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Oscillations of Third Order Nonlinear Neutral Differential Equations with Positive and Negative Coefficients

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Abstract

In this paper oscillation criterion is investigated for all solutions of the third-order non linear neutral differential equations with positive and negative coefficients:

$[x(t)+r(t)f(x(\tau(t)))]'''+p(t)g\left(x\left(\sigma(t)\right)\right)-q(t)g\left(x\left(\alpha(t)\right)\right)=0, t\geq t_0$ (1.1) Some sufficient conditions are established so that every solution of eq.(1.1) oscillate. We improved theorem 2.4 and theorem 2.10 in [5]. Examples are given to illustrated our main results.

Keyword: Neutral differential equations, Oscillations and nonoscillation.

1. Introduction

The study of neutral differential equations with positive and negative coefficients has been recently considered the attention of many authors see [2]- [5] and [7]-[9] a few of them have been investigated the case with variable coefficients , see [2]-[3], [5],[7]- [9]. The authors in [5] investigated eq.(1.1) of higher order where $f(u)=u$ and established necessary and sufficient conditions to ensure the oscillation of every solution of eq.(1.1). In this paper we give a different structure of [5] and established necessary and sufficient conditions for the oscillation of every solution of eq.(1.1).

Consider the nonlinear neutral differential equation with positive and negative coefficients of third order.

$$[x(t)+r(t)f(x(\tau(t)))]'''+p(t)g(x(\sigma(t)))-q(t)g(x(\alpha(t)))=0, t\geq t_0 \quad (1.1)$$

Where $r,p,q\in C\left[[t_0,\infty);R^+\right],\tau,\sigma,\alpha$ are continuous functions with

$\lim_{t\to\infty}\tau(t)=\infty$, $\lim_{t\to\infty}\sigma(t)=\infty$, $\lim_{t\to\infty}\alpha(t)=\infty$, and $\alpha(t)$ is increasing function, $\alpha^{-1}(t)$,will denote the inverse function of $\alpha(t)$. (1.2)

By a solution of eq.(1.1) we mean a function $x(t)\in([t_x,\infty);R)$ such that

$x(t)+r(t)f(x(\tau(t)))$ is three times continuously differentiable and $x(t)$ satisfies eq. (1.1) where $t_x=\max\{\tau(t),\sigma(t),\alpha(t)\}$. In initial interval solution of eq.(1.1) is said to be oscillatory if it has arbitrarily large zeros , otherwise is said to be non oscillatory .The purpose of this paper is to obtain sufficient conditions for the oscillation of all solutions of eq. (1.1).

2. Some Basic Lemmas

The following lemmas will be useful in the proof of the main results:

Lemma 1 (Lemma 2.1.2 [1])

Suppose that $p, q \in C[R^+, R^+], q(t) < t$ for $t \geq t_0$,$\lim_{t\to\infty} q(t) = \infty$ and

$$\liminf_{t\to\infty} \int_{q(t)}^{t} p(s)ds > \frac{1}{e} \qquad (2.1)$$

Then the inequality $y'(t) + p(t)y(q(t)) \leq 0$ has no eventually positive solutions, and the inequality $y'(t) + p(t)y(q(t)) \geq 0$ has no eventually negative solutions.

Lemma 2 (Lemma 2.1.2 [1])

Suppose that $p, q \in C[R^+, R^+], q(t) > t$ for $t \geq t_0$, and

$$\liminf_{t\to\infty} \int_{t}^{q(t)} p(s)ds > \frac{1}{e} \qquad (2.1)$$

Then the inequality $y'(t) - p(t)y(q(t)) \geq 0$ has no eventually positive solutions, and the inequality $y'(t) - p(t)y(q(t)) \leq 0$ has no eventually negative solutions.

Lemma 3(Lemma 1.5.5[2])

let $g(t) \in C[R, R^+], R^+ = [0, \infty), \sigma(t), \alpha(t)$ be continuous strictly increasing functions

with $\lim_{t\to\infty} \sigma(t) = \infty$, $\lim_{t\to\infty} \alpha(t) = \infty$ and $\sigma(t) < \alpha(t)$, for $\sigma(t) \geq t_0$,

if $\int_{t_0}^{\infty} g(t)dt < \infty$ then $\lim_{t\to\infty} \int_{\sigma(t)}^{\alpha(t)} g(s)ds = 0$.

