

Bioeconomic Model for Tilapia – Nile Perch Fishery in Polluted Environment with Constant Harvesting Efforts in Tanzanian Waters of Lake Victoria

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Abstract

In this paper, bioeconomic model for Tilapia (*Oreochromis niloticus*) – Nile perch (*Lates niloticus*) fishery in polluted environment with the constant harvesting efforts is developed. The model analysed to get maximum sustainable yield (MSY) points and the corresponding conditions for existence established. The equilibrium points of the model found and the conditions for their existence established. The stability analysis of the interior equilibrium point is investigated by using Jacobian matrix method. Later, numerical simulations and their corresponding graphs revealed that water pollution has significant effects on the maximum sustainable yields of both Tilapia and Nile perch produces. These effects also manifests the rapid changes of species population at the interior equilibrium point.

Keywords: bioeconomic model, fishery, Tilapia, Nile perch, harvesting and water pollution.

1. Introduction

Lake Victoria is a three internationally shared lake which covers an area of approximately **68,000 Km²** and adjoining catchment area of **193,000Km²**. It is the largest freshwater lake in Africa, and the second largest freshwater body in the world. It is sharing its resources with Tanzania, Kenya and Uganda. The lake has substantial fish resources for both Tilapia and Nile perch and other species. Tanzania controls 51% of the lake's surface area, Kenya (6%) and Uganda (43%) but its catchment area extends to Rwanda and Burundi (Matsuishi, *et al.*, 2006). According to the National Economic Survey of 2009, the fisheries sector is among the important economic sub-sectors of the economy in Tanzania. The sector provides substantial employment, income, livelihood, foreign earnings to the country. The industry employs more than four million people engaged in fisheries and fisheries related activities while more than 400,000 fisheries operators are directly employed in the sector. In 2009 the fisheries sector contributed 1.3% to GDP. The per capita fish consumption is **8.0 Kg** and about **30%** of animal protein consumption in Tanzania is from fish (MoFEA, 2010).

Nile perch is a vital foreign exchange earner for riparian states in international market, while Tilapia and other fishery resource have ready regional local markets. Nile perch dominated the fisheries of Lake Victoria since

1980's contributing to about 85% of the total catch by weight. It is estimated that the total annual catch of Nile perch ranges between 200,000 to 250,000 tons (Twong'o and Sikoyo, 2004). Tilapia fishery takes the third position in fishery dominance in Lake Victoria next to Dagaa fishery. The fisheries in the Lake Victoria especially that of Tilapia and Nile perch are affected by the environmental degradation. The fishing mortality rate was reported to be at its highest in Tanzania (Twong'o and Sikoyo, 2004).

The network of interactions among organisms and between them and the environment can come in any size but usually encompasses specific limited spaces (Willis, 1997). Fishes, as one of the components in an ecosystem like animals, are categorized into two major groups that are prey and predators, whereby the predators eat prey. The number of prey is controlled by the predators (Curtis, 1972).

The water quality of the Lake Victoria has been declining due to both point and non-point sources from domestic, industrial and agricultural activities (Rwetabula, *et al.*, 2005). Pollution from agricultural activities is mainly from fertilizers and pesticides (Scheren, *et al.*, 2000). This pollution has adverse effects to the health of Tilapia and Nile perch, and other delicate species in the lake and thus degrading the lake ecosystem. The Simiyu catchment which is relatively large catchment with the area of approximately $10\,800\text{ km}^2$, with many agricultural activities using agrochemicals is in record of being one of the main contributors to the deterioration of Lake Victoria quality (LVEMP, 2000). It is generating a high yield of sediments (Machiwa, 2002). Contaminants are carried away from agricultural fields mainly by water runoff to the lake in terms of sediments. Sorption, desorption and binding are considered to be the most important processes determining transport and affecting biodegradation of organic micro pollutants like pesticides (Rav-Acha and Rebhun, 1992).

The population dynamics of Tilapia and Nile Perch suffers from a number of factors. This study engages in studying the dynamical population of Tilapia and Nile perch species which are harvested continuously where by the effects of water pollution on the species population are incorporated. The three factors have a significant role in regulating population size of Tilapia. Thus, it is worthwhile to study the combined effects of epidemiological and demographic features on the real ecological populations of Tilapia and Nile perches.

