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Comparative Analysis of Stratified Randomized Response Models for HIV Seroprevalence Surveys

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Abstract
Seroprevalence surveys of HIV pandemic are highly sensitive especially in Africa. The objective of this study is to reach research frontier to devise a two-way randomized response model (RRM) in stratification and use same to estimate HIV seroprevalence rates in a given population and compare results with the existing seroprevalence rates. The randomized response techniques (RRT) guarantees the anonymity of respondents in surveys aimed at determining the frequency of stigmatic, embarrassing or criminal behaviour where direct techniques for data collection may induce respondents to refuse to answer or give false responses. The motivation was to improve upon the existing RRMs as well as to apply them to estimate HIV seroprevalence rates. Warner proposed the pioneering RRM for estimating the proportion of persons bearing a socially disapproved character. Quatember produced unified criteria for all RRTs, Kim and Warde proposed a stratified RRM and so many others. The proposed two-way RRM in stratification for HIV seroprevalence surveys was relatively more efficient than the Kim and Warde stratified estimator for a fixed sample size. The chosen design parameter was 0.7, using the criteria of Quatember who derived the statistical properties of the standardized estimator for general probability sampling and privacy protection. Furthermore, the model was used to estimate the HIV seroprevalence rate in a sampled population of adults 3,740 people aged 18 years and above attending a clinic in Kaduna, Nigeria using a sample size of 550. The findings revealed that HIV seroprevalence rate, as estimated by Model I, stood at 6.1% with a standard error of 0.0082 and a 95% confidence interval of [4.5%, 7.7%]. These results are consistent with that of Nigerian sentinel survey (2003) conducted by NACA, USAID and CDC which estimated the HIV seroprevalence in Kaduna State as *6.0%*. Hence, the RRTs herein can serve as new viable methods for HIV seroprevalence surveys. Similarly, the result for model II show that, using the survey data, the model estimated the HIV seroprevalence rate as *8.74%* with a standard error of *0.0134* and 95% confidence bands of [*6.1%*, *11.4%*]. Accordingly, the sentinel projected seroprevalence rate, using the EPP Package, for the next ten years (2013) was 9.7%; very consistent with the 95% confidence interval. Hence, the RRTs herein can also serve as new viable methods for HIV seroprevalence surveys. Model II has a better chance of estimating HIV seroprevalence because it has higher privacy preservation.

Key words
Randomized response techniques, stratified randomized response models, seroprevalence rates, design parameter, efficiency, sentinel surveys, stratified random sampling

1. Introduction

When sensitive topics are studied, respondents often react in ways that negatively affect the validity of the data. Such a threat to the validity of the results is the respondents' tendency to give socially desirable answers to avoid social embarrassment and to project a positive self-image (Rasinski, 1999). Warner (1965) reasoned that the reluctance of the respondents to reveal sensitive or probably harmful information would diminish when respondents could be convinced that their anonymity was guaranteed. Hence, Warner (1965) designed the first randomized response model (RRM). The crux of his method and all other RRTs that followed is that the meaning of the respondents' answers is hidden by a deliberate contamination of the data collection settings (Lee, 1993).

Studies with RRTs have been conducted in the areas of healthcare (Volicer & Volicer, 1982), on alcohol, drug abuse and sexual behaviour (Jarman, 1997), on child molestation (Fox and Tracy, 1986), on tax evasion (Houston & Tran, 2008), among others. Meta-analysis on 42 comparative studies showed that RRTs resulted in more valid population estimates than direct question–answer techniques (Lensvelt-Mulders et al., 2005). An advantage of using RRT when conducting sensitive research is that, the individual 'yes'-answer becomes meaningless as it is only a 'yes-answer' to the random device (Van der Hout, et al., 2002). The randomized response design is more effective than the direct question-answer design (Lensvelt-Mulders et al., 2005). The loss of efficiency in RR designs could be compensated when the results prove to be more valid (Kuk, 1990). When the loss in efficiency can be kept as small as possible the use of a RR design to study sensitive questions will become more profitable.

2. Methodology

In order apply the Strafified RRMs; a study was conducted in Gwamna Awan General Hospital, Kaduna, Nigeria in November, 2011. With a carefully coordinated field work and sampling design on a population of *3,740*

adults aged 18 years and above attending the Hospital using a sample size of *550* for each model. Furthermore, each model was used to estimate the HIV seroprevalence rate in the same population. Quatember (2009) both theoretically and empirically analyzed the effect of different design parameters on the performance of RRTs using different levels of privacy protection. Quatember (2009) suggested that 0.7 approximately works well for most RRM where the questions are regarded as highly sensitive. Hence, 0.7 is the chosen design parameter and deck of 50 cards as our random device throughout.

