

Application of Marks to Computation of Ranks and Subdegrees of the Symmetric Group Acting on Ordered Pairs and on Ordered Triples

Patrick Kimani*, Jane Rimberia, Geoffrey Muthoka, Hussein Lao

Department of Pure and Applied Mathematics, Kenyatta University, P.O. Box 43844-00100, Nairobi, Kenya

*Email of the corresponding author: patkim32@gmail.com

Abstract

Ranks and subdegrees can be computed using combinatorial arguments, the Cauchy-Frobenius lemma and use of the concept of marks. However the concept of Marks has been given very little attention. In this paper we will apply the concept of marks to compute the ranks and subdegrees of the symmetric group S_n ($n = 5, 6, 7$) and S_n ($n = 6, 7, 8$) acting on ordered pairs and triples respectively.

Keywords: Ranks, Subdegrees, Marks

1. Introduction

Let G be transitive on a set X and let G_x be the stabilizer in G of a point $x \in X$. The orbits of G_x on X are known as suborbits of G . The number of these suborbits is known as the rank of G on X and the lengths of the suborbits is known as the subdegrees of G on X . If G acts on X transitively, then its action is equivalent to the action on the cosets of $H = G_x$, while that of H on any suborbit is equivalent to its action on the cosets of some subgroup F of H .

2. Notations and Preliminary Results

Notation 2.1

In this paper, $X^{[r]}$, $[a_1, a_2, \dots, a_n]$, $R(G)$ and $m(A, B, G)$ denote the set of all ordered r -element subsets from the set X , ordered r -element subset of X the rank of a permutation Group G and the mark of subgroup A in the representation of G on the cosets of subgroup B respectively.

Theorem 2.1(Rose [5])

Let G act on the set X , and let $x \in X$. Then

$$|Orb_G(x)| = |G:Stab_G(x)|.$$

Definition 2.2(Ivanov [2])

If $G_j \leq G_i \leq G$ and $(G_{j_1}, G_{j_2}, \dots, G_{j_n})$ is a complete set of conjugacy class representatives of subgroups of G_i that are conjugate to G_j in G , then

$$m(G_j, G_i, G) = \sum_{k=1}^n |N_G(G_{j_k}) : N_{G_i}(G_{j_k})|.$$

Definition 2.3

Let H_1, H_2, \dots, H_t be a set of representatives of all distinct conjugacy classes of subgroups of H in G , ordered such that $|H_1| \leq |H_2| \leq \dots \leq |H_t| = |H|$ Form a matrix $M = (m_{ij})$, where

$m_{ij} = m(H_j, H_i, H)$. We call M the table of marks of H ;

If we denote by Q_i the number of suborbits Δ_j on which the action of H is equivalent to its action on the cosets of H_i ($i = 1, 2, \dots, t$), by computing all Q_i we get the subdegrees of G on X .

Theorem 2.4 (Ivanov [2])

The numbers Q_i satisfy the system of linear equations

$$\sum_{i=j}^t Q_i m(H_j, H_i, H) = m(H_j, H, G), \text{ for each } j = 1, 2, \dots, t.$$

Theorem 2.5 (Rimberia [4])

Let G be the symmetric group S_n , then G acts transitively on $X^{[r]}$.

Theorem 2.6(Rimberia [4])

Let $G = S_n$ be the symmetric group on the set $X = \{a_1, a_2, \dots, a_n\}$. The stabilizer of an ordered subset $[a_1, a_2, \dots, a_r]$ in G is isomorphic to S_{n-r} , the symmetric group on the set $\{a_{r+1}, a_{r+2}, \dots, a_n\}$.