2. Model and its properties

In this section, we consider the Tilapia – Nile perch model in prey-predator interaction. The ecological set up considers the following. The Tilapia and Nile perch are continually harvested. The Nile perch feeds on the Tilapia as its favourite food in the lake. However, in the absence of Tilapia, the Nile perch feeds on the other prey species available in the lake. Both species are affected by water pollution. In the absence of pollution, harvesting and predation the growth of both Tilapia and Nile perch is assumed to be logistic. Let $x(t)$ and $y(t)$ be, respectively, the size of Tilapia and Nile perch population at time t . Then the Tilapia – Nile perch fishery with time subscripts being suppressed, can be described by:

$$(1) \quad \frac{dx}{dt} = r_1 \left(1 - \frac{x}{k_1} \right) x - h_1(t) - \frac{\alpha y x}{A+x} - d_1 x$$

$$(2) \quad \frac{dy}{dt} = r_2 \left(1 - \frac{y}{k_2} \right) y - h_2(t) + \frac{\beta y x}{A+x} - d_2 y$$

where r_1 and r_2 are the intrinsic growth rates of Tilapia and Nile perch respectively; k_1 and k_2 are the carrying capacity of Tilapia and Nile perch respectively; $h_1(t)$ and $h_2(t)$ are the harvesting of Tilapia and Nile perch respectively at time t ; α is the maximal relative increase of predation, β is the effect of a unit change in Tilapia on the percentage growth rate of Nile perch (conversion factor), A is the half – saturation constant, d_1 and d_2 are the mortality rate of Tilapia and Nile perch respectively due to water pollution. Due to the preference of Nile perch to feed on the Tilapia other than other species, we assume that the intrinsic growth rate of Nile perch is relatively very smaller than that of Tilapia (say $r_2 < r_1$).

The breeding of Tilapia specie takes place at exclusively near – shore, confined to shallow waters of up to about **20 m** deep (Balirwa *et al.*, 2004). Contrary to Tilapia, the breeding of Nile perch is in record of taking place throughout the lake (including the shallow waters) (Balirwa *et al.*, 2004). They reached this conclusion basing on the fact that in the catch assessment survey conducted the Nile perch juveniles were caught in both near shore waters and in the off shore waters. The near shore waters region is at high risk of water pollution, thus suggesting that Tilapia receives relatively greater effects from water pollution than that of Nile perch (say $d_2 < d_1$).

We consider the functional form of harvesting using the *catch-per-unit-effort (CPUE)* phenomenon to describe an assumption that CPUE is proportional to the stock level. Thus we have:

$$(3) \quad h_1(t) = q_1 E_1 x(t)$$

$$(4) \quad h_2(t) = q_2 E_2 y(t)$$

Where q_1 and q_2 are the catchability co-efficient of Tilapia and Nile perch respectively, E_1 and E_2 are the harvesting effort used to harvest Tilapia and Nile perch population respectively.

Thus model (1) – (2) can be extended to:-

$$(5) \quad \frac{dx}{dt} = \left(r_1 - q_1 E_1 - d_1 - \frac{r_1}{k_1} x \right) x - \frac{\alpha y x}{A+x}$$

$$(6) \quad \frac{dy}{dt} = \left(r_2 - q_2 E_2 - d_2 - \frac{r_2}{k_2} y \right) y + \frac{\beta y x}{A+x}$$

with initial conditions $x(0) \geq 0$, and $y(0) \geq 0$. all parameters are considered to be non-negative.

3. Maximum sustained yield points

The right hand side of the model equation (5) can be viewed as the functional sum of $f(x)$ and $F(x, y)$. We have:

$$(7) \quad \frac{dx}{dt} = f(x) + F(x, y)$$

such that

$$f'(x) = \begin{cases} > 0, & x < x_{msy} \\ 0, & x = x_{msy} \end{cases}$$

where $msy = \text{maximum sustained yield}$ (single stock definition), it follows that:

$$f(x) = \left(r_1 - q_1 E_1 - d_1 - \frac{r_1}{k_1} x \right) x$$

Thus

$$f'(x) = (r_1 - q_1 E_1 - d_1) - \frac{2r_1 x}{k_1}$$

At $x = x_{msy}$ we get $f'(x) = 0$ implying that

$$(8) \quad x_{msy} = \frac{k_1}{2r_1} (r_1 - q_1 E_1 - d_1)$$

In order to get the positive value of x_{msy} the following inequality has to be satisfied.

$$(9) \quad r_1 > q_1 E_1 + d_1$$

This suggests that the intrinsic growth rate of Tilapia must exceed the sum of harvesting rate and the death rate of Tilapia due to water pollution, otherwise, the Tilapia specie will go to extinction.