2.1 The Proposed HIV Seroprevalence Model I

The proposed HIV seroprevalence surveys Model I requires that a sample respondent in stratum *h* to answer an innocuous direct question and asked to use the random device R_{h1} if his/her answer to direct question is "yes". If answer to the direct question is "no", he/she is requested to use another random device R_{h2} twice. Both random devices R_{h1} and R_{h2} consist of two statements (i) "I am HIV positive" and (ii) "I am HIV negative", presented with probabilities P_{h1} and $(1-P_{h1})$ respectively. Here the random device R_{h2} would to be answered twice. Hence, we can obtain the estimator of population proportion π_h in *hth* stratum based on the responses from R_{h1} as follows. The probability of a 'yes' response from the respondents using R_{h1} is given by:

$$\lambda_{h1} = P_{h1}\pi_h^* + (1-P_{h1})\pi_{hy} = P_{h1}\pi_h^* + (1-P_{h1}) \tag{1}$$

Also, the probability of a 'no' response from the respondents using R_{h1} is given by:

$$\lambda'_{h1} = P_{h1}(1-\pi_h^*) + (1-P_{h1})(1-\pi_{hy}) = P_{h1}(1-\pi_h^*) \tag{2}$$

Since the respondent using R_{h1} has already answered yes to the direct question, $\pi_{hy} = 1$.

Among those that answered 'yes' to the innocuous questions in stratum *h*; suppose that n_{h1} report 'yes' and $(n_h - n_{h1})$ report 'no', the likelihood of the sample in the same stratum is given below:

$$\xi = \left[P_{h1}\pi_h^* + (1-P_{h1})\right]^{n_{h1}} \times \left[P_{h1}(1-\pi_h^*)\right]^{n_h - n_{h1}} \tag{3}$$

We obtain the maximum likelihood estimate (MLE) of π_h^* as follows:

$$\therefore \quad \pi_h^* = \frac{n_h P_{h1} - n_h + n_{h1}}{n_h P_{h1}} \tag{4}$$

Hence, the unbiased estimators in terms of the responses of the respondents using R_{h1} is given by:

$$\hat{\pi}_{h1} = \frac{\hat{\lambda}_{h1} - (1-P_{h1})}{P_{h1}} \tag{5}$$

Where; the proportion of 'yes' answers from R_{h1} in the sample is given as;

$$\hat{\lambda}_{h1} = \frac{n_{h1}}{n_h}$$

The variance of is obtained as follows:

$$Var(\hat{\pi}_{h1}) = \left[\frac{1}{P_{h1}}\right]^2 Var(\hat{\lambda}_{h1})$$

$$= \left[\frac{1}{P_{h1}}\right]^2 \left(\frac{\hat{\lambda}_{h1}(1-\hat{\lambda}_{h1})}{n_{h1}}\right)$$

$$\therefore \quad Var(\hat{\pi}_{h1}) = \frac{(1-\pi_{h1})(P_{h1}\pi_{h1} + 1 - P_{h1})}{n_{h1}P_{h1}} \tag{6}$$

The respondent, in *hth* stratum, giving a "no" answer to the question are to use R_{h2} twice to report two answers, where R_{h2} consists of the two statement of Warner's RR method. To have the first response reported the probabilities of the two statements are P_{h2} and $(1-P_{h2})$ whereas to get the second response from the responses

these probabilities are P_{h2}^{*} and $(1-P_{h2}^{*})$. Two unbiased estimators based on the two set of responses from respondents using R_{h2} can be defined as follows:

$$\pi_{h12}=\frac{\hat{\lambda}_{h2}-(1-P_{h2})}{(2P_{h2}-1)} \tag{7}$$

and $$\pi_{h22}=\frac{\hat{\lambda}_{h2}^{*}-(1-P_{h2}^{*})}{(2P_{h2}^{*}-1)} \tag{8}$$

where; $$\lambda_{h1}=P_{h2}\pi_h+(1-P_{h1})(1-\pi_h)=(2P_{h2}-1)\pi_h+(1-P_{h1}) \tag{9}$$

$$\lambda_{h2}^{*}=P_{h2}^{*}\pi_h+(1-P_{h2}^{*})(1-\pi_h)=(2P_{h2}^{*}-1)\pi_h+(1-P_{h2}^{*}) \tag{10}$$

