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Effects Of Some Thermo-Physical Properties On Force Convective Stagnation Point On A Stretching Sheet With Convective Boundary Conditions In The Presence Of Thermal Radiation And Magnetic Field

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Abstract

Of concern in this paper is an investigation of force convective stagnation point on a stretching sheet with convective boundary conditions in the presence of thermal radiation and uniform magnetic field. Rosseland approximation is used to model the radiative heat transfer. The nonlinear partial differential equations arising from the flow modeling were transformed by similarity transformation. The resulting coupled nonlinear ordinary differential equations were then solved numerically by employing the Adaptive Runge-Kutta together with shooting technique. Finally the effects of the pertinent thermo-physical parameters are presented in graphs and tables.

Keywords: Plane Stagnation Point, Thermal Radiation, Convective Boundary Conditions, Magnetic Field.

1. Introduction

Stagnation point flow has over the years become an interesting research area to scientist and engineers due to its prodigious scientific and industrial applications. It is found useful in nuclear reactors, extrusion of polymers, cooling of electronics devices, etc. Crane (1970) the initiator of the study of convection boundary layer flow over a stretching sheet, studied the two dimensional steady flow of an incompressible viscous fluid caused by a linearly stretching plate. Remachndran (1988) studied laminar mixed convection in two dimensional stagnation flows around heated surfaces. He considered both cases of an arbitrary wall temperature and arbitrary heat flux variation. Their results showed that for a specified range of buoyancy parameter dual solutions existed and a reversed flow developed in the buoyancy opposing flow region. Sharma and Singh (2009) studied the effects of variable thermal conductivity and heat source / sink on MHD flow near a stagnation point on a linearly stretching sheet. They observed that the rate of heat transfer at the sheet increases due to increase in the thermal conductivity parameter, while it decreases due to increase in the ratio of free stream velocity parameter to stretching sheet parameter, in absence of magnetic field and volumetric rate of heat source/sink parameter.

In the glass industry, processes such as, blowing, floating or spinning of fibers involves flow due to a stretching surface. Thermal radiation effects plays an important role in scientific processes such as cooling of a metal or glass sheet. Its effect cannot be neglected because this process takes place at high temperatures. Samad and Rahman (2006) investigated the thermal radiation interaction on an absorbing emitting fluid past a vertical porous plate immersed in a porous medium. Steady radiative free convective flow along a vertical flat plate in the presence of magnetic field was investigated by Enamuel and Uddin (2011). Their result showed that magnetic field can control the heat transfer and radiation has a significant effect on the velocity as well as temperature distributions.

Convective boundary condition is used mostly to describe a linear convective heat exchange condition for one or more algebraic entities in thermal. Thermal energy storage, nuclear plants, gas turbines, etc are processes that defines heat transfer analysis with convective boundary conditions. Aziz (2009) presented a similarity solution for Blasius flow of viscous fluid with convective boundary conditions. Okedayo et al (2011) presented the effects of viscous dissipation on the mixed convection heat transfer over a flat plate with internal heat generation and convective boundary condition. Okedayo et al (2012) presented the similarity solution to the plane stagnation point flow with convective boundary conditions. Adeniyan and Adegun (2013) studied the effects of convective plane stagnation point MHD flow with convective boundary conditions in the presence of a uniform magnetic field. They observed that the convective conditions, the stagnation point and the magnetic field have significant effects on the heat transfer rate, velocity boundary layer thickness and the thermal boundary layer thickness.

In the present paper, force convective stagnation point on a stretching sheet with convective boundary conditions in the presence of thermal radiation and magnetic fields is investigated. The inclusion of thermal radiation and viscous dissipation extends the work of Adeniyan and Adigun (2013).

2. Governing Equations

Let us consider a steady two-dimensional MHD flow of a viscous, incompressible and electrically conducting fluid of temperature T_∞ along a heated vertical flat plate under the influence of a uniform magnetic field. The flow is assumed to be in the x-direction, which is chosen along the plate in the upward direction and y-axis normal to the plate.