Similarly, considering RHS of a model equation (3.2) as the functional sum of $g(y)$ and $G(x, y)$ we have

$$(10) \quad \frac{dy}{dt} = g(y) + G(x, y)$$

such that

$$g'(y) = \begin{cases} m > 0, & y < y_{msy} \\ 0, & y = y_{msy} \end{cases}$$

It follows that:

$$g(y) = \left(r_2 - q_2 E_2 - d_2 - \frac{r_2}{k_2} y \right) y$$

Thus,

$$g'(y) = \left(r_2 - q_2 E_2 - d_2 \right) - \frac{2r_2 y}{k_2}$$

At $y = y_{msy}$ we have $g'(y) = 0$ implying that

$$(11) \quad y_{msy} = \frac{k_2}{2r_2} (r_2 - q_2 E_2 - d_2)$$

In order to get the positive value of y_{msy} the following inequality has to be satisfied.

$$(12) \quad r_2 > q_2 E_2 + d_2$$

Inequality condition (12) implies that the rate at which the Nile perch grow must exceed the sum of harvesting rate and death rate of Nile perch due to water pollution. When this condition is violated the specie will go to extinction.

We assume that the marginal cost of harvest is independent of specie population size (perhaps 0). The objective of this model is to maximize the present value of profit (Π).

$$\Pi = \int_0^{\infty} e^{-\rho t} (p_1 h_1 + p_2 h_2) dt,$$

where ρ is the discount rate and p_1 and p_2 are the unit profit of harvested Tilapia and Nile perch respectively.

It is convenient to assume that there exist maximum harvest values h_{1max} and h_{2max} for Tilapia and Nile perch respectively, which satisfy the following inequalities

$$0 \leq h_1 \leq h_{1max}$$

$$0 \leq h_2 \leq h_{2max}$$

The current value Hamiltonian function for the simple maximization problem is hereby formed

$$(13) \quad H = p_1 h_1 + p_2 h_2 + \lambda_1 \left[f(x) - \frac{\alpha xy}{A+x} \right] + \lambda_2 \left[g(y) + \frac{\beta xy}{A+x} \right]$$

with λ_1 and λ_2 are the adjoint variables for equation (5) and (6) respectively.

Substituting (3) and (4) we get:

$$H = p_1 q_1 E_1 x + p_2 q_2 E_2 y + \lambda_1 \left[\left(r_1 - q_1 E_1 - d_1 - \frac{r_1}{k_1} x \right) x - \frac{\alpha xy}{A+x} \right] + \lambda_2 \left[\left(r_2 - q_2 E_2 - d_2 - \frac{r_2}{k_2} y \right) y + \frac{\beta xy}{A+x} \right]$$

Now, applying the Bang-bang control for maximization of H we get the following values

$$(14) \quad q_1 E_1 x = \begin{cases} 0 & \lambda_1 > p_1 \\ h_{1max} & \lambda_1 < p_1 \end{cases}$$

$$(15) \quad q_2 E_2 y = \begin{cases} 0 & \lambda_2 > p_2 \\ h_{2max} & \lambda_2 < p_2 \end{cases}$$

The singular control gives:

$$(16) \quad \lambda_1 = p_1$$

$$(17) \quad \lambda_2 = p_2$$

The adjoint equations are:

$$(18) \quad \dot{\lambda}_1 - \rho\lambda_1 = -p_1q_1E_1 - \lambda_1\left(a_1 - \frac{2r_2x}{k_1} - \frac{\alpha yA}{(A+x)^2}\right) - \lambda_2\left(\frac{\beta yA}{(A+x)^2}\right)$$

$$(19) \quad \dot{\lambda}_2 - \rho\lambda_2 = -p_2q_2E_2 + \frac{\lambda_1\alpha x}{A+x} - \lambda_2\left(a_2 - \frac{2r_2y}{k_2} + \frac{\beta x}{A+x}\right)$$

where $a_i = r_i - (q_iE_i + d_i)$ for $i = 1, 2$.