Which are the probabilities of "yes" responses for the first and second use of R_{h2}. The variances of the estimators $\hat{\pi}_{h12}$ and $\hat{\pi}_{h22}$ are given by:

$$Var(\hat{\pi}_{h21})=\frac{\lambda_{h1}(1-\lambda_{h1})}{n_{h2}(2P_{h2}-1)^2}=\frac{\pi_{h1}(1-\pi_{h1})}{n_{h2}}+\frac{P_{h2}(1-P_{h2})}{n_{h2}(2P_{h2}-1)^2} \tag{11}$$

and $$Var(\hat{\pi}_{h22})=\frac{\lambda_{h2}(1-\lambda_{h2})}{n_{h2}(2P_{h2}^{*}-1)^2}=\frac{\pi_{h2}(1-\pi_{h2})}{n_{h2}}+\frac{P_{h2}^{*}(1-P_{h2}^{*})}{n_{h2}(2P_{h2}^{*}-1)^2} \tag{12}$$

These were obtained from Warner's RR model as given below. The first responses from respondents using R_{h2} can be defined as follows. The probability of a 'yes' response from the respondents using R_{h2} in the first response is given by:

$$\lambda_{h12}=P_{h1}\pi_h+(1-P_{h1})(1-\pi_h) \tag{13}$$

Also, the probability of a 'no' response from the respondents using R_{h2} in the first response is given by:

$$\lambda'_{h12}=P_{h1}(1-\pi_h)+(1-P_{h1})\pi_h \tag{14}$$

Among those that answered 'no' to the innocuous questions in stratum *h*; suppose that n_{h2} report 'yes' and (n_h-n_{h2}) report 'no' in first case, the likelihood of the sample in the same stratum is as follows:

$$\xi=\left[P_{h1}\pi_h+(1-P_{h1})(1-\pi_h)\right]^{n_{h2}}\times\left[P_{h1}(1-\pi_h)+(1-P_{h1})\pi_h\right]^{n_h-n_{h2}} \tag{15}$$

We also obtain the MLE of π_h, as follows:

$$\pi_{h12}=\frac{\hat{\lambda}_{h2}-(1-P_{h2})}{(2P_{h2}-1)} \tag{16}$$

Where; the proportion of 'yes' answers from R_{h1} in the sample is given as;

$$\hat{\lambda}_{h2}=\frac{n_{h2}}{n_h}$$

The variance of is obtained as follows:

$$Var(\pi_{h21})=\left[\frac{1}{n_h(2P_{h2}-1)}\right]^2 Var(n_{h2}) \tag{17}$$

Since; $$\pi_{h21}=\frac{\frac{n_{h2}}{n_h}-(1-P_{h2})}{(2P_{h2}-1)}=\frac{n_{h2}}{n_h(2P_{h2}-1)}+\frac{P_{h2}-1}{2P_{h2}-1}$$

Then; $$Var(\pi_{h21})=\left[\frac{1}{n_h(2P_{h2}-1)}\right]^2 Var(n_{h2})$$

$$=\frac{Var(X_{i2})}{n_h^2(2P_{h2}-1)^2}$$

$$= \frac{[P_{h1}\pi_h + (1-P_{h1})(1-\pi_h)][P_{h1}(1-\pi_h) + (1-P_{h1})\pi_h]}{n_h(2P_{h2}-1)^2}$$

Hence; $$Var(\hat{\pi}_{h21}) = \frac{\pi_{h2}(1-\pi_{h2})}{n_{h2}} + \frac{P_{h2}(1-P_{h2})}{n_{h2}(2P_{h2}-1)^2} = \frac{\lambda_{h1}(1-\lambda_{h1})}{n_{h2}(2P_{h2}-1)^2} \quad (18)$$

Where; $$\lambda_{h1} = P_{h2}\pi_h + (1-P_{h1})(1-\pi_h) = (2P_{h2}-1)\pi_h + (1-P_{h1})$$

The second response from R_{h2} have similar parameters; so that we have:

$$\pi_{h22} = \frac{\hat{\lambda}^*_{h2} - (1-P^*_{h2})}{(2P^*_{h2}-1)}$$

and $$Var(\hat{\pi}_{h22}) = \frac{\pi_h(1-\pi_h)}{n_{h2}} + \frac{P^*_{h2}(1-P^*_{h2})}{n_{h2}(2P^*_{h2}-1)^2} = \frac{\lambda_{h2}(1-\lambda_{h2})}{n_{h2}(2P^*_{h2}-1)^2}$$

where; $$\lambda^*_{h2} = P^*_{h2}\pi_h + (1-P^*_{h2})(1-\pi_h) = (2P^*_{h2}-1)\pi_h + (1-P^*_{h2})$$

