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Survival and Hazard Estimation of Weibull Distribution Based on Interval Censored Data

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Abstract

This paper describes the Maximum Likelihood Estimator and the Bayesian using gamma prior Estimator of the Weibull distribution based on interval censored data. The Bayesian estimates of the survival and hazard functions we can't solve it analytical for that Gauss Quadrature method is used to estimate the survival and hazard functions. Also Lindley's approximation is used. The two methods are compared to maximum likelihood counterparts and the comparisons are made with respect to the Mean Square Error (MSE) and absolute bias to determine the best for estimating of the survival and hazard function.

Keywords: Weibull distribution, survival and hazard functions, Bayesian method, interval censored data, Gauss Quadrature, Lindley's approximation.

1. Introduction

One of the most appealing classical statistical techniques used for fitting statistical models to data as well as providing estimates for the parameters of a model are the Bayesian approach and maximum likelihood estimation (MLE) methods. (Sinha 1986) discussed a variety of situations where Weibull distribution has been used to analyze other types of survival data. The maximum likelihood method is the usual frequentist approach in the estimation of parameters for parametric survival data. Bayesian was employed to estimate the survival function and hazard rate of Weibull distribution using Lindley's approximation method.(Smith 1987) developed the maximum likelihood and Bayesian estimators and compared them using the three-parameter Weibull distribution. The application of the Weibull distribution in modeling and analyzing survival data has also been described extensively by (Mudholkar *et al*. 1996). (Hossain and Zimmer 2003) estimated the scale and shape parameters of Weibull distribution using complete and censored samples by maximum likelihood estimator and least squares method. (Nassar & Eissa 2005) used the Bayesian approach for type II censored data to estimate the two shape parameters and the reliability function of the Exponentiated Weibull distribution with Lindely approximations. In numerical analysis, the Gauss quadrature method is very useful in solving Bayesian parameter and survival and hazard functions. (Singh *et al*. 2002) estimated the Exponentiated Weibull shape parameters by maximum likelihood estimators and Bayesian estimator whereby in the Bayes estimation approach they solved it numerically by the use of 16 points Gauss-Legendre quadrature formula to estimate the parameters. (Singh *et al.* 2005) obtained Bayesian and Maximum likelihood estimation for the two-parameter Exponentiated Weibull distribution when sample was available from type-II censoring scheme. They first obtained the estimates under maximum likelihood by making use of the Newton-Raphson method. Secondly, the Bayesian estimator could not also be obtained in close form. For this they proposed and used the 16-point Gaussian quadrature formulas. For more detail in Gauss Quadrature Method see (Richard & Douglas 1989).

The objective of this paper is to estimate the survival and hazard functions of the Weibull distribution based interval censored data by using Bayesian approach with help of the Gauss Quadrature Method and Lindley's approximation and compared to maximum likelihood estimator by using mean square error (MSE) and absolute bias to determine the best estimator under several conditions.

2. Maximum Likelihood Estimation

Let $(t_1, t_2, \ldots, t_n)$ be the set of n random lifetimes from Weibull distribution with parameters λ and α .

The probability density function (pdf) of Weibull distribution is

$$f(x;\lambda,\alpha) = \frac{\alpha}{\lambda} x^{\alpha-1} \exp\left(-\frac{x^{\alpha}}{\lambda}\right)$$

The cumulative distribution function (cdf) of the Weibull distribution is given as

$$F(x;\lambda,\alpha)=1-\exp\left(-\frac{x^{\alpha}}{\lambda}\right)$$

The likelihood function based on interval censored data as given in Flygare *et al*., (1985) is

$$L(\lambda,\alpha\mid l_i,u_i)=\prod_{i=1}^{n}[F(u_i;\lambda,\alpha)-F(l_i;\lambda,\alpha)],$$

$$=\prod_{i=1}^{n}\left[\exp\left(-\frac{l_i^{\alpha}}{\lambda}\right)-\exp\left(-\frac{u_i^{\alpha}}{\lambda}\right)\right] \tag{1}$$

The logarithm of the likelihood function

$$\ln L(\lambda,\alpha\mid l_i,u_i)=\sum_{i=1}^{n}\log\left[\exp\left(-\frac{l_i^{\alpha}}{\lambda}\right)-\exp\left(-\frac{u_i^{\alpha}}{\lambda}\right)\right] \tag{2}$$