The fluid is considered to be gray, absorbing emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The radiative heat flux in the x-direction is negligible to the flux in the y-direction. A uniform magnetic field of strength $\mathbf{B_0}$ is applied normal to the plate parallel to y-direction. The plate temperature is initially raised to T_w(where $T_w > T_\infty$) which is thereafter maintained constant.

The flow and heat transfer equations relevant for the model, namely the Continuity equation, momentum equation and energy equation are;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \gamma\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \propto\frac{\partial^2 T}{\partial y^2} + \frac{\gamma}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

Boundary Conditions:

$$\left.\begin{aligned} u(x,0) &= v(x,0) = 0 \\ u(x,\infty) &= xa \\ -k\frac{\partial T}{\partial y}(x,0) &= h_f\left(T_f - T(x,0)\right) \\ T(x,\infty) &= 0 \end{aligned}\right\} \tag{4}$$

In equations 1 – 4 the velocity field is $(u(x,y), v(x,y))$, where u, v are the velocity along and normal to the plate respectively. T(x,y) is the temperature field, T_∞ is the free stream temperature, T_f is the temperature of stretching sheet, a is the stretching rate constant, γ is the kinematic viscosity, $\propto$ is the thermal diffusivity of the fluid, h_f is the heat transfer coefficient ,p is the fluid pressure, ρ is the density, k is the thermal conductivity, σ is electrical conductivity of the fluid, $B = (0, B_0)$ is the imposed magnetic field and θ is dimensionless fluid temperature.

The term $\frac{\partial q_r}{\partial y}$ is introduced for radiation effect, q_r is the radiative heat flux and c_p is the specific heat at constant pressure. Within the frame work of the above mentioned assumptions, we assume that the boundary layer and Boussinesq approximations hold and the flow and heat transfer in the presence of radiation are governed by the equations Eq. (1), Eq. (2) and Eq. (3). By using Rosseland approximation q_r takes the form

$$q_r = -\frac{4\sigma_1}{3k_1}\frac{\partial T^4}{\partial y} \tag{5}$$

where, σ_1 is the Stefan-Boltzmann constant and k_1 is the mean absorption coefficient. It is assumed that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong T_\infty^4 + (T - T_\infty)4T_\infty^3 = 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

3. Similarity Transformation

The suitable similarity variables, for the problem under consideration, are

$$\left.\begin{aligned} &\eta = \sqrt{\frac{a}{\gamma}}, && \psi(x,y) = x\sqrt{a\gamma}f(\eta) \\ &\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, && P_r = \frac{\gamma}{\alpha} \\ &M = \frac{\sigma B_0^2}{\rho\alpha}, && B_i = \frac{h_f}{k}\sqrt{\frac{\gamma}{\alpha}} \\ &E_c = \frac{\alpha^2 x^2}{c_p(T_f - T_\infty)}, && N = \frac{kk_1}{4\sigma_1 T_\infty^3} \end{aligned}\right\} \tag{7}$$

Here P_r, M, B_i, E_c, N signifies respectively the Prandtl number, the Magnetic parameter, the Biot number, the Eckert number and the Radiation parameter. In terms of the stream function $\psi(x,y)$, the velocity components are

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x} \tag{8}$$

Which in essence satisfy Eq. (1) identically. With the use of the similarity transformation, momentum and energy equations become

$$f''' + ff' - (f')^2 + 1 - Mf' = 0 \tag{9}$$

$$(3N+4)\theta'' + 3NP_r f\theta' + 3NP_r E_c (f'')^2 = 0 \tag{10}$$

Subject to the transformed boundary conditions

$$\left.\begin{aligned} &f(0) = 0, \ f'(0) = 0, \quad f'(\infty) = 1 \\ &\theta'(0) = -B_i\big(1-\theta(0)\big), \quad \theta(\infty) = 0 \end{aligned}\right\} \tag{11}$$

Here f is the dimensionless stream function and prime denotes the derivative with respect to η.