In this paper we will assume that the following conditions hold.

H_1. $\liminf_{t\to\infty}[h(t) = p(t) - q(\alpha^{-1}(\sigma(t)))[\alpha^{-1}(\sigma(t))]'] > 0, \qquad t \geq t_0$.

H_1'. $\limsup_{t\to\infty}[h(t) = p(t) - q(\alpha^{-1}(\sigma(t)))[\alpha^{-1}(\sigma(t))]'] \leq 0$

H_2. $f \in C(R; R),\ 0 < \frac{f(u)}{u} \leq M, u \neq 0$.

H_3. $g \in C(R, R), 0 < N_1 \leq \frac{g(u)}{u} \leq N_2$, $u \neq 0$.

3. Main results

In this section we present the following results. For simplicity let

$$z(t) = x(t) + r(t)f(x(\tau(t))) \qquad (3.1)$$

$$w(t) = z(t) + \int_t^\infty \int_s^\infty \int_u^{\alpha^{-1}(\sigma(u))} q(v)g\left(x(\alpha(v))\right) dvduds, \qquad (3.5)$$

Theorem 3.1. Assume that H_1- H_3 hold ,$\tau(t) < t, \sigma(t) < t, \alpha(t) < t$, $r(t)$ is bounded and there exists a continuous function $\beta(t) > t$ with $\sigma(\beta(t)) < t$

$$\int_{t_2}^\infty \int_s^\infty \int_u^{\alpha^{-1}(\sigma(u))} q(v)dvduds < \infty \qquad (3.3)$$

$$\liminf_{t\to\infty} N_1 \int_{\sigma(\beta(t))}^t \int_u^{\beta(u)} (v-u)h(v)\left[1 - Mr\left(\sigma(v)\right)\right]dvdu > \frac{1}{e} \qquad (3.4)$$

Then every solution of (1.1) is oscillatory.

Proof. Let $x(t)$ be nonoscillatory solution of eq.(1.1). for the sake of contradiction suppose that $x(t) > 0, x\left(\tau(t)\right) > 0, x\left(\sigma(t)\right) > 0, x(\alpha(t)) > 0$ for $t \geq t_0$.

By condition (3.3) and Lemma(3) we get

$$\lim_{t\to\infty} \int_t^\infty \int_s^\infty \int_u^{\alpha^{-1}(\sigma(u))} q(v)dvd\,uds = 0 \qquad (3.5)$$

Differentiating eq.(3.2) three times reduce to

$$w'''(t) = -\left[p(t) - q\left(\alpha^{-1}\left(\sigma(t)\right)\right)\left[\alpha^{-1}\left(\sigma(t)\right)\right]'\right] g\left(x\left(\sigma(t)\right)\right) \leq 0 \qquad (3.6)$$

Hence $w(t), w'(t), w''(t)$ are monotonic functions.

Since $w(t) > 0$ then we have only two cases to consider for $t \geq t_1 \geq t_0$:

i. $w(t) > 0,\ w'(t) > 0,\ w''(t) > 0,\ w'''(t) \leq 0$

ii. $w(t) > 0,\ w'(t) < 0,\ w''(t) > 0,\ w'''(t) \leq 0$

Case i. In this case it follows that $\lim_{t\to\infty} w(t) = \infty$.

On the other hand

By H1 there exist $\lambda > 0$ such that $h(t) = p(t) - q(\alpha^{-1}(\sigma(t)))[\alpha^{-1}(\sigma(t))]' \geq \lambda$ for $t \geq t_1 \geq t_2$. Integrating (3.6) from t_2 to ∞ we get:

$$-w''(t_2) \leq -\int_{t_2}^\infty h(s)g\left(x\left(\sigma(s)\right)\right) ds$$

$$w''(t_2) \geq \int_{t_2}^{\infty} h(s) g\left(x(\sigma(s))\right) ds \geq N_1 \int_{t_2}^{\infty} h(s) x(\sigma(s)) ds$$

$$w''(t_2) \geq N_1 \lambda \int_{t_2}^{\infty} x(\sigma(s)) ds$$

$\int_{t_2}^{\infty} x(\sigma(s)) ds \leq \frac{w''(t_2)}{N_1 \lambda}$ then $x(t) \in L[t_2, \infty)$ implies that $x(t)$ is bounded then there exist a constant $k > 0$ such that $x(t) \leq k$. From (3.2) it follows

$$w(t) \leq x(t) + Mr(t)x(\tau(t)) + N_2 \int_t^{\infty} \int_s^{\infty} \int_u^{\alpha^{-1}(\sigma(u))} q(v) x(\alpha(v)) dv du ds$$

$$\leq k + Mr(t)k + N_2 k \int_t^{\infty} \int_s^{\infty} \int_u^{\alpha^{-1}(\sigma(u))} q(v) dv du ds$$