To obtain the equations for the unknown scale and shape parameters, we differentiate Eq. (2) partially with respect to the parameters λ and α and equal it to zero. The resulting equations are given respectively as,

$$\frac{\partial L(\lambda,\alpha\mid l_i,u_i)}{\partial\lambda_i}=\sum_{i=1}^{n}\frac{l_i^{\alpha}}{\lambda^2}\exp\left(-\frac{l_i^{\alpha}}{\lambda}\right)-\frac{u_i^{\alpha}}{\lambda^2}\exp\left(-\frac{u_i^{\alpha}}{\lambda}\right)/D_i \tag{3}$$

$$\frac{\partial L(\lambda,\alpha\mid l_i,u_i)}{\partial\alpha_i}=\sum_{i=1}^{n}\frac{u_i^{\alpha}}{\lambda}\ln(u_i)\exp\left(-\frac{u_i^{\alpha}}{\lambda}\right)-\frac{l_i^{\alpha}}{\lambda}\ln(l_i)\exp\left(-\frac{l_i^{\alpha}}{\lambda}\right)/D_i \tag{4}$$

Where,

$$D_i=\exp\left(-\frac{l_i^{\alpha}}{\lambda}\right)-\exp\left(-\frac{u_i^{\alpha}}{\lambda}\right)$$

The Equations 3 and 4 cannot be solved analytically, and for that we employed Newton Raphson method to find the numerical solution.

The estimate of the survival function and hazard function of Weibull distribution are

$$\hat{S}_M(t)=\exp\left(-\frac{l^{\hat{\alpha}_M}}{\hat{\lambda}_M}\right)-\exp\left(-\frac{u^{\alpha_M}}{\lambda_M}\right) \tag{5}$$

$$\hat{h}_M(t)=\frac{\hat{\alpha}_M}{\hat{\lambda}_M}\left(l^{\hat{\alpha}_M-1}-u^{\alpha_M-1}\right) \tag{6}$$

Where $\hat{\lambda}_M$ is the scale parameter estimated by maximum likelihood estimator and the $\hat{\alpha}_M$ is the shape parameter of Weibull distribution

3. Bayesian approach

We consider the both the scale and shape parameters are unknown, and we compute the Bayesian estimates of the scale and shape parameters. It is assumed that λ and α each have independent gamma (a, b), and gamma(c, d) priors respectively

$$\pi_1(\lambda / a,b) = \lambda^{a-1}\exp(-b\lambda)$$

$$\pi_2(\alpha / c,d) = \alpha^{c-1}\exp(-d\alpha)$$

The posterior is given as

$$\prod\nolimits_1(\lambda,\alpha \mid l_i,u) = \frac{L(\lambda,\alpha \mid l_i,u_i)\pi_1(\lambda / a,b)\pi_2(\alpha / c,d)}{\int_0^\infty\int_0^\infty L(\lambda,\alpha \mid l_i,u_i)\pi_1(\lambda / a,b)\pi_2(\alpha / c,d)\, d\lambda d\alpha}$$

$$= \frac{\prod_{i=1}^{n}\left[\exp\left(-l_i^{\alpha}/\lambda\right)-\exp\left(-u_i^{\alpha}/\lambda\right)\right]\lambda^{a-1}\alpha^{c-1}\exp(-(b\lambda+d\alpha))}{\int_0^\infty\int_0^\infty\prod_{i=1}^{n}\left[\exp\left(-l_i^{\alpha}/\lambda\right)-\exp\left(-u_i^{\alpha}/\lambda\right)\right]\lambda^{a-1}\alpha^{c-1}\exp(-(b\lambda+d\alpha))\, d\lambda d\alpha} \qquad (8)$$

The Bayesian estimates for the survival and hazard functions under squared error loss function are given as:

$$\hat{S}_B(t) = \frac{\int_0^\infty\int_0^\infty \lambda^{a-1}\alpha^{c-1}\exp\left(-\frac{t^{\alpha}}{\lambda}\right)\prod_{i=1}^{n}\left[\exp\left(-l_i^{\alpha}/\lambda\right)-\exp\left(-u_i^{\alpha}/\lambda\right)\right]\exp(-(b\lambda+d\alpha))d\lambda d\alpha}{\int_0^\infty\int_0^\infty\prod_{i=1}^{n}\left[\exp\left(-l_i^{\alpha}/\lambda\right)-\exp\left(-u_i^{\alpha}/\lambda\right)\right]\lambda^{a-1}\alpha^{c-1}\exp(-(b\lambda+d\alpha))\, d\lambda d\alpha} \qquad (9)$$