3. Ranks And Subdegrees Of The Symmetric Group S_n ($N = 5, 6, 7$) Acting On $X^{[2]}$

3.1 Rank and Subdegrees of S_5 acting on $X^{[2]}$

We first construct the table of marks of $H = \text{Stab}_G\{[1,2]\}$, where $G = S_5$. Then by Theorem 2.6, H is isomorphic to S_3 , the symmetric group on the set $\{3,4,5\}$. We will use GAP software to determine the conjugacy classes of subgroups and construct the table of marks by using the algorithms shown on the Appendix 1. The conjugacy classes of subgroups of S_3 are as shown below and its table of marks is as shown in Table 1

- i) H_1 Identity
- ii) H_2 3 conjugate subgroups of order 2
- iii) H_3 A normal cyclic group of order 3
- iv) H_4 H

For $F \leq H$, with the order they are represented in Table 1, the values of $m(F, H, G) = m(F)$ are displayed in Table 3.1.1.

Table 3.1.1: Marks of F in G(H), where $G = S_5$ and $H = \text{Stab}_G\{[1, 2]\}$

F	H_1	H_2	H_3	H_4
$m(F)$	20	6	2	2

Let $Q = (Q_1, Q_2, Q_3, Q_4)$ and the matrix obtained from Table 1 be M . By Theorem 2.3 and using Table 1 we obtain the following system of linear equations,

$$\begin{aligned} 6Q_1 + 3Q_2 + 2Q_3 + Q_4 &= 20 \\ Q_2 + Q_4 &= 6 \\ 2Q_3 + Q_4 &= 2 \\ Q_4 &= 2 \dots \dots \dots (1) \end{aligned}$$

On solving (1), we get $Q = (1, 4, 0, 2)$.

Therefore the subdegrees of G acting on $X^{[2]}$ are as shown in Table 3.1.2.

Table 3.1 2: subdegrees of S_5 acting on $X^{[2]}$

Suborbit length	1	3	6
No. of suborbits	2	4	1

From Table 3.1.2 we obtain, the rank of $G, R(G) = 1 + 4 + 2 = 7$.

We now apply the same procedure for the other actions.

3.2. Rank and Subdegrees of $G = S_6$ acting on $X^{[2]}$

Let $H = \text{Stab}_G\{[1,2]\}$, where $G = S_6$. By Theorem 3.1.1 H is isomorphic to S_4 which has 11 conjugacy classes of subgroups. These are;

- i). H_1 identity.
- ii). H_2 conjugate subgroups of order 2 generated by permutations of the form (ab)(cd).
- iii). H_3 conjugate subgroups of order 2 generated by permutations of the form (ab).
- iv). H_4 conjugate cyclic subgroups of order 3.
- v). H_5 a normal subgroups of order 4 generated by permutations of the form (ab)(cd) and (ac)bd).
- vi). H_6 3 conjugate subgroups of order 4 generated by permutations of the form (ab)(cd) and (cd).
- vii). H_7 3 conjugate cyclic subgroups of order 4.
- viii). H_8 4 conjugate subgroups of order 6 isomorphic to S_3 generated by permutations of the form (abc) and (cd).
- ix). H_9 3 conjugate subgroups of order 8 generated by permutations of the form (ab)(cd), (ac)(bd) and (bd).

- x). H_{10} a normal subgroup of order 12 isomorphic to A_4
- xi). H_{11} S_4

Its table of marks is as shown Table 2.

For $F \leq H$, the values of $m(F, H, G) = m(F)$ are displayed in Table 3.

Table 3.2.1: Marks of F in $G(H)$, where $G = S_6$ and $H = \text{Stab}_G\{[1, 2]\}$

F	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8	H_9	H_{10}	H_{11}
$m(F)$	30	12	2	6	2	2	2	6	2	2	2

Let $Q = (Q_1, Q_2, \dots, Q_{11})$ By Theorem 2.3 and using Table 3 we obtain the following system of linear equations in matrix form,

$$\begin{pmatrix}
 24 & 12 & 12 & 8 & 6 & 6 & 6 & 4 & 3 & 2 & 1 \\
 0 & 2 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 0 & 1 \\
 0 & 0 & 4 & 0 & 2 & 6 & 2 & 0 & 3 & 2 & 1 \\
 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 2 & 1 \\
 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 3 & 2 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
 \end{pmatrix}
 \begin{pmatrix}
 Q_1 \\
 Q_2 \\
 Q_3 \\
 Q_4 \\
 Q_5 \\
 Q_6 \\
 Q_7 \\
 Q_8 \\
 Q_9 \\
 Q_{10} \\
 Q_{11}
 \end{pmatrix}
 =
 \begin{pmatrix}
 30 \\
 12 \\
 2 \\
 6 \\
 2 \\
 2 \\
 2 \\
 6 \\
 2 \\
 2 \\
 2
 \end{pmatrix}
 \dots\dots\dots(2)$$

On solving (2), we obtain $Q = (0, 1, 0, 0, 0, 0, 0, 4, 0, 0, 2)$.