From Lanke (1976), to provide equal protection in R_{h1} and R_{h2} it can be shown that we must have either of the following:

$$P_{h2} = \frac{1}{2-P_{h1}}$$

or $$P^*_{h2} = \frac{1}{2-P_{h1}}$$

With this restriction the variance of the estimators $\hat{\pi}_{h12}$ and $\hat{\pi}_{h22}$ become same. To estimate π_h from the information collected by the double use of R_{h2} , we defined an unbiased estimator as follows:

$$\hat{\pi}_{hP} = \lambda_1\hat{\pi}_{h21} + \lambda_2\hat{\pi}_{h22}$$

where; λ_1 and λ_2 are the weights assuming value 0.5 when $Var(\hat{\pi}_{hP})$ is optimized.

Thus the $\hat{\pi}_{hP}$ becomes:

$$\hat{\pi}_{hP} = \frac{\hat{\pi}_{h21} + \hat{\pi}_{h22}}{2} \quad (19)$$

Its variance is given by:

$$Var(\hat{\pi}_{hP}) = \frac{Var(\hat{\pi}_{h21})}{2} = \frac{1}{2}\left[\frac{\pi_h(1-\pi_h)}{n_{h2}} + \frac{P_{h2}(1-P_{h2})}{n_{h2}(2P_{h2}-1)^2}\right] \quad (20)$$

Since; $$Var(\hat{\pi}_{h21}) = Var(\hat{\pi}_{h22})$$

and $$P_{h2} = 1 - P^*_{h2}$$

An unbiased estimator in terms of all the information collected by both the random devices R_{h1} and R_{h2} in the *hth* stratum is defined as follows:

$$\hat{\pi}_{hP(tot)} = \pi_h = \frac{n_{h1}}{n_h}\hat{\pi}_{h1} + \frac{n_{h2}}{n_h}\hat{\pi}_{hP} \quad (21)$$

As both the random devices R_{h1} and R_{h2} are independent, the variance of $\hat{\pi}_{hP(tot)}$ under the restriction by Lanke (1976):

$$P_{h2} = \frac{1}{2-P_{h1}}$$

Thus is given by:

$$Var(\hat{\pi}_{hP(tot)}) = \left(\frac{\lambda_h + 1}{2}\right)\frac{\pi_h(1-\pi_h)}{n_h} + \frac{\lambda_h(1-\pi_h)(1-P_{h1})}{n_h P_{h1}} + \frac{(1-\lambda_h)(1-P_{h1})}{2n_h P^2_{h1}} \quad (22)$$

where; $\lambda_h = \frac{n_{h1}}{n_h}$

A stratified proportion estimator of the population proportion of the individuals with sensitive trait is defined as:

where; $\hat{\pi}_{Sero} = \sum_{i=1}^{L} W_h \hat{\pi}_{hP(tot)}$ (23)

Its variance is given by:

$$Var(\hat{\pi}_{Sero}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[\pi_h(1-\pi_h)\left(\frac{\lambda_h+1}{2}\right) + \frac{\lambda_h(1-\pi_h)(1-P_{h1})}{P_{h1}} + \frac{(1-\lambda_h)(1-P_{h1})}{2P_{h1}^2}\right] \quad (24)$$

Its variance under the optimum allocation of total sample size into different strata is given by:

$$Var(\hat{\pi}_{Sero}) = \frac{1}{n}\left[\sum_{h=1}^{L} W_h\left[\left(\frac{\lambda_h+1}{2}\right)\pi_h(1-\pi_h) + \frac{\lambda_h(1-P_{h1})(1-\pi_h)}{P_{h1}} + \frac{(1-P_{h1})(1-\lambda_h)}{2P_{h1}^2}\right]^{\frac{1}{2}}\right]^2 \quad (25)$$