4. Result and Discussion

The numerical solutions of the nonlinear differential equations Eq. (9) & Eq. (10) under the boundary conditions Eq. (11) have been performed by applying a shooting method along with Adaptive Runge-Kutta integration scheme. The technique was implemented on a computer program written in Matlab. A convenient step size was chosen to obtain the desired accuracy.

Figure 1 represents the velocity profile (f') for various values of the Magnetic field parameter while Pr, Bi, N and Ec are kept constant at 0.71, 0.1, 1 and 0.025 respectively. It is observed that increase in the Magnetic parameter leads to a corresponding decrease in the velocity profile. Fig 2 reflect the Temperature profile (θ) for different values of the Biot number while Pr, M, N and Ec are kept constant at 0.71, 0.1, 1 and 0.025 respectively. It is observed that increase in the Biot number leads to a corresponding increase in the temperature profile.

The effect of Eckert number (Ec) on the temperature profile is shown in Fig. 3. The temperature profile decrease with an increase in the Eckert number while Pr, M, N and Bi are kept constant at 0.71, 0.1, 1 and 0.1 respectively. In Fig. 4, we depict the temperature profile for different values of the Prandtl number. It is clearly seen that increase in the Prandtl number leads to decrease in the temperature profile. In Fig. 5, the influence of the Radiation number on the temperature profile is investigated; the values of Pr, M, Ec and Bi are kept fixed at 0.71, 0.1, 0.025 and 0.1 respectively. It is observed that as the radiation parameter increases the temperature profile decreases.

The parameters of engineering interest for the present problem are the Skin friction coefficient and Local Nusselt number which indicate physically wall shear stress and rate of heat transfer respectively. The numerical values proportional to C_f and N_u are shown in Table1. In the absence of thermal radiation and viscous dissipation, the results of the present paper are reduced to those obtained by Adeniyan and Adegun (2013). To this end, a comparison of the two results is presented in Table 2 and 3.

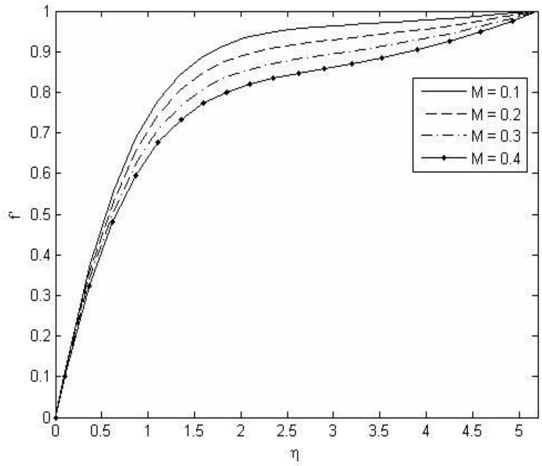

Fig. 1. Velocity profile for various values of Magnetic field parameter.

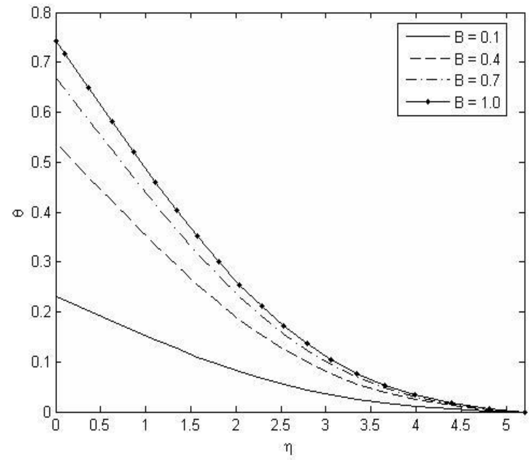

Fig. 2. Temperature profile for various values of Biot number.