Using (3.5) then for t large enough it follows that for each $\varepsilon > 0$

$$w(t) \leq k(1 + Mr(t) + N_2 \varepsilon)$$

Which is a contradiction.

Case ii. $w(t) > 0,\ w'(t) < 0,\ w''(t) > 0,\ w'''(t) \leq 0$. Integrating (3.6) from $t\ to\ \beta(t),$ where $\beta(t) \geq t$ we get

$$-w''(t) \leq -\int_t^{\beta(t)} h(s) g\left(x(\sigma(s))\right) ds$$

$$w''(t) \geq \int_t^{\beta(t)} h(s) g\left(x(\sigma(s))\right) ds$$

$$\geq N_1 \int_t^{\beta(t)} h(s) x(\sigma(s)) ds$$

Integrating the last inequality from t to $\beta(t)$

$$-w'(t) \geq N_1 \int_t^{\beta(t)} \int_s^{\beta(s)} h(v) x(\sigma(v)) dv ds$$

$$-w'(t) \geq N_1 \int_t^{\beta(t)} (v - t) h(v) x(\sigma(v)) dv$$

$$w'(t) \leq -N_1 \int_t^{\beta(t)} (v - t) h(v) x(\sigma(v)) dv \qquad (3.7)$$

From (3.2)

$$x(t) = w(t) - r(t) f\left(x(\tau(t))\right) - \int_t^{\infty} \int_s^{\infty} \int_u^{\alpha^{-1}(\sigma(u))} q(v) g\left(x(\alpha(v))\right) dv du ds$$

$$\geq w(t) - Mr(t)x(\tau(t)) - N_2 \int_t^{\infty} \int_s^{\infty} \int_u^{\alpha^{-1}(\sigma(u))} q(v) x(\alpha(v)) dv du ds$$

$$\geq w(t) - Mr(t)w\big(\tau(t)\big) - N_2 \int_t^{\infty} \int_s^{\infty} \int_u^{\alpha^{-1}(\sigma(u))} q(v)w\big(\alpha(v)\big)dvduds$$

$$\geq w(t) - Mr(t)w\big(\tau(t)\big) - N_2 w(\alpha(t)) \int_t^{\infty} \int_s^{\infty} \int_u^{\alpha^{-1}(\sigma(u))} q(v)dvduds$$

$x(t) \geq w(t) - Mr(t)w\big(\tau(t)\big) - N_2 \varepsilon w\big(\alpha(t)\big)$, for each $\varepsilon > 0$

$x(t) \geq w(t)[1 - Mr(t)] - K\varepsilon$ where $N_2 w(\alpha(t)) \leq K$, hence for large enough t

$$x(\sigma(t)) \geq w\big(\sigma(t)\big)[1 - Mr(\sigma(t))] \tag{3.8}$$

Substituting (3.8) into (3.7) yields

$$w'(t) \leq -N_1 \int_t^{\beta(t)} (v-t)h(v)w\big(\sigma(v)\big)\big[1 - Mr\big(\sigma(v)\big)\big]dv \tag{3.9}$$

$$w'(t) + N_1 w\Big(\sigma\big(\beta(t)\big)\Big) \int_t^{\beta(t)} (v-t)h(v)\big[1 - Mr\big(\sigma(v)\big)\big]dv \leq 0$$

Then by Lemma 1 with condition (3.4) the last inequality has no eventually positive solution, which is a contradiction. □

Remark 3.2. We can see that theorem 3.1 remain true if we replace condition (3.4) by

$$\liminf_{t\to\infty} \frac{N_1}{2} \int_t^{\beta(t)} (v-t)^2\, h(v)\Big(1 - Mr\big(\sigma(v)\big)\Big)\,dv > 1 \tag{3.10}$$

Proof. The proof is similar to that in case i (Theorem 3.1), while in case ii Integrating (3.9) from t to $\beta(t)$ we get

$$-w(t) \leq\ -N_1 \int_t^{\beta(t)} \int_s^{\beta(s)} (v-s)h(v)w\big(\sigma(v)\big)\Big(1 - Mr\big(\sigma(v)\big)\Big)\,dvds$$

$$\leq -\frac{N_1}{2} \int_t^{\beta(t)} (v-t)^2\, h(v)w\big(\sigma(v)\big)\Big(1 - Mr\big(\sigma(v)\big)\Big)\,dv$$

$$w(t) \geq \frac{N_1}{2} w(\sigma(\beta(t))) \int_t^{\beta(t)} (v-t)^2\, h(v)\Big(1 - Mr\big(\sigma(v)\big)\Big)\,dv$$

$$1 \geq \frac{N_1}{2} \int_t^{\beta(t)} (v-t)^2\, h(v)\Big(1 - Mr\big(\sigma(v)\big)\Big)\,dv$$

Which is a contradiction with (3.10).

Example 1. Consider the third order neutral differential equation:

$$\left[x(t)+\tfrac{1}{2}x(t-4\pi)\right]''' + \left(e^{-t}+\tfrac{3}{2}\left(1-\frac{1}{\cos^2 t+2}\right)\right)\frac{x\left(t-\frac{3\pi}{2}\right)[x^2\left(t-\frac{3\pi}{2}\right)+2]}{x^2\left(t-\frac{3\pi}{2}\right)+1}$$
$$-e^{-t}\frac{x\left(t-\frac{7\pi}{2}\right)[x^2\left(t-\frac{7\pi}{2}\right)+2]}{x^2\left(t-\frac{7\pi}{2}\right)+1}=0 \quad ,t\ge t_0, \qquad (E1)$$

where $r(t)=\frac{1}{2}$, $p(t)=e^{-t}+\frac{3}{2}\left(1-\frac{1}{\cos^2 t+2}\right)$, $q(t)=e^{-t}$

$\tau(t)=t-4\pi$, $\sigma(t)=t-\frac{3\pi}{2}$, $\alpha(t)=t-\frac{7\pi}{2}$, $\alpha^{-1}(\sigma(t))=t+2\pi$

$f(u)=u$, $M=1$

$g(u)=\frac{u(u^2+2)}{u^2+1}$, $\frac{g(u)}{u}=\frac{u^2+2}{u^2+1}=1+\frac{1}{u^2+1}$ hence $N_1=1$, $N_2=2$

(i) $h(t)=p(t)-q(\alpha^{-1}(\sigma(t))[\alpha^{-1}(\sigma(t))]'=e^{-t}+\frac{3}{2}\frac{\cos^2 t+1}{\cos^2 t+2}-e^{-t-2\pi}$

$\liminf_{t\to\infty}[p(t)-q\left(\alpha^{-1}(\sigma(t))[\alpha^{-1}(\sigma(t))]'\right]=1>0$

(ii) $\int_{t_0}^{\infty}\int_s^{\infty}\int_u^{u+2\pi}e^{-v}dvduds=e^{-t_0}(1-e^{-2\pi})<\infty$

(iii) $\liminf_{t\to\infty}\int_{t-\frac{\pi}{2}}^{t}\int_u^{u+\pi}(v-u)[e^{-v}+\frac{3}{2}\left(1-\frac{1}{\cos^2 v+2}\right)-e^{-v-2\pi}]\left(1-\frac{1}{2}\right)dvdu=\frac{3\pi^3}{32}\approx$ 2.9

Then according to Theorem 3.1 every solution of equation (E1) is oscillatory, for instance $x(t)=\sin t$ is such solution.

Theorem 3.3 : Assume that H_1', H_2 - H_3, and (3.3) are hold, $\tau(t)<t,\sigma(t)>t,$ $r(t)$ is bounded, and there exists a continuous functions $\beta(t)>t$ such that

$$\liminf_{t\to\infty} N_1\int_t^{\sigma(t)}\int_u^{\beta(u)}(v-u)|h(v)|[1-Mr(\sigma(v))]dvdu>\frac{1}{e} \qquad (3.4)'$$

Then every bounded solution of eq.(1.1) is oscillatory.

Proof. Let $x(t)$ be nonoscillatory solution of eq.(1.1), for the sake of contradiction suppose that $x(t)>0, x(\tau(t))>0, x(\sigma(t))>0, x(\alpha(t))>0$ for $t\ge t_0$.

By condition (3.3) and Lemma(3) implies that (3.5) holds.

Differentiating eq.(3.2) three times reduce to
$w'''(t)=-\left[p(t)-q\left(\alpha^{-1}(\sigma(t))\right)[\alpha^{-1}(\sigma(t))]'\right]g\left(x(\sigma(t))\right)\ge 0$

Hence $w(t), w'(t), w''(t)$ are monotonic functions.

Since $w(t) > 0$ then we have two cases:

i. $w(t) > 0,\ w'(t) > 0, w''(t) > 0, w'''(t) \geq 0$

ii. $w(t) > 0, w'(t) > 0, w''(t) < 0, w'''(t) \geq 0$

Case i. In this case it follows that $\lim_{t\to\infty} w(t) = \infty$.

On the other hand since $x(t)$ is bounded then there exist a constant $k > 0$ such that $x(t) \leq k$. From (3.2) it follows

$$w(t) \leq x(t) + Mr(t)x\big(\tau(t)\big) + N_2 \int_t^{\infty}\int_s^{\infty}\int_u^{\alpha^{-1}(\sigma(u))} q(v)x\big(\alpha(v)\big)dvduds$$

$$\leq k + Mr(t)k + N_2 k \int_t^{\infty}\int_s^{\infty}\int_u^{\alpha^{-1}(\sigma(u))} q(v)dvduds$$

Using (3.5) it follows that for t large enough we have for each $\varepsilon > 0$

$$w(t) \leq k(1 + Mr(t) + N_2\varepsilon)$$

Which as $t \to \infty$ we get a contradiction.

Case ii. Let $\lim_{t\to\infty} w(t) = l\,,\ \ 0 < l \leq \infty$

If $l = \infty$ we can treat in similar way as in case i and get a contradiction.

If $0 < l < \infty,\ w(t) \leq l$

Integrating (3.6) from t to $\beta(t)$, where $\beta(t) > t$ we obtain

$$-w''(t) \geq -\int_t^{\beta(t)} h(s)g(x(\sigma(s)))ds$$

$$w''(t) \leq \int_t^{\beta(t)} h(s)g(x(\sigma(s)))ds$$

Integrating the last inequality from t to $\beta(t)$ yields

$$-w'(t) \leq \int_t^{\beta(t)}\int_s^{\beta(s)} h(v)g(x(\sigma(v)))dvds$$

$$w'(t) \geq -\int_t^{\beta(t)}\int_s^{\beta(s)} h(v)g\Big(x\big(\sigma(v)\big)\Big)dvds$$

$$w'(t) \geq N_1 \int_t^{\beta(t)} (v - t)|h(v)|x\big(\sigma(v)\big)dv \tag{3.11}$$

From (3.2) we get

$$x(t) = w(t) - r(t)f\Big(x\big(\tau(t)\big)\Big) - \int_t^{\infty}\int_s^{\infty}\int_u^{\alpha^{-1}(\sigma(u))} q(v)g\Big(x\big(\alpha(v)\big)\Big)dvduds$$

$$\geq w(t) - r(t)Mx\big(\tau(t)\big) - N_2 \int_t^\infty \int_s^\infty \int_u^{\alpha^{-1}(\sigma(u))} q(v)x\big(\alpha(v)\big)dvduds$$

$$\geq w(t) - Mr(t)w\big(\tau(t)\big) - N_2 \int_t^\infty \int_s^\infty \int_u^{\alpha^{-1}(\sigma(u))} q(v)w\big(\alpha(v)\big)dvduds$$

$$\geq w(t) - Mr(t)w\big(\tau(t)\big) - N_2 l \int_t^\infty \int_s^\infty \int_u^{\alpha^{-1}(\sigma(u))} q(v)dvduds.$$

By (3.5) for each $\varepsilon > 0$ the last inequality reduce to

$x(t) \geq w(t)[1 - Mr(t)] - N_2 l\varepsilon$, then for *t* large enough it follows that

$x(t) \geq w(t)[1 - Mr(t)]$ hence

$$x\big(\sigma(t)\big) \geq w\big(\sigma(t)\big)\big[1 - Mr\big(\sigma(t)\big)\big] \tag{3.12}$$

Substituting (3.12) into (3.11) we get

$$w'(t) \geq N_1 \int_t^{\beta(t)} (v - t)|h(v)|w\big(\sigma(v)\big)\big[1 - Mr\big(\sigma(v)\big)\big]dv \tag{3.13}$$ *

$$w'(t) - N_1 w\big(\sigma(t)\big) \int_t^{\beta(t)} (v - t)|h(v)|(1 - Mr(\sigma(v)))dv \geq 0$$

Then by Lemma 2 with condition $(3.4)'$ the last inequality has no eventually positive solution, which is a contradiction. □

Remark 3.4. We can see that theorem 3.3 remain true if we replace condition $(3.4)'$ by

$$\liminf_{t\to\infty} \frac{N_1}{2} \int_t^{\beta(t)} (v - t)^2\, |h(v)| \Big(1 - Mr\big(\sigma(v)\big)\Big)\, dv > 1 \tag{3.14}$$

And $\sigma(t) \geq \beta(t)$.

Proof. The proof is similar to that in case i Theorem 3.3, and in case ii Integrating (3.13) from t to $\beta(t)$ we get

$$w(\beta(t)) - w(t) \geq N_1 \int_t^{\beta(t)} \int_s^{\beta(s)} (v - s)|h(v)|w\big(\sigma(v)\big)\big[1 - Mr\big(\sigma(v)\big)\big]dvds$$

$$w(\beta(t)) \geq \frac{N_1 w\big(\sigma(t)\big)}{2} \int_t^{\beta(t)} (v - t)^2|h(v)|\big[1 - Mr\big(\sigma(v)\big)\big]dv,\ \ \sigma(t) \geq \beta(t)$$

$$1 \geq \frac{N_1}{2} \int_t^{\beta(t)} (v - t)^2|h(v)|[1 - Mr(\sigma(v))]dv$$

Which contradicts (3.14).

Example2. Consider the neutral differential equation of third order

$$\big[x(t) + \tfrac{1}{4}x(t - 2\pi)\big]''' + \frac{1}{t^4}\frac{x(2t)[x^2(2t)+3]}{x^2(2t)+2} - \frac{2\pi}{t^3}\frac{x(2t-\pi)[x^2(2t-\pi)+3]}{x^2(2t-\pi)+2} = 0$$

$$, t \geq t_1 > 0, \qquad (E2)$$

where $r(t) = \frac{1}{4}$, $p(t) = \frac{1}{t^4}$, $q(t) = \frac{2\pi}{t^3}$,

$\tau(t) = t - 2\pi$, $\sigma(t) = 2t$, $\alpha(t) = 2t - \pi$, $\alpha^{-1}(\sigma(t)) = t + \frac{\pi}{2}$, $\beta(t) = 2t$

$f(u) = u$, $M = 1$

$g(u) = \frac{u(u^2+3)}{u^2+2}$, $\frac{g(u)}{u} = \frac{u^2+3}{u^2+2} = 1 + \frac{1}{u^2+2}$ hence $N_1 = 1$, $N_2 = \frac{3}{2}$

(i) $h(t) = p(t) - q\left(\alpha^{-1}(\sigma(t))\right)\frac{d}{dt}\left[\alpha^{-1}(\sigma(t))\right] = \frac{1}{t^4} - \frac{2\pi}{\left(t+\frac{\pi}{2}\right)^3} \leq 0$, for $t > 0$

(ii) $\int_{t_1}^{\infty}\int_{s}^{\infty}\int_{u}^{u+\frac{\pi}{2}} \frac{2\pi}{v^3}\, dvduds = \pi \ln \frac{t_1+\frac{\pi}{2}}{t_1} < \infty$

(iii) $$\liminf_{t\to\infty} \int_t^{2t}\int_u^{2u} (v-u)\left[\frac{2\pi}{\left(v+\frac{\pi}{2}\right)^3} - \frac{1}{v^4}\right]\left(1-\frac{1}{4}\right) dvdu = \frac{3\pi}{4}\ln 2 \approx 1.6 > \frac{1}{e}$$

Then according to Theorem 3.3 every solution of equation (E2) is oscillatory.

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