2.2 The Proposed HIV Seroprevalence Model II

The HIV seroprevalence surveys Model II requires that a sample respondent in stratum *h* to answer an innocuous direct question and asked to use the random device R_{h1} if his/her answer to direct question is "yes". If answer to the direct question is "no", he/she is requested to use another random device R_{h2}. The random device R_{h1} consists of two statements (i) "I am HIV positive" and (ii) "I am HIV negative", presented with probabilities P_{h1} and $(1-P_{h1})$ respectively. Similarly, the random device R_{h2} consists of the two statements (i) "I am HIV positive" and (ii) "I am HIV negative", presented with probabilities and P_{h2} and $(1-P_{h2})$ respectively. The probabilities of a 'yes' response from the respondents using R_{h1} and R_{h2} are respectively given by:

$$\lambda_{h1} = P_{h1}\pi_h + (1-P_{h1})\pi_{hy} = P_{h1}\pi_h + (1-P_{h1}) \quad (26)$$

And

$$\lambda_{h2} = P_{h2}\pi_h + (1-P_{h2}) \quad (27)$$

On the other hand, the probabilities of a 'no' response from the respondents using R_{h1} and R_{h2} are respectively given by:

$$\lambda'_{h1} = P_{h1}(1-\pi_h) + (1-P_{h1})(1-\pi_{hy}) = P_{h1}(1-\pi_h) \quad (28)$$

And

$$\lambda'_{h2} = P_{h2}(1-\pi_h) \quad (29)$$

Since the respondent using R_{h1} has already answered yes to the direct question, $\pi_{hy} = 1$.

Among those that answered 'yes' to the innocuous questions in stratum *h*; suppose that n_{h1} report 'yes' and $(n_h - n_{h1})$ report 'no', the likelihood of the sample in the same stratum is as follows:

$$\xi = \left[P_{h1}\pi_h + (1-P_{h1})\right]^{n_{h1}} \times \left[P_{h1}(1-\pi_h)\right]^{n_h - n_{h1}} \quad (30)$$

The natural log of the likelihood is given below:

$$\log\xi = n_{h1}\log\left[P_{h1}\pi_h + (1-P_{h1})\right] + (n_h - n_{h1})\log\left[P_{h1}(1-\pi_h)\right] \quad (31)$$

To obtain the value of π_h, differentiate $\log\xi$ w.r.t. π_h and equate to zero as follows:

$$\frac{\partial\log\xi}{\partial\pi_h} = \frac{n_{h1}P_{h1}}{P_{h1}\pi_h + (1-P_{h1})} - \frac{(n_h - n_{h1})P_{h1}}{P_{h1}(1-\pi_h)} = 0 \quad (32)$$

$$\frac{n_{h1}P_{h1}}{P_{h1}\pi_h + (1-P_{h1})} = \frac{(n_h - n_{h1})P_{h1}}{P_{h1}(1-\pi_h)}$$

$$n_{h1}P_{h1}(1-\pi_h) = (n_h - n_{h1})\left[P_{h1}\pi_h + (1-P_{h1})\right]$$

$$n_{h1}P_{h1} - n_{h1}P_{h1}\pi_h = n_hP_{h1}\pi_h + n_h - n_hP_{h1} - n_{h1}P_{h1}\pi_h - n_{h1} + n_{h1}P_{h1}$$
$$n_hP_{h1}\pi_h = n_hP_{h1} - n_h + n_{h1}$$
$$\pi_h = \frac{n_hP_{h1} - n_h + n_{h1}}{n_hP_{h1}}$$

Hence, the unbiased estimators in terms of the responses of the respondents using R_{h1} is given by:

$$\hat{\pi}_{h1} = \frac{\hat{\lambda}_{h1} - (1 - P_{h1})}{P_{h1}} \tag{33}$$

Where the proportion of 'yes' answers from R_{h1} in the sample is $\hat{\lambda}_{h1} = n_{h1}/n_h$ .The variance of $\hat{\pi}_{h1}$ is obtained as follows:

$$Var(\hat{\pi}_{h1}) = \left[\frac{1}{P_{h1}}\right]^2 Var(\hat{\lambda}_{h1}) \tag{34}$$
$$= \left[\frac{1}{P_{h1}}\right]^2 \left(\frac{\hat{\lambda}_{h1}(1-\hat{\lambda}_{h1})}{n_{h1}}\right)$$
$$= \left[\frac{1}{P_{h1}}\right]^2 \frac{[P_{h1}\pi_h + (1-P_{h1})][P_{h1}(1-\pi_h)]}{n_{h1}}$$
$$= \frac{[P_{h1}\pi_h + (1-P_{h1})](1-\pi_h)}{n_{h1}P_{h1}}$$

Hence;

$$Var(\hat{\pi}_{h1}) = \frac{(1-\pi_{h1})(P_{h1}\pi_{h1} + 1 - P_{h1})}{n_{h1}P_{h1}}$$