Therefore subdegrees of S_6 acting on $X^{[2]}$ are as shown in Table 3.2.2.

Table 3.2.2: Subdegrees of $G = S_6$ acting on $X^{[2]}$

Suborbit length	1	4	12
No. of suborbits	2	4	1

From Table 4 we see that, the rank of $G, R(G) = 2 + 4 + 1 = 7$.

3.3 Rank and Subdegrees of $G = S_7$ acting on $X^{[2]}$

Let $H = \text{Stab}_G\{[1,2]\}$, where $G = S_7$. By Theorem 2.6 H is isomorphic to S_5 which has 19 conjugacy classes of subgroups (See Appendix 3). Its table of marks is as shown by Table 3.

For $F \leq H$, with the order they are represented in Table 3, the values of $m(F, H, G) = m(F)$ are displayed in Table 3.3.1.

Table 3.3.1: Marks of F in $G(H)$ where $G = S_7$ and $H = \text{Stab}_G\{[1, 2]\}$

F	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8	H_9	H_{10}
$m(F)$	42	20	6	12	6	6	6	2	12	2

F	H_{11}	H_{12}	H_{13}	H_{14}	H_{15}	H_{16}	H_{17}	H_{18}	H_{19}
$m(F)$	2	6	2	6	2	2	6	2	2

Let $Q = (Q_1, Q_2, \dots, Q_{19})$. By Theorem 2.3 and using Table 5, we obtain the following system of linear equations in matrix form,

$$\begin{pmatrix}
 120 & 60 & 60 & 40 & 30 & 30 & 30 & 24 & 20 & 20 & 20 & 15 & 12 & 10 & 10 & 6 & 5 & 2 & 1 \\
 0 & 6 & 0 & 0 & 0 & 6 & 0 & 6 & 2 & 0 & 3 & 0 & 0 & 4 & 0 & 3 & 0 & 1 \\
 0 & 0 & 4 & 0 & 2 & 6 & 2 & 0 & 0 & 4 & 3 & 4 & 2 & 2 & 2 & 1 & 2 & 1 \\
 0 & 0 & 0 & 4 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 0 & 4 & 1 & 0 & 2 & 2 & 1 \\
 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & 1 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 3 & 0 & 2 & 0 & 0 & 1 & 2 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 1 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 2 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 2 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 2 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
 \end{pmatrix} \cdot \begin{pmatrix} Q_1 \\ Q_2 \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ Q_{18} \\ Q_{19} \end{pmatrix} = \begin{pmatrix} 42 \\ 20 \\ 6 \\ 12 \\ 6 \\ 6 \\ 6 \\ 6 \\ 2 \\ 12 \\ 2 \\ 2 \\ 6 \\ 2 \\ 6 \\ 2 \\ 6 \\ 2 \\ 2 \\ 6 \\ 2 \\ 2 \\ 6 \\ 2 \\ 2 \\ 2 \\ 6 \\ 2 \\ 2 \\ 6 \\ 2 \\ 2 \end{pmatrix} \dots(3)$$

On solving (3) we obtain $Q = (0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 4, 0, 2)$.

Therefore, the Subdegrees of G acting on $X^{[2]}$ are as shown in Table 3.3.2.

Table 3.3.2: Subdegrees of $G = S_7$ acting on $X^{[2]}$

Suborbit length	1	5	20
No. of suborbits	2	4	1

From Table 6 we see that the Rank of $G, R(G) = 2 + 4 + 1 = 7$.