$$\hat{h}_B(t) = \frac{\int_0^\infty\int_0^\infty \lambda^{a-2}\alpha^{c}t^{\alpha-1}\prod_{i=1}^{n}\left[\exp\left(-l_i^{\alpha}/\lambda\right)-\exp\left(-u_i^{\alpha}/\lambda\right)\right]\exp(-(b\lambda+d\alpha))d\lambda d\alpha}{\int_0^\infty\int_0^\infty\prod_{i=1}^{n}\left[\exp\left(-l_i^{\alpha}/\lambda\right)-\exp\left(-u_i^{\alpha}/\lambda\right)\right]\lambda^{a-1}\alpha^{c-1}\exp(-(b\lambda+d\alpha))\, d\lambda d\alpha} \qquad (10)$$

The Equations 9 and 10 we can't solve it analytical for that we used Gaussian quadrature method and Lindley's approximation to solve the problem.

3.1 Gaussian Quadrature Formulas

Following Richard L. Burden & J. Douglas Faires (1989)
The Gauss quadrature rule are used to solve our problems for each estimators are a mention in this paper, where the double integrations as follow

$$I = \int_{A1}^{B_1}\int_{A_2}^{B_2} f(x_1,x_2) \quad dx_1 dx_2$$

The Gauss Legendre quadrature rules for single integration is

$$\int_{A}^{B} f(x)\,dx \approx \sum_{i=1}^{N} C_i f(z_i)$$

This is the n-point Gauss quadrature rule, where the C_i is the coefficients and x_i are called the function arguments

Now to solve the double integration be the Gauss quadrature rule the follow step are needs:

$$I = \frac{B_1 - A_1}{2} \int_{-1}^{1} \int_{A_2}^{B_2} \cdots f(v_1 z_1 + \beta_1, x_2) \quad dz_1 dx_2$$

Where

$$v_1 = \frac{B_1 - A_1}{2}, \beta_1 = \frac{B_1 + A_1}{2}$$

$$I \simeq \frac{B_1 - A_1}{2} \sum_{i=1}^{N} C_i \int_{a_2}^{b_2} \cdots f(v_1 z_i + \beta, x_2) \quad dx_2 dx_2$$

C_i are the weighting factors and z_i are the function arguments

To apply the Gauss quadrature rule for the second integration as follows

$$I \simeq \frac{B_1 - A_1}{2} \sum_{i=1}^{N} C_i \left[\frac{B_2 - A_2}{2} \right] \sum_{j=1}^{N} C_i f(v_1 z_i + \beta_1, v_2 z_j + \beta_2)$$

$$v_2 = \frac{B_2 - A_2}{2}, \beta_2 = \frac{B_2 + A_2}{2}$$

The techniques can be applied for estimated the survival and hazard functions (see for example Rathod *et al*, 2007)

3.2 Lindley's Approximation

As shown the integrals involved in (9) and (10) cannot be solved analytically and for that we obtained Lindley's Expansion to solve the parameters approximation.

According to Al Omari (2012), Lindley proposed a ratio of integral of the form

$$\int w(\lambda) \exp\{L(\lambda)\}\, d\lambda \,/ \int \upsilon(\lambda) \exp\{L(\lambda)\}\, d\lambda$$

Where $L(\lambda)$ is the log-likelihood and $w(\lambda), \upsilon(\lambda)$ are arbitrary functions of λ In applying this procedure, it is assumed that $\upsilon(\lambda)$ is the prior distribution for λ and $w(\lambda) = u(\lambda).\upsilon(\lambda)$ with $u(\lambda)$ being some function of interest.

The posterior expectation according to Sinha (1986) is

$$E(u(\lambda) \mid t) = \int \upsilon(\lambda) \exp\{L(\lambda) + \rho(\lambda)\}\, d\lambda \,/ \int \exp\{L(\lambda) + \rho(\lambda)\}\, d\lambda$$

where $\rho = \log(\upsilon(\lambda))$

Lindley expansion is therefore approximated asymptotically by

$$E(u(\lambda) \mid t) = u + \frac{1}{2}(u_{11}\sigma_{11} + u_{22}\sigma_{22}) + u_1\rho_1\sigma_{11} + u_2\rho_2\sigma_{22} + \frac{1}{2}(L_{30}u_1\sigma_{11}^2 + L_{03}u_2\sigma_{22}^2)$$

where L is the log-likelihood equation in (2). Taking the survival function estimation, where

$$u(\lambda)=\exp\left(-\frac{l^{\alpha}}{\lambda}\right)-\exp\left(-\frac{u^{\alpha}}{\lambda}\right),$$

$$u_1=\frac{\partial u}{\partial\lambda}=\frac{(l^{\alpha}-\mathrm{u}^{\alpha})}{\lambda^2}u(\lambda),\quad u_2=\frac{\partial u}{\partial\alpha}=-\frac{(\mathrm{l}^{\alpha}\,\mathrm{lnl}-u^{\alpha}\,\mathrm{lnu})}{\lambda}u(\lambda),$$

and

$$u_{11}=\frac{\partial^2 u}{\partial\lambda^2}=\frac{l^{\alpha}}{\lambda^3}u(\lambda)\left(\frac{l^{\alpha}}{\lambda}-2\right)-\frac{u^{\alpha}}{\lambda^3}u(\lambda)\left(\frac{u^{\alpha}}{\lambda}-2\right)$$
$$u_{22}=\frac{\partial^2 u}{\partial\alpha^2}=\frac{l^{\alpha}(\mathrm{lnl})^2}{\lambda}u(\lambda)\left(\frac{l^{\alpha}}{\lambda}-1\right)-\frac{u^{\alpha}(\mathrm{lnu})^2}{\lambda}u(\lambda)\left(\frac{u^{\alpha}}{\lambda}-1\right).$$

For the Hazard function, set

$$u(\lambda)=\frac{\alpha}{\lambda}(\mathrm{l}^{\alpha-1}-u^{\alpha-1}),$$
$$u_1=\frac{\partial u}{\partial\lambda}=\frac{-u(\lambda)}{\lambda},\quad u_2=\frac{\partial u}{\partial\alpha}=-\frac{(\mathrm{l}^{\alpha-1}+\alpha l^{\alpha-1}\,\mathrm{lnl})-(\mathrm{u}^{\alpha-1}+\alpha u^{\alpha-1}\,\mathrm{lnu})}{\lambda}$$
$$u_{11}=\frac{\partial^2 u}{\partial\lambda^2}=\frac{2u(\lambda)}{\lambda^2},\quad u_{22}=\frac{\partial^2 u}{\partial\alpha^2}=\frac{[\mathrm{l}^{\alpha-1}\,\mathrm{lnl}\left(2+\alpha\,\mathrm{lnl}\right)]-[\mathrm{u}^{\alpha-1}\,\mathrm{lnu}\left(2+\alpha\,\mathrm{lnu}\right)]}{\lambda},$$
$$\rho_1=\frac{\partial\rho}{\partial\lambda}=\frac{a-1}{\lambda}-b,\quad \rho_2=\frac{\partial\rho}{\partial\alpha}=\frac{c-1}{\alpha}-d$$

$$\sigma_{11}=(-L_{20})^{-1},\quad \sigma_{22}=(-L_{02})^{-1}$$

$$L_{20}=\frac{\partial^2 L}{\partial\lambda^2}=\sum_{i=1}^{n}-\frac{2l_i^{\alpha}e^{-\frac{l_i^{\alpha}}{\lambda}}}{\lambda^3}+\frac{(l_i^{\alpha})^2e^{-\frac{l_i^{\alpha}}{\lambda}}}{\lambda^4}+\frac{2u_i^{\alpha}e^{-\frac{u_i^{\alpha}}{\lambda}}}{\lambda^3 D_i}-\frac{(u_i^{\alpha})^2e^{-\frac{u_i^{\alpha}}{\lambda}}}{\lambda^4 D_i}+\frac{u_i^{\alpha}e^{-\frac{u_i^{\alpha}}{\lambda}}(l_i^{\alpha}e^{-\frac{l_i^{\alpha}}{\lambda}}-u_i^{\alpha}e^{-\frac{u_i^{\alpha}}{\lambda}})}{D_i^2}$$

$$L_{02}=\frac{\partial^2 L}{\partial\alpha^2}=\sum_{i=1}^{n}\frac{u_i^{\alpha}\ln\left(u_i\right)^2\mathrm{e}^{-\frac{u_i^{\alpha}}{\lambda}}}{\lambda}-\frac{\left(u_i^{\alpha}\right)^2\ln\left(u_i\right)^2\mathrm{e}^{-\frac{u_i^{\alpha}}{\lambda}}}{\lambda^2}-\frac{l_i^{\alpha}\ln\left(l_i\right)^2}{\lambda}+\frac{u_i^{\alpha}\ln\left(u_i\right)\mathrm{e}^{-\frac{u_i^{\alpha}}{\lambda}}}{\lambda}$$

$$L_{30}=\frac{\partial^3 L}{\partial\theta^3}=\sum_{i=1}^{n}\frac{6l_i^{\alpha}e^{-\frac{l_i^{\alpha}}{\lambda}}}{\lambda^4}-\frac{6(l_i^{\alpha})^2e^{-\frac{l_i^{\alpha}}{\lambda}}}{\lambda^5}+\frac{(l_i^{\alpha})^3e^{-\frac{l_i^{\alpha}}{\lambda}}}{\lambda^6}+\frac{6u_i^{\alpha}e^{-\frac{u_i^{\alpha}}{\lambda}}}{\lambda^4 D_i}+\frac{6(u_i^{\alpha})^2e^{-\frac{u_i^{\alpha}}{\lambda}}}{\lambda^5 D_i}-\frac{4u_i^{\alpha}e^{-\frac{u_i^{\alpha}}{\lambda}}(l_i^{\alpha}e^{-\frac{l_i^{\alpha}}{\lambda}}-u_i^{\alpha}e^{-\frac{u_i^{\alpha}}{\lambda}})}{\lambda D_i^2}$$
$$-\frac{(u_i^{\alpha})^3e^{-\frac{u_i^{\alpha}}{\lambda}}}{\lambda^6 D_i}+\frac{2\left(u_i^{\alpha}\right)^2e^{-\frac{u_i^{\alpha}}{\lambda}}(l_i^{\alpha}e^{-\frac{l_i^{\alpha}}{\lambda}}-u_i^{\alpha}e^{-\frac{u_i^{\alpha}}{\lambda}})}{\lambda^2 D_i^2}-\frac{2u_i^{\alpha}e^{-\frac{u_i^{\alpha}}{\lambda}}(l_i^{\alpha}e^{-\frac{l_i^{\alpha}}{\lambda}}-u_i^{\alpha}e^{-\frac{u_i^{\alpha}}{\lambda}})^2}{D_i^3}+$$
$$\frac{1}{\lambda^2 D_i^2}\left(u_i^{\alpha}e^{-\frac{u_i^{\alpha}}{\lambda}}\left(-\frac{2l_i^{\alpha}e^{-\frac{l_i^{\alpha}}{\lambda}}}{\lambda^3}+\frac{\left(l_i^{\alpha}\right)^2e^{-\frac{l_i^{\alpha}}{\lambda}}}{\lambda^4}+\frac{2u_i^{\alpha}e^{-\frac{u_i^{\alpha}}{\lambda}}}{\lambda^3}-\frac{\left(u_i^{\alpha}\right)^2e^{-\frac{u_i^{\alpha}}{\lambda}}}{\lambda^4}\right)\right)$$

$$L_{03} = \frac{\partial^3 L}{\partial \alpha^3} = \sum_{i=1}^{n} \frac{u_i^\alpha \ln(u_i)^3 e^{-\frac{u_i^\alpha}{\lambda}}}{\lambda} - \frac{3(u_i^\alpha)^2 \ln(u_i)^3 e^{-\frac{u_i^\alpha}{\lambda}}}{\lambda^2} + \frac{(u_i^\alpha)^3 \ln(u_i)^3 e^{-\frac{u_i^\alpha}{\lambda}}}{\lambda^3} - \frac{l_i^\alpha \ln(l_i)^3}{\lambda} +$$
$$\frac{u_i^\alpha \ln(u_i)^2 e^{-\frac{u_i^\alpha}{\lambda}}}{\lambda} - \frac{(u_i^\alpha)^2 \ln(u_i)^2 e^{-\frac{u_i^\alpha}{\lambda}}}{\lambda^2}$$

4. Simulation Study

In this section, we compare the Maximum likelihood estimation and Bayesian using gamma prior with help of the Gaussian Quadrature method and Lindley's Approximation, The mean squared errors (MSE) and absolute bias for each method were calculated using 10,000 replications for sample size n=25, 50 and 100 of Weibull distribution based on interval censored data for different value of parameters were the scale parameter λ = 2 and shape parameter α = 0.5, 1.5 and 3. The results are presented in Tables for different selections of the parameters.

Table 1: Estimated of the survival function and (MSE, absolute bias) of Weibull distribution.