Similarly, the unbiased estimators in terms of the responses of the respondents using R_{h2} is given by:

$$\hat{\pi}_{h2} = \frac{\hat{\lambda}_{h2} - (1 - P_{h2})}{P_{h2}} \tag{35}$$

Where the proportion of 'yes' answers from R_{h2} in the sample is $\hat{\lambda}_{h2} = n_{h2}/n_h$ .The variance of $\hat{\pi}_{h2}$ is obtained as follows:

$$Var(\hat{\pi}_{h2}) = \frac{(1-\pi_{h2})(P_{h2}\pi_{h2} + 1 - P_{h2})}{n_{h2}P_{h2}}$$

In stratum *h* two randomization devices R_{h1} and R_{h2} are equally protective against the privacy of the respondents if $P_{h1} = P_{h2} = P_h$. Under this setting, the variances of the two unbiased estimators $\hat{\pi}_{h1}$ and $\hat{\pi}_{h2}$ become the same. We can also propose an estimator based on all the information collected in stratum *h* which we can use to estimate seroprevalence rates in stratum *h* as follows:

$$\hat{\pi}_h = \frac{n_{h1}}{n_h}\hat{\pi}_{h1} + \frac{n_{h2}}{n_h}\hat{\pi}_{h2} \tag{36}$$

Its variance is given by:

$$Var(\hat{\pi}_h) = \left(\frac{n_{h1}}{n_h}\right)^2 Var(\hat{\pi}_{h1}) + \left(\frac{n_{h2}}{n_h}\right)^2 Var(\hat{\pi}_{h2}) \tag{37}$$
$$= \left(\frac{n_{h1}}{n_h}\right)^2 \left[\frac{(1-\pi_h)(P_{h1}\pi_h + 1 - P_{h1})}{n_{h1}P_{h1}}\right] + \left(\frac{n_{h2}}{n_h}\right)^2 \left[\frac{(1-\pi_h)(P_{h2}\pi_h + 1 - P_{h2})}{n_{h2}P_{h2}}\right]$$

$$=\left(\frac{n_{h1}}{n_h^2}\right)\left[\frac{(1-\pi_h)(P_{h1}\pi_h+1-P_{h1})}{P_{h1}}\right]+\left(\frac{n_{h2}}{n_h^2}\right)\left[\frac{(1-\pi_h)(P_{h2}\pi_h+1-P_{h2})}{P_{h2}}\right]$$

If we decide that $P_{h1}=P_{h2}=P_h$ thus we get:

$$Var(\hat{\pi}_h)=\left(\frac{n_{h1}+n_{h2}}{n_h^2}\right)\left[\frac{(1-\pi_h)(P_h\pi_h+1-P_h)}{P_h}\right] \quad (38)$$

$$=\left(\frac{1}{n_h}\right)\left[\frac{(1-\pi_h)[P_h\pi_h+(1-P_h)]}{P_h}\right]$$

Hence;

$$Var(\hat{\pi}_h)=\frac{\pi_h(1-\pi_h)}{n_h}+\frac{(1-P_h)(1-\pi_h)}{n_hP_h}$$

An unbiased stratified seroprevalence rates estimator is given by:

$$\hat{\pi}_{sero}=\sum_{h=1}^{L}W_h\hat{\pi}_h \quad (39)$$

where; $W_h=N_h/N$ for is $h=1,2,...,L$

N_h is the total number of individuals in the stratum h

N is the total number of individuals in the population

obviously $\sum_{h=1}^{L}W_h=1$

Its variance is given by:

$$Var(\hat{\pi}_{Sero})=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\pi_h(1-\pi_h)+\frac{(1-P_h)(1-\pi_h)}{P_h}\right]^2 \quad (40)$$

Cochran (1977) established that the sampling fraction n_h/n is ignorable, then $Var(\hat{\pi}_{Sero})$ is minimized for a fixed total sample size n if:

$$n_h=\frac{nW_h\left[\pi_h(1-\pi_h)+\frac{(1-P_h)(1-\pi_h)}{P_h}\right]^{\frac{1}{2}}}{\sum_{h=1}^{L}W_h\left[\pi_h(1-\pi_h)+\frac{(1-P_h)(1-\pi_h)}{P_h}\right]^{\frac{1}{2}}} \quad (41)$$

where; $n_h=n_{h1}+n_{h2}$

$$\sum_{h=1}^{L}n_h=n$$

Thus substituting the optimum value of n_h in (15) we get:

$$Var(\hat{\pi}_{Sero})=\frac{1}{n}\left[\sum_{h=1}^{L}W_h\left[\pi_h(1-\pi_h)+\frac{(1-P_h)(1-\pi_h)}{P_h}\right]^{\frac{1}{2}}\right]^2 \quad (42)$$