In steady – state interior equilibrium point, we set $\dot{\lambda}_1 = \dot{\lambda}_2 = 0$ and substituting the values of λ_1 and λ_2 in (16) and (17). Equations (18) and (19) take the form

$$(20) \quad \rho = \left(a_1 + q_1E_1 - \frac{2r_2x}{k_1}\right) + \left(\frac{\beta y}{p_1(A+x)^2}\right) (\beta p_2 - \alpha p_1)$$

$$(21) \quad \rho = \left(a_2 + q_2E_2 - \frac{2r_2y}{k_2}\right) + \frac{x}{p_2(A+x)} (\beta p_2 - \alpha p_1)$$

The last additional terms in both (20) and (21) arise from the predator – prey interactions which vanishes when the species' interaction coefficients are zero (i.e. $\alpha = \beta = 0$). This can be thought of as the biological technical externality (Brown, *at al.*, 2005). The biological technical externality has the common terms $(\beta p_2 - \alpha p_1)$ in each of the equations. When $\beta p_2 > \alpha p_1$ we get the positive value of the biological technical externality which suggests that both optimal steady – state Tilapia and Nile perch stock are larger than for a model with independent stocks ($\alpha = \beta = 0$).

Rewriting the inequality as $p_2\left(\frac{\beta}{\alpha}\right) > p_1$ we get the left hand side indicating the marginal product of Tilapia converted into Nile perch whereas in the right hand we have the unit price (marginal cost) of Tilapia. The equilibrium condition for efficient factor use is $p_2\left(\frac{\beta}{\alpha}\right) - p_1 = 0$. For inequality condition to hold, we see that the more inefficient Tilapia is converted into Nile perch biomass, the more valuable Nile perch must be

relative to the value of Tilapia.

Now, we see that the biological technical externality is increasing in p_2 by the factor of β and decreasing in

p_1 by a factor of α . Referring $\frac{(\beta p_2 - \alpha p_1)}{p_1} > 0$ as appeared in (20) we know that an increase in the price of

Tilapia (p_1) decreases $(\beta p_2 - \alpha p_1)$ and contributes to the reduction of the optimal stock of Tilapia specie.

Using the steady – state equations (20) and (21) the population dynamics becomes

$$(22) \quad \frac{k_1}{2r_1} (a_1 + q_1 E_1 - \rho) + \frac{k_1 A y}{2r_1 (A+x)^2} \left(\frac{\beta p_2 - \alpha p_1}{p_1} \right) - x = 0$$

$$(23) \quad \frac{k_2}{2r_2} (a_2 + q_2 E_2 - \rho) + \frac{k_2 x}{2r_2 (A+x)} \left(\frac{\beta p_2 - \alpha p_1}{p_2} \right) - y = 0$$

The model equation (22) illustrate that Nile perch steady – state population (y) enters in the Tilapia steady – state population (x) linearly with the positive effects. Similarly the model equation (23) illustrate that the steady – state population of Tilapia (x) is a linear and positive function of the Nile perch (y), for $\beta p_2 - \alpha p_1 > 0$.

4. Equilibrium points

In this section, we establish conditions for existence of equilibrium points of the model equations (5) and (6).

The system has at least four equilibrium points obtained by setting $\frac{dx}{dt} = \frac{dy}{dt} = 0$ and solving the system of simultaneous equations we get:

i. $E_0(0,0)$ – Extinction of both species.

ii. $E_1 \left(\frac{a_1 k_1}{r_1}, 0 \right)$ – Predator extinction.

iii. $E_2 \left(0, \frac{a_2 k_2}{r_2} \right)$ – Prey extinction.

Assuming that both $x \neq 0$ and $y \neq 0$ the interior equilibrium point becomes

iv. $E^*(x^*, y^*) = \left(\frac{A(b_4 - 9a_2 k_2)}{9k_2 \beta - (b_4 - 9a_2 k_2)}, \frac{b_4}{9r_2} \right)$ – Coexistence

where $b_0 = -\frac{A}{\alpha k_1 r_2^2} (k_1(\beta + a_2)a_1 + Aa_2 r_1) \beta k_2^2$

$$b_1 = \frac{k_2}{\alpha k_1 r_2^2} ((\alpha(\beta + a_2)^2 k_2 + A\beta a_1 r_2) k_1 + A^2 \beta r_1 r_2)$$

$$b_2 = -\frac{2k_2}{r_2} (\beta + a_2)$$

$$b_3 = \frac{1}{6} \left(-8b_2^3 + 36b_1 b_2 - 108b_0 + 12 \sqrt{12b_0 b_2^3 - 3b_1^2 b_2^2 - 54b_0 b_1 b_2 + 12b_1^3 + 81b_1^2} \right)^{\frac{1}{3}}$$

$$b_4 = r_2 \left(3(3b_3 - b_2) - \frac{1}{b_3} (3b_1 - b_2^2) \right)$$

The existence of the equilibrium point $E_0(0, 0)$ is trivial. The predator extinction equilibrium point

