(x,\alpha,\beta,\lambda)\ dx = \frac{2}{B^2(\alpha,\beta)}\ .\sum_{k=0}^{\infty}\binom{-\alpha-\beta}{k}\ .A(i,j).[\frac{1}{B_1}\left(\mu-\frac{1}{B_1}\right)e^{\mu B_1}-\frac{1}{{B_1}^2}+\frac{1}{{B_2}^2}] \tag{5.6}$$

Substituting from (5.2),(5.5), and (5.6) in (5.1) the result of $\Delta_{11}(\mu)$ is obtained.
The second case, for $\mu > 0$, we get

$$F(\mu) = 1-\frac{2}{B^2(\alpha,\beta)}\ .\sum_{k=0}^{\infty}\binom{-\alpha-\beta}{k}\ .A(i,j).\frac{1}{B_2}.e^{-\mu B_2} \tag{5.7}$$

The integrals in (5.1) are evaluating as:

$$\int_{-\infty}^{\mu} x\ f(x,\alpha,\beta,\lambda)\ dx = \frac{2}{B^2(\alpha,\beta)}\ .\sum_{k=0}^{\infty}\binom{-\alpha-\beta}{k}\ .A(i,j).[\frac{1}{{B_2}^2}+\frac{1}{B_2}\left(\mu+\frac{1}{B_2}\right)e^{\mu B_2}-\frac{1}{{B_1}^2}] \tag{5.8}$$

$$\int_{\mu}^{\infty} x\ f(x,\alpha,\beta,\lambda)\ dx = \frac{2}{B^2(\alpha,\beta)}\ .\sum_{k=0}^{\infty}\binom{-\alpha-\beta}{k}\ .A(i,j).\frac{1}{B_2}\left(\frac{1}{B_2}-\mu\right)e^{\mu B_2} \tag{5.9}$$

Employing (5.7),(5.8), and (5.9) in (5.1) the results of $\Delta_{12}(\mu)$ is obtained, which complete the proof of theorem.

6. Transformation of variables

In practice, one often works with the family of distribution generated by linear transformation $Z = \mu + \eta X$, where X has GSLDIV(α,β,λ). The random variable Z gives the general class of the GSLDIV(α,β,λ) by inclusion of the location parameter μ and the scale parameter η. It is easy to see that the random variable Z having also GSLDIV$(\alpha,\beta,\lambda,\mu,\eta)$.

Theorem (6.1):
Let X be a random variable having GSLDIV(α,β,λ) , and $Z = \mu + \eta X$. Then the n<u>th</u> moment of the random variable Z is given by

$$E(Z)^n = \sum_{j=0}^{n}\binom{n}{j}\mu^{n-j}\,\eta^j E(X^j)$$

By elementary calculation, we can prove the theorem. Therefore by illustrating the first four moments of a random variable X given in section (4)

(i) $E(Z) \quad = \mu + \eta E(X)$
(ii) $var(Z) = \eta^2\ var(X)$
(iii) $\gamma_1(Z) \quad = \eta^3\ \gamma_1(X)$
(iv) $\gamma_2(Z) \quad = \eta^4\ \gamma_2(X)$

7. Numerical Example

In this section, we express the flexibility of the distribution to account for wide ranges of the skewness and the kurtosis coefficients γ_1, γ_2 respectively. The mean deviation about the mean is also given for ($\beta = 1$ and different values of α and λ).

λ / α	0	1	2	3	4	15	20
1	**0.00397921**	**-1.06715**	**0.688399**	**0.879374**	**0.963878**	**1.55392**	**1.95642**
2	**0**	**1.21645×10^{-9}**	**1.09019×10^{-8}**	**1.09019×10^{-8}**	**1.61090×10^{-7}**	**0.00431492**	**0.00546951**
3	**-2.32976×10^{-14}**	**-8.25856×10^{-16}**	**-1.26204×10^{-15}**	**-1.84072×10^{-15}**	**-2.43318×10^{-15}**	**3.00771×10^{-13}**	**1.75283×10^{-12}**
4	**-5.85248×10^{-21}**	**-4.05034×10^{-21}**	**-6.90184×10^{-21}**	**-1.08120×10^{-20}**	**-1.57064×10^{-20}**	**-1.04786×10^{-19}**	**-5.93160×10^{-20}**
15	**-2.20439×10^{-60}**	**-6.82711×10^{-60}**	**-1.41885×10^{-59}**	**-2.42661×10^{-59}**	**-3.70563×10^{-59}**	**-3.56666×10^{-58}**	**-6.10366×10^{-58}**
20	**-1.01189×10^{-72}**	**-3.32695×10^{-72}**	**-7.03555×10^{-72}**	**-1.21327×10^{-71}**	**-1.86177×10^{-71}**	**-1.81517×10^{-70}**	**-3.11053×10^{-70}**