3. Results

Computations from Model I

$$\hat{\pi}_{Sero}=\sum_{i=1}^{L}W_h\hat{\pi}_{hP(tot)} \qquad \text{where; } W_h=N_h/N \text{ for is } h=1,2,...,L$$

Its variance is given by:

$$Var(\hat{\pi}_{Sero}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[\pi_h(1-\pi_h)\left(\frac{\lambda_h+1}{2}\right) + \frac{\lambda_h(1-\pi_h)(1-P_{h1})}{P_{h1}} + \frac{(1-\lambda_h)(1-P_{h1})}{2P_{h1}^2}\right]$$

The computations for the model to estimate HIV seroprevalence rate give the following results:

$$\phi = \pi_h(1-\pi_h)\left(\frac{\lambda_h+1}{2}\right) + \frac{\lambda_h(1-\pi_h)(1-P_{h1})}{P_{h1}} + \frac{(1-\lambda_h)(1-P_{h1})}{2P_{h1}^2}$$

$$\hat{\pi}_{Sero} = \sum_{i=1}^{L} W_h \hat{\pi}_{hP(tot)} = 0.0612$$

$$Var(\hat{\pi}_{Sero}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[\pi_h(1-\pi_h)\left(\frac{\lambda_h+1}{2}\right) + \frac{\lambda_h(1-\pi_h)(1-P_{h1})}{P_{h1}} + \frac{(1-\lambda_h)(1-P_{h1})}{2P_{h1}^2}\right]$$

$$Var(\hat{\pi}_{Sero}) = 0.000067$$

$$SE(\hat{\pi}_{Sero}) = \sqrt{Var(\hat{\pi}_{Sero})} = 0.0082$$

The *95%* confidence interval for HIV seroprevalence rate using the two-way RR Model in stratification is given by:

$$(\hat{\pi}_{Sero}) \pm 1.96 \times SE(\hat{\pi}_{Sero}) = 0.0612 \pm 1.96 \times 0.0082 = [0.045, 0.077]$$

Computations from Model I

$$\phi = \pi_h(1-\pi_h) + (1-P_h)(1-\pi_h)/P_h$$

$$\hat{\pi}_{sero} = \sum_{h=1}^{L} W_h \hat{\pi}_h = 0.0874$$

$$Var(\hat{\pi}_{Sero}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[\pi_h(1-\pi_h) + \frac{(1-P_h)(1-\pi_h)}{P_h}\right]^2 = 0.00018$$

$$SE(\hat{\pi}_{Sero}) = \sqrt{Var(\hat{\pi}_{Sero})} = 0.0134$$

The *95%* confidence interval for HIV seroprevalence rate is given by:

$$\hat{\pi}_{sero} \pm 1.96 \times SE(\hat{\pi}_{Sero}) = 0.0874 \pm 1.96 \times 0.0134 = [0.061, 0.114]$$

4. Conclusion

This study was motivated by the fact that conventional data collection techniques usually cause evasive or untruthful responses when people are asked sensitive questions like their HIV serostatus. As a result, it is difficult to make accurate inferences from such unreliable data. Hence a two-way RR Model in stratification was devised using the work of Warner (1965), Arnab (2004), Quatember (2009), among others particularly for HIV seroprevalence surveys.

Furthermore, the model was used to estimate HIV seroprevalence rate in a small adult population using a sample size of *550* and a design parameter of *0.7*. The result for model I show that, using the survey data, the model estimated the HIV seroprevalence rate as *6.1%* with a standard error of *0.0082*and 95% confidence bands of [*4.5%*, *7.7%*]. These estimates are for adults who are 18 years and above who attend a hospital. These results are consistent with that of Nigerian sentinel survey (2003) conducted by NACA, USAID and CDC which estimated the HIV seroprevalence in Kaduna State as *6.0%*. Hence, the RRTs herein can serve as new viable methods for HIV seroprevalence surveys.

Similarly, the result for model II show that, using the survey data, the model estimated the HIV seroprevalence rate as *8.74%* with a standard error of *0.0134* and 95% confidence bands of [*6.1%*, *11.4%*]. These estimates are for adults who are 18 years and above who attend a hospital. Accordingly, the sentinel projected seroprevalence rate, using the EPP Package, for the next ten years (2013) was 9.7%; very consistent with the 95% confidence interval. Hence, the RRTs herein can also serve as new viable methods for HIV seroprevalence surveys. Model II has a better chance of estimating HIV seroprevalence because it has higher privacy preservation.