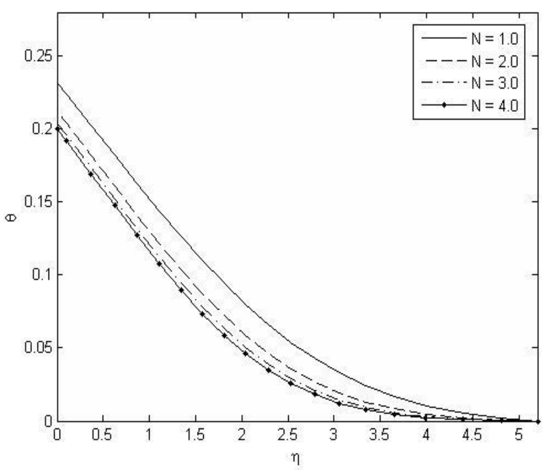

Fig. 3. Temperature profile for various values of Radiation number.

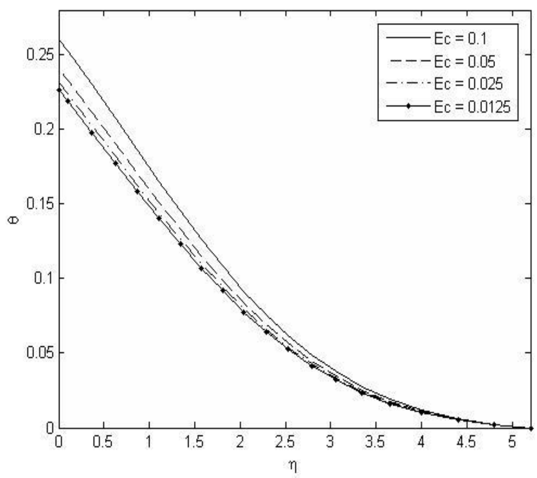

Fig. 3. Temperature profile for various values of Eckert number.

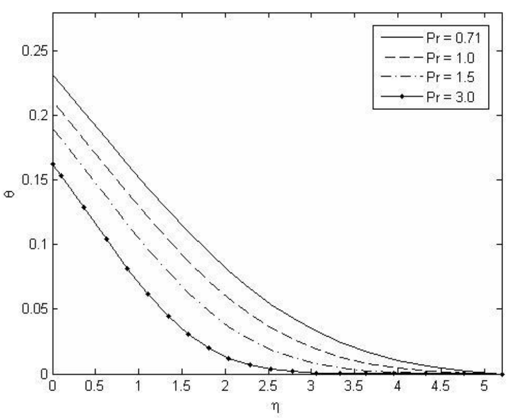

Fig. 5. Temperature profile for various values of Prandtl number.

Table 1 Values of Skin friction and Nusselt number for various values of the pertinent thermophysical properties.

M	Bi	Ec	Pr	N	Skin Friction	Nusselt Number
0.1	0.1	0.025	0.71	1.0	1.18405903005266	0.0768625203195186
0.2	0.1	0.025	0.71	1.0	1.13849227354205	0.0766398395257563
0.3	0.1	0.025	0.71	1.0	1.09576066840255	0.0764133797552343
0.4	0.1	0.025	0.71	1.0	1.05573012884600	0.0761843929477461
0.1	0.4	0.025	0.71	1.0	1.18405903111620	0.184599588733999
0.1	0.7	0.025	0.71	1.0	1.18405903124815	0.230818951997864
0.1	1.0	0.025	0.71	1.0	1.18405903127899	0.256508402597919
0.1	0.1	0.1	0.71	1.0	1.18405903008162	0.0739997922586484
0.1	0.1	0.05	0.71	1.0	1.18405903010019	0.0759082776328551
0.1	0.1	0.0125	0.71	1.0	1.18405903001451	0.0773396416626939
0.1	0.1	0.025	1.0	1.0	1.18405903123802	0.0788381133835695
0.1	0.1	0.025	1.5	1.0	1.18405903080678	0.0809583422080608
0.1	0.1	0.025	3.0	1.0	1.18405902288151	0.0837748081796424
0.1	0.1	0.025	0.71	2.0	1.18405903122752	0.0788044923649799
0.1	0.1	0.025	0.71	3.0	1.18405903135932	0.0795886932352367
0.1	0.1	0.025	0.71	4.0	1.18405903131575	0.0800119647511661