4. Ranks and Subdegrees Of $S_n (n = 6, 7, 8)$ Acting On $X^{[3]}$

4.1 Rank and Subdegrees of $G = S_6$ acting on $X^{[3]}$

Let $H = \text{Stab}_G\{[1,2,3]\}$, where $G = S_6$. By Theorem 2.6, H is isomorphic to S_3 , the symmetric group on the set $\{4, 5, 6\}$. Its table of marks is as shown by Table 1.

For $F \leq H$, with the order they are represented in Table 1, the values of $m(F, H, G) = m(F)$ are displayed in Table 4.1.1.

Table 4.1.1: Marks of F in $G(H)$, where $G = S_6$ and $H = \text{Stab}_G\{[1, 2, 3]\}$

F	H_1	H_2	H_3	H_4
$m(F)$	120	24	6	6

Let $Q = (Q_1, Q_2, Q_3, Q_4)$. By Theorem 2.3 and using Table 4.1.1, we obtain the following system of linear equations.

$$\begin{aligned}
 6Q_1 + 3Q_2 + 2Q_3 + Q_4 &= 120 \\
 Q_2 + Q_4 &= 24 \\
 2Q_3 + Q_4 &= 6 \\
 Q_4 &= 6 \dots\dots\dots(4)
 \end{aligned}$$

On solving (4) we obtain $Q = (10,18,0,6)$.

Therefore the subdegrees of G acting on $X^{[3]}$ are as shown in Table 4.1.2.

Table 4.1.2: subdegrees of $G = S_6$ acting on $X^{[3]}$

Suborbit length	1	3	6
No. of suborbits	6	18	10

From Table 8 we see that the rank of $G, R(G) = 10 + 18 + 6 = 34$.

4.2. Rank and Subdegrees of $G = S_7$ acting on $X^{[3]}$

Let $H = \text{Stab}_G\{[1,2,3]\}$, where $G = S_7$. By Theorem 2.6 H is isomorphic to S_4 , the symmetric group on the set $\{4,5,6,7\}$. Its table of marks is as shown by Table 2.

For $F \leq H$, with the order they are represented in Table 2. The values of $m(F, H, G) = m(F)$ are displayed in Table 4.2.1.

Table 4.2.1: Marks of F in $G(H)$, where $G = S_7$ and $H = \text{Stab}_G\{[1, 2, 3]\}$

F	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8	H_9	H_{10}	H_{11}
m(F)	210	60	6	24	6	6	6	24	6	6	6

Let $Q = (Q_1, Q_2, \dots, Q_{11})$. By Theorem 2.3 and using Table 4.2.1, we obtain the following system of linear equations in matrix form.

$$\begin{pmatrix} 24 & 12 & 12 & 8 & 6 & 6 & 6 & 4 & 3 & 2 & 1 \\ 0 & 2 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 0 & 1 \\ 0 & 0 & 4 & 0 & 2 & 6 & 2 & 0 & 3 & 2 & 1 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 3 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \\ Q_{10} \\ Q_{11} \end{pmatrix} = \begin{pmatrix} 210 \\ 60 \\ 6 \\ 24 \\ 6 \\ 6 \\ 6 \\ 24 \\ 6 \\ 6 \\ 6 \end{pmatrix} \dots\dots\dots(5)$$

On solving (5) we obtain $Q = (1, 9, 0, 0, 0, 0, 0, 18, 0, 0, 6)$.

Therefore the subdegrees of G acting on $X^{[3]}$ are as shown in Table 4.2.2.

Table 4.2.2: Subdegrees of $G = S_7$ acting on $X^{[3]}$

Suborbit length	1	4	12	24
No. of suborbits	6	18	9	1

From Table 4.2.2 we see that the rank of $G, R(G) = 6 + 18 + 9 + 1 = 34$.

4.3 Rank and Subdegrees of $G = S_8$ acting on $X^{[3]}$

Let $H = \text{Stab}_G\{[1,2,3]\}$, where $G = S_8$. By Theorem 2.6 H is isomorphic to S_5 , the symmetric group on the set $\{4,5,6,7\}$. Its table of marks is as shown by Table 3.