Table 1: Skewness coefficients of GSLDIV(α, β, λ) for β = 2.

λ / α	0	1	2	3	4	15	20
1	**-0.0000178965**	**8.43576**	**4.34649**	**4.35356**	**4.61145**	**8.41985**	**11.4415**
2	**-0.000217223**	**1.14279×10^{-11}**	**1.00814×10^{-10}**	**6.48378×10^{-10}**	**3.12726×10^{-9}**	**0.00238411**	**0.00325009**
3	**-7.07013×10^{-20}**	**-3.09378×10^{-22}**	**-3.70565×10^{-22}**	**2.15101×10^{-22}**	**4.08639×10^{-21}**	**4.13294×10^{-17}**	**3.80039×10^{-16}**
4	**-1.51547×10^{-29}**	**-2.62460×10^{-30}**	**-5.20149×10^{-30}**	**-9.65149×10^{-30}**	**-1.62266×10^{-29}**	**2.16821×10^{-27}**	**2.13053×10^{-26}**
15	**-1.41954×10^{-89}**	**-7.25928×10^{-89}**	**-2.13505×10^{-88}**	**-4.73348×10^{-88}**	**-8.88671×10^{-88}**	**-2.61773×10^{-86}**	**-5.85189×10^{-86}**
20	**-3.84837×10^{-108}**	**-2.20924×10^{-107}**	**-6.71880×10^{-107}**	**-1.51348×10^{-106}**	**-2.86799×10^{-106}**	**-8.65918×10^{-105}**	**-1.94078×10^{-104}**

Table 2: Kurtosis coefficients of GSLDIV(α, β, λ) for β = 2.

λ / α	0	1	2	3	4	15
1	1.49214×10^{-13}	4.3259×10^{14}	6.56916×10^{62}	7.83093×10^{101}	3.87547×10^{139}	8.26094×10^{665}
2	1.66624×10^{15813}	9.83283×10^{47637}	8.68520×10^{42835}	1.42721×10^{36948}	4.00755×10^{32627}	1.12668×10^{34922}
3	$2.20260\times10^{4735296}$ 3	$9.28478\times10^{57099403}$	$1.50457\times10^{4556817}$ 2	$2.78712\times10^{3700061}$ 2	$7.07169\times10^{3097385}$ 0	$2.85089\times10^{1147529}$ 9
4	$2.31443\times10^{9538625}$ 0	$8.23480\times10^{10933742}$ 2	$1.31518\times10^{8668566}$ 9	$2.06428\times10^{7019979}$ 6	$1.64426\times10^{5867195}$ 0	$1.25610\times10^{2132769}$ 5

Table 3: Mean deviation about the mean of GSLDIV(α, β, λ) for β =2.

8. Conclusion

From figure 3, table 1 and table 2, we see that (i) the expectation value E(X) increases as λ increases and decreases as α increases. (ii) var(x) decreases as |λ| increases and decreases as α increases. (iii) Skewness coefficient γ_1 increases as λ increases and decreases as α increase. (iv) kurtosis coefficients γ_2 initially decreases before increasing as |λ| increases and decreases as α increases. (v) from for the standard logistic distribution γ_1 = 0 and γ_2 =2 which means it is symmetric platykurtic (vi) from table 3, it is clear that the mean deviation about the mean increases when λ increases and α increases. (vii) the flexibility of SGLD(λ,α, β) in terms of accommodating more general types of skewness than the ordinary SDL(λ) is illustrated by computing moments and, in particular, skewness and kurtosis coefficients.

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