$E_1 \left(\frac{a_1 k_1}{r_1}, 0 \right)$ exists if and only if $a_1 > 0$; hence the following condition follows;

$$(24) \quad r_1 - (q_1 E_1 + d_1) > 0$$

Similarly, the prey extinction equilibrium point $E_2 \left(0, \frac{a_2 k_2}{r_2} \right)$ exists if and only if $a_2 > 0$; hence the condition

$$(25) \quad r_2 - (q_2 E_2 + d_2) > 0$$

The existence of the interior equilibrium point E^* is subject to the satisfaction of the inequalities

$$(26) \quad b_4 > 0$$

$$(27) \quad 0 < (b_4 - 9a_2 k_2) < 9k_2 \beta$$

5. Stability Analysis

The stability analysis of the interior equilibrium point (co-existence point E^*) was investigated by using the Jacobian matrix J of a linearized system evaluated at each point E^* .

$$(28) \quad J = \begin{pmatrix} a_1 - \frac{2r_1 x}{k_1} - \frac{\alpha A y}{(A+x)^2} & -\frac{\alpha x}{A+x} \\ \frac{\beta A y}{(A+x)^2} & a_2 - \frac{2r_2 y}{k_2} + \frac{\beta x}{A+x} \end{pmatrix}$$

The Jacobian matrix evaluated at E^* is given by

$$(29) \quad J^* = \begin{pmatrix} a_1 - \frac{2r_1 x^*}{k_1} - \frac{\alpha A y^*}{(A+x^*)^2} & -\frac{\alpha x^*}{A+x^*} \\ \frac{\beta A y^*}{(A+x^*)^2} & a_2 - \frac{2r_2 y^*}{k_2} + \frac{\beta x^*}{A+x^*} \end{pmatrix} \quad \text{where}$$

$$x^* > 0 \text{ and } y^* > 0$$

The trace and determinant of matrix J^* are given by $Trace(J^*)$ and $Det(J^*)$ respectively, where

$$Trace(J^*) = a_1 - \frac{2r_1 x^*}{k_1} - \frac{\alpha A y^*}{(A+x^*)^2} + a_2 - \frac{2r_2 y^*}{k_2} + \frac{\beta x^*}{A+x^*}$$

and

$$\begin{aligned} \text{Det}(J^*) &= \frac{k_2}{k_1(A+x^*)^2} \left((-2(\beta+a_2)k_2+4r_2y^*)r_1x^{*3} \right. \\ &\quad + (r_1(-2(\beta+2a_2)k_2+8r_2y^*)A + ((\beta+a_2)k_2-2r_2y^*)a_1k_1)x^{*2} \\ &\quad + 2(r_1(2r_2y^*-a_2k_2)A + a_1((\beta+2a_2)k_2-4r_2y^*)k_1)Ax^* \\ &\quad \left. - A(r_2y^*-a_2k_2)(Aa_1-y^*\alpha)k_1 \right) \end{aligned}$$

We know that for the interior equilibrium point to be asymptotically stable the trace of Jacobian matrix evaluated at the point must be negative while its determinant value maintains positive value (Say $\text{Trace}(J^*) < 0$ while $\text{Det}(J^*) > 0$). Thus, we have the following conditions.

$$(30) \quad 2 \left(\frac{r_1}{k_1}x^* + \frac{r_2}{k_2}y^* \right) - \left(a_1 + a_2 + \frac{\beta}{A+x^*}x^* \right) > 0$$

(31)

$$\begin{aligned} &\left((-2(\beta+a_2)k_2+4r_2y^*)r_1x^{*3} + (r_1(-2(\beta+2a_2)k_2+8r_2y^*)A + ((\beta+a_2)k_2- \right. \\