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Appendix Tables

Table 1: Samples and Strata Sizes Model I

Strata	Strata Description	N_h	n_h	n_{h1}	n_{h21}	n_{h22}	W_h
1	Married (Men/ Women)	1,285	189	32	42	38	0.344
2	Unmarried (Men/ Women)	2,020	297	56	55	63	0.540
3	Divorced/Separated/Widowed	435	64	12	10	11	0.116
Total		3,740	550	100	107	112	1.000

Table 2: Summary of Results of the Random Devices Model I

Strata	$\hat{\lambda}_{h1}$	$\hat{\pi}_{h1}$	$V(\hat{\pi}_{h1})$	$\hat{\lambda}_{h21}$	$\hat{\pi}_{h21}$	$V(\hat{\pi}_{h21})$	$\hat{\lambda}_{h22}$	$\hat{\pi}_{h22}$	$V(\hat{\pi}_{h22})$
1	0.376	0.109	0.0150	0.402	0.255	0.0358	0.365	0.163	0.0381
2	0.350	0.071	0.0083	0.401	0.253	0.0273	0.460	0.256	0.0246
3	0.343	0.061	0.0383	0.345	0.113	0.1412	0.379	0.198	0.0902

Table 3: Summary of Computations Model I

Strata	$\frac{n_{h1}}{n_h}$	$\hat{\pi}_{hP}$	$\frac{n_{h2}}{n_h}$	$\hat{\pi}_h$	$W_h\hat{\pi}_h$	$\frac{W_h^2}{n_h}$	$\hat{\pi}_h(1-\hat{\pi}_h)$	ϕ	$\sum_{h=1}^{L}\frac{W_h^2}{n_h}\phi$
1	0.169	0.209	0.201	0.060	0.0206	0.00063	0.056	0.037	0.000023
2	0.189	0.255	0.212	0.067	0.0362	0.00098	0.063	0.041	0.000040
3	0.188	0.156	0.172	0.038	0.0044	0.00021	0.037	0.019	0.000004
Total					0.0612				0.000067

Table 4: Summary of Seroprevalence Results Model I

N	n	$\hat{\pi}_{Sero}$	$Var(\hat{\pi}_{Sero})$	95% confidence interval	
				Lower limit	Upper limit
3,740	550	0.0610	0.000067	0.045	0.077

Table 5: Samples and Strata Sizes Model II

Strata	Strata Description	N_h	n_h	n_{h1}	n_{h2}	W_h
1	Married (Men/ Women)	1,285	189	35	38	0.344
2	Unmarried (Men/ Women)	2,020	297	57	58	0.540
3	Divorced/Separated/Widowed	435	64	11	9	0.116
Total		3,740	550	103	105	1.000

Table 6: Summary of Results of the Random Devices Model II

Strata	$\hat{\lambda}_{h1}$	$\hat{\pi}_{h1}$	$V(\hat{\pi}_{h1})$	$\hat{\lambda}_{h2}$	$\hat{\pi}_{h2}$	$V(\hat{\pi}_{h2})$	$\hat{\pi}_h$	$V(\hat{\pi}_h)$
1	0.365	0.093	0.0135	0.409	0.156	0.0130	0.098	0.0052
2	0.383	0.119	0.0085	0.392	0.131	0.0838	0.097	0.0033
3	0.324	0.034	0.0406	0.300	0.000	0.0476	0.011	0.0156

Table 7: Summary of Computations Model II

Strata	W_h	$\hat{\pi}_h$	$W_h\hat{\pi}_h$	W_h^2/n_h	$\hat{\pi}_h(1-\hat{\pi}_h)$	$\sum_{h=1}^{L}\frac{W_h^2}{n_h}\phi^2$
1	0.344	0.098	0.0337	0.00063	0.156	0.000056
2	0.540	0.097	0.0524	0.00098	0.131	0.000088
3	0.116	0.011	0.0013	0.00021	0.000	0.000036
Total	1.000		0.0874			0.000180

Table 8: Summary of Seroprevalence Results Model II

N	n	$\hat{\pi}_{Sero}$	$Var(\hat{\pi}_{Sero})$	95% confidence interval	
				Lower limit	Upper limit
3,740	550	0.0874	0.00018	0.061	0.113

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