For $F \leq H$, with the order they are represented in Table 3, the values of $m(F, H, G) = m(F)$ are as displayed in Table 4.3.1.

Table 4.3.1: Marks of F in $G(H)$, where $G = S_8$ and $H = \text{Stab}_G\{[1, 2, 3]\}$

F	H ₁	H ₂	H ₃	H ₄	H ₅	H ₆	H ₇	H ₈	H ₉	H ₁₀
m(F)	336	120	24	60	24	24	24	6	60	6

F	H ₁₁	H ₁₂	H ₁₃	H ₁₄	H ₁₅	H ₁₆	H ₁₇	H ₁₈	H ₁₉
m(F)	6	24	6	24	6	6	24	6	6

Let $Q = (Q_1, Q_2, \dots, Q_{19})$. By Theorem 2.3 and using Table 4.3.1, we obtain the following system of linear equations in matrix form,

$$\begin{pmatrix}
 120 & 60 & 60 & 40 & 30 & 30 & 30 & 24 & 20 & 20 & 20 & 15 & 12 & 10 & 10 & 6 & 5 & 2 & 1 \\
 0 & 6 & 0 & 0 & 0 & 0 & 6 & 0 & 6 & 2 & 0 & 3 & 0 & 0 & 4 & 0 & 3 & 0 & 1 \\
 0 & 0 & 4 & 0 & 2 & 6 & 2 & 0 & 0 & 0 & 4 & 3 & 4 & 2 & 2 & 2 & 1 & 2 & 1 \\
 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 0 & 4 & 1 & 0 & 2 & 2 & 1 \\
 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & 1 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 2 & 0 & 0 & 1 & 2 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 1 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 2 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 2 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
 \end{pmatrix}
 \begin{pmatrix}
 Q_1 \\
 Q_2 \\
 \vdots \\
 Q_{18} \\
 Q_{19}
 \end{pmatrix}
 =
 \begin{pmatrix}
 336 \\
 120 \\
 24 \\
 60 \\
 24 \\
 24 \\
 24 \\
 6 \\
 60 \\
 6 \\
 6 \\
 24 \\
 6 \\
 24 \\
 6 \\
 24 \\
 6 \\
 6 \\
 24 \\
 6 \\
 6
 \end{pmatrix} \dots (6)$$

On solving (6) we obtain $Q = (0, 1, 0, 0, 0, 0, 0, 0, 9, 0, 0, 0, 0, 0, 0, 0, 18, 0, 6)$

Therefore the subdegrees of S_8 acting on $X^{[3]}$ are as shown in the Table 4.3.2.

Table 4.3.2: Subdegrees of $G = S_8$ acting on $X^{[3]}$

Suborbit length	1	5	20	60
No. of suborbits	6	18	9	1

From Table 12, we see that rank of $G, R(G) = 6 + 18 + 9 + 1 = 34$

References

- [1] Burnside, W. (1911), "Theory of groups of finite order", Cambridge University Press, Cambridge.
- [2] Ivanov, A. A., Klin, M. H., Tsaranov, S. V. and Shpektorov, S. V. (1983), "The problem of computing subdegrees of transitive permutation groups", *Soviet Mathematical Survey* 38: 123 – 124.
- [3] Kamuti, I. N. (1992), "Combinatorial formulas, invariants and structures associated with primitive permutation representations of PSL (2, q) and PGL (2, q)", Ph. D. Thesis, Southampton University, U.K.
- [4] Rimberia J.K, Kamuti I.N, Kivunge B.M and Kinyua F. (2013), "Rank and Subdegrees of the symmetric group S_n acting on ordered r-element subsets", *Internantonal Journal of sciences, commerce and Humanities* 6 :112-118.
- [5] Rose, J. S. (1978), "A course on group theory", Cambridge University Press, Cambridge.

*******APPENDICES*******

Appendix 1: GAP algorithmis used to generate conjugacy classes and table of marks of $H = S_3$

To generate $H = S_3$ the symmetric group on the set $\{3,4,5\}$ on the GAP, we have used its generators; $(3,4,5)$ and $(3,4)$ and the command;

```
gap> H:=Group([(3,4,5),(3,4)]);
Group([ (3,5), (4,5) ])
```

To get the conjugacy classes of subgroups of H. here we use the command;

```
gap> ConjugacyClassesSubgroups(H);
[ Group( () )^G, Group( [ (4,5) ] )^G, Group( [ (3,5,4) ] )^G, Group( [ (3,5,4),
(4,5) ] )^G ]
```

To construct the table of marks of H, we use the command;

```
gap> Tom1:=TableOfMarks(H);
TableOfMarks( Sym( [ 3 .. 5 ] ) )
```

To view it we use the command;

```
gap> Display(Tom1);
1: 6
2: 3 1
3: 2 . 2
4: 1 1 1 1.....Table 1: Table of marks of S3
```

To get the sizes number of conjugate subgroups in each conjugacy class of subgroups we use the following command;

```
gap> LengthsTom(Tom1);
[ 1, 3, 1, 1 ]
```

To get the orders of the conjugacy classes of subgroups we use the following command;

```
gap> OrdersTom(Tom1);
[ 1, 2, 3, 6 ]
```

Let the matrix obtained from expressing T in matrix form be M. to get M we use the command;

```
gap> M:=MatTom(Tom1);
[[ 6, 0, 0, 0 ], [ 3, 1, 0, 0 ], [ 2, 0, 2, 0 ], [ 1, 1, 1, 1 ] ]
```

To solve the system of linear equation, we use the command;

```
gap> SolutionMat(M, [20,6,2,2]);
[ 1, 4, 0, 2 ]
```

For the other cases the similar algorithms were used to generate the other results.

Appendix 2: GAP algorithms used to generate conjugacy classes and table of marks of $H = S_4$

S_4 can be generated by (ab) and $(abcd)$. To generate $H = S_4$ the symmetric group on the set $\{3,4,5,6\}$ on the GAP, we have used its generators; $(3,4,5,6)$ and $(3,4)$ and the command;

```
gap> H:=Group([(3,4,5,6),(3,4)]);
Group([ (3,6), (4,6), (5,6) ])
```

We now use a similar procedure as in Appendix 1.

```
gap> ConjugacyClassesSubgroups(H);
gap> ConjugacyClassesSubgroups(H);
[ Group( () )^G, Group( [ (3,4)(5,6) ] )^G, Group( [ (5,6) ] )^G, Group( [ (4,6,5)
] )^G,
Group( [ (3,5)(4,6), (3,4)(5,6) ] )^G, Group( [ (5,6), (3,4)(5,6) ] )^G, Group( [
(3,5,4,6), (3,4)(5,6) ] )^G,
Group( [ (5,6), (4,6,5) ] )^G, Group( [ (3,5)(4,6), (3,4)(5,6), (5,6) ] )^G,
Group( [ (3,5)(4,6), (3,4)(5,6), (4,6,5) ] )^G, Group( [ (3,5)(4,6), (3,4)(5,6),
(4,6,5), (5,6) ] )^G ]
gap> Tom2:=TableOfMarks(H);
TableOfMarks( Sym( [ 3 .. 6 ] ) )
```

```
gap> Display(Tom2);
1: 24
2: 12 4
3: 12 . 2
4: 8 . . 2
5: 6 6 . . 6
6: 6 2 2 . . 2
7: 6 2 . . . 2
8: 4 . 2 1 . . . 1
9: 3 3 1 . 3 1 1 . 1
10: 2 2 . 2 2 . . . 2
11: 1 1 1 1 1 1 1 1 1 1.....Table 2: Table of marks of S4
gap> LengthsTom(Tom2);
[ 1, 3, 6, 4, 1, 3, 3, 4, 3, 1, 1 ]
gap> OrdersTom(Tom2);
[ 1, 2, 2, 3, 4, 4, 4, 6, 8, 12, 24 ]
```

Appendix 3: GAP algorithms used to generate conjugacy classes and table of marks of $H = S_5$

S_4 can be generated by (ab) and (abcd,e). To generate $H = S_5$ the symmetric group on the set {3,4,5,6,7} on the GAP, we have used its generators; (3,4,5,6,7) and (3,4) using the command;

```

gap> H:=Group([(3,4,5,6,7),(3,4)]);
Group([ (3,7), (4,7), (5,7), (6,7) ])
gap> ConjugacyClassesSubgroups(H);
[ Group( () )^G, Group( [ (6,7) ] )^G, Group( [ (4,5)(6,7) ] )^G, Group( [ (5,6,7) ] )^G, Group( [ (4,5)(6,7), (4,6)(5,7) ] )^G,
  Group( [ (4,5)(6,7), (4,6,5,7) ] )^G, Group( [ (6,7), (4,5) ] )^G, Group( [ (3,4,5,6,7) ] )^G, Group( [ (5,6,7), (6,7) ] )^G,
  Group( [ (5,6,7), (3,4)(6,7) ] )^G, Group( [ (6,7), (3,4,5) ] )^G, Group( [ (6,7), (4,5), (4,6)(5,7) ] )^G,
  Group( [ (3,4,5,6,7), (4,7)(5,6) ] )^G, Group( [ (4,5)(6,7), (4,6)(5,7), (5,6,7) ] )^G, Group( [ (6,7), (3,4,5), (4,5) ] )^G,
  Group( [ (3,4,5,6,7), (4,7)(5,6), (4,5,7,6) ] )^G, Group( [ (4,7)(5,6), (4,5)(6,7), (5,6,7), (6,7) ] )^G,
  Group( [ (3,4,5,6,7), (5,6,7) ] )^G, SymmetricGroup( [ 3 .. 7 ] )^G ]
gap> Tom3:=TableOfMarks(H);
TableOfMarks( Sym( [ 3 .. 7 ] ) )
gap> Display(T);
1: 120
2: 60 6
3: 60 . 4
4: 40 . . 4
5: 30 . 6 . 6
6: 30 . 2 . . 2
7: 30 6 2 . . . 2
8: 24 . . . . . 4
9: 20 6 . 2 . . . . 2
10: 20 . 4 2 . . . . . 2
11: 20 2 . 2 . . . . . 2
12: 15 3 3 . 3 1 1 . . . . 1
13: 12 . 4 . . . . 2 . . . . 2
14: 10 . 2 4 2 . . . . . 2
15: 10 4 2 1 . . 2 . 1 1 1 . . 1
16: 6 . 2 . . 2 . 1 . . . . 1 . . 1
17: 5 3 1 2 1 1 1 . 2 . . 1 . 1 . . 1
18: 2 . 2 2 2 . . 2 . 2 . . 2 2 . . . 2
19: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 .....
Table 3: Table of marks of  $S_5$ 
gap> LengthsTom(Tom3);
[ 1, 10, 15, 10, 5, 15, 15, 6, 10, 10, 10, 15, 6, 5, 10, 6, 5, 1, 1 ]
gap> OrdersTom(Tom3);
[ 1, 2, 2, 3, 4, 4, 4, 5, 6, 6, 6, 8, 10, 12, 12, 20, 24, 60, 120 ]
    
```


The IISTE is a pioneer in the Open-Access hosting service and academic event management. The aim of the firm is Accelerating Global Knowledge Sharing.

More information about the firm can be found on the homepage:
<http://www.iiste.org>

CALL FOR JOURNAL PAPERS

There are more than 30 peer-reviewed academic journals hosted under the hosting platform.

Prospective authors of journals can find the submission instruction on the following page: <http://www.iiste.org/journals/> All the journals articles are available online to the readers all over the world without financial, legal, or technical barriers other than those inseparable from gaining access to the internet itself. Paper version of the journals is also available upon request of readers and authors.

MORE RESOURCES

Book publication information: <http://www.iiste.org/book/>

IISTE Knowledge Sharing Partners

EBSCO, Index Copernicus, Ulrich's Periodicals Directory, JournalTOCS, PKP Open Archives Harvester, Bielefeld Academic Search Engine, Elektronische Zeitschriftenbibliothek EZB, Open J-Gate, OCLC WorldCat, Universe Digital Library, NewJour, Google Scholar