Size	Estimators	MLE	BL	BG
25	$\alpha = 0.5$	0.0891 (0.1231, 0.1945)	0.0889 (0.1134, 0.1722)	0.0890 (0.1035, 0.1642)
	$\alpha = 1.5$	0.1102 (0.0612, 0.0699)	0.1119 (0.0522, 0.0654)	0.1241 (0.0501, 0.0631)
	$\alpha = 3$	0.2211 (0.0519, 0.0711)	0.2155 (0.0535, 0.0832)	0.2302 (0.0481, 0.0679)
50	$\alpha = 0.5$	0.0893 (0.0963, 0.1342)	0.0892 (0.1040, 0.1466)	0.0895 (0.0935, 0.1278)
	$\alpha = 1.5$	0.1135 (0.0502, 0.0628)	0.1034 (0.0514, 0.0644)	0.1271 (0.0481, 0.0589)
	$\alpha = 3$	0.2225 (0.0419, 0.0611)	0.2135 (0.0505, 0.0708)	0.2301 (0.0412, 0.0573)
100	$\alpha = 0.5$	0.0897 (0.0531,0.1052)	0.0891 (0.0540, 0.1172)	0.0907 (0.0489, 0.0934)
	$\alpha = 1.5$	0.1150 (0.0491, 0.0576)	0.1031 (0.0498, 0.0591)	0.1241 (0.0421, 0.0522)
	$\alpha = 3$	0.2345 (0.0369, 0.0519)	0.2157 (0.0412, 0.0580)	0.2338 (0.0371, 0.527)

Table 2: Estimated of the hazard function and (MSE, absolute bias) of Weibull distribution

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Size	Estimators	MLE	BL	BG
25	$\alpha = 0.5$	1.8123 (1.0311, 0.9345)	1.8288 (0.9146, 0.8766)	1.8910 (0.8935, 0.8621)
	$\alpha = 1.5$	0.4512 (0.9612, 0.8994)	0.4602 (0.9532, 0.8854)	0.4218 (0.9401, 0.8631)
	$\alpha = 3$	0.1719 (0.8713, 0.8211)	0.1655 (0.8835, 0.8732)	0.1022 (0.8381, 0.7869)
50	$\alpha = 0.5$	1.8452 (0.9041, 0.8592)	1.8392 (0.9090, 0.8616)	1.8955 (0.8735, 0.8128)
	$\alpha = 1.5$	0.4815 (0.8952, 0.8681)	0.4794 (0.9054, 0.8764)	0.4315 (0.8681, 0.8566)
	$\alpha = 3$	0.2125 (0.8549, 0.7901)	0.2035 (0.8755, 0.8068)	0.2301 (0.8242, 0.7557)
100	$\alpha = 0.5$	1.8685 (0.8991,0.8412)	1.8499 (0.9014, 0.8602)	1.9011 (0.8649, 0.8034)
	$\alpha = 1.5$	0.5100 (0.8649, 0.8361)	0.5001 (0.8798, 0.8651)	0.5421 (0.8421, 0.8402)
	$\alpha = 3$	0.2315 (0.8061, 0.7419)	0.2157 (0.8412, 0.7810)	0.2311 (0.8097, 0.7471)

5. Discussion.

In Table 1, when we compare the mean squared error (MSE) and absolute bias of the survival function of Weibull distribution based on interval censored data by maximum likelihood (MLE), Bayesian using Lindley's approximation (BL) Bayesian using Gaussian Quadrature (BM), we found that Bayesian using Gaussian Quadrature is better compare to the others for all cases except when the *n*=100 with $\alpha = 3$. Moreover, maximum likelihood is better than Bayesian using Lindley's approximation for all cases except when the *n*= 25 with $\alpha = 0.5$ and 1.5.

In Table 2, when we compared the mean squared error (MSE) and absolute bias of the hazard function of Weibull distribution with interval censored data by maximum likelihood (MLE), Bayesian using Lindley's approximation (BL) and Bayesian using Gaussian Quadrature (BG), we found that Bayesian using Gaussian Quadrature is better compare to the others for all cases except when the *n*=100 with $\alpha = 3$. Moreover, maximum likelihood is better than Bayesian using Lindley's approximation for all cases except when the *n*= 25 with $\alpha = 0.5$ and 1.5.

5. Conclusion

In this paper we have considered Bayesian using Lindley's approximation (BL) and Bayesian using Gaussian Quadrature problems of the Weibull distribution based on interval censored data to estimate survival and hazard functions. Comparisons are made between the Bayesian and maximum likelihood estimators based on simulation study and we observed that, the Weibull survival and hazard functions are better estimated by Bayesian using Gaussian Quadrature.

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