Table 2 Comparison of the Skin friction coefficient values with Adeniyan's paper

Skin Friction for Ec = 0.025 and N = 1				
M	**Bi**	**Pr**	**Current Paper**	**Adeniyan's Paper**
0.1	0.1	0.71	1.18405903005266	1.18405903060128
0.2	0.1	0.71	1.13849227354205	1.13849227348308
0.3	0.1	0.71	1.09576066840255	1.09576066854401
0.4	0.1	0.71	1.05573012884600	1.05573012886324
0.1	0.4	0.71	1.18405903111620	1.18405903241679
0.1	0.7	0.71	1.18405903124815	1.18405903464824
0.1	1.0	0.71	1.18405903127899	1.18405903530630
0.1	0.1	1.0	1.18405903123802	1.18405903003880
0.1	0.1	3.0	1.18405902288151	1.18405902765236

Table 3 Comparison of the Nusselt number values with Adeniyan's paper

Nusselt Number for Ec = 0.025 and N = 14.5				
M	**Bi**	**Pr**	**Current Paper**	**Adeniyan's Paper**
0.1	0.1	0.71	0.0810131341128705	0.0830413824446776
0.2	0.1	0.71	0.0808572460989608	0.0827839701938565
0.3	0.1	0.71	0.0806911278679854	0.0825256014419438
0.4	0.1	0.71	0.0805160044807970	0.0822668367685374
0.1	0.4	0.71	0.212678443555220	0.220158167787210
0.1	0.7	0.71	0.276988631990510	0.288121287645617
0.1	1.0	0.71	0.315101014435776	0.328710654162741
0.1	0.1	1.0	0.0825301037425521	0.0848542422936972
0.1	0.1	3.0	0.0856376539807322	0.0894799245968819

5. Conclusion

In this paper we have studied the convective stagnation point on a stretching sheet with thermal radiation in the presence of magnetic field. Using similarity transformations the governing equations have been transformed into non-linear ordinary differential equations and were solved numerically. We can infer from the present study that; increasing the value of magnetic field parameter resulted in increases in the skin-friction coefficients, whereas the Nusselt number decreased with the increasing values of the magnetic parameter and the radiation number.

References

Aziz, A. (2009). A similarity solution for a laminar thermal boundary layer over a flat plate with convective surface boundary conditions. *Common Nonlinear Sci. Simul.* 14: 1064 – 1068

Enamuel, k. M. and M.J. Uddin (2011). Steady Radiative Free Convective Flow Along a Vertical Flat Plate in the Presence of Magnetic Field. *Daffodil Int. Uni. Journal. Sci. Tech*. 6(2): 55 - 62

Crane, I.J. (1970) Flow past a stretching plate, *Journal of Applied Maths. Phys. (ZAMP),* 21: 645 -647

Ramachadam, N., S. Chen. and B.F. Armaly (1988). Mixed convection in stagnation flows adjacent to a vertical plate. *Acta Mech.* 89: 123 – 132

Okedayo, G.T., P.O. Olanrewaju and J.A. Gbadeyan (2012). Analysis of convective plane stagnation point flow with convective boundary conditions . *Int. Jornal. Sci. Tech.* 15 – 17

Okedayo, G.T., Y.M. Aiyesimi and B.I. Mundi (2011). Viscous dissipation effect on the boundary layer radiative MHD flow over a vertical plate in a porous medium with internal heat generation and convective boundary condition. *Int. Journal. Eng. Tech (IJEST),* 3(8): 6430 – 6440

Samad, M.A. and M.M. Rahman (2006). Thermal Radiation Interaction with Unsteady MHD Flow Past a Vertical Porous Plate Immersed in a Porous Medium. *Journal. Nav. Arc. Mar. Eng*. 3: 7-14.

Sharma, P.R. and G. Singh (2009). Effects of Variable Thermal Conductivity and Heat Source / Sink on MHD Flow Near a Stagnation Point on a Linearly Stretching Sheet. *Journal of Fluid Mechanics*, 2, 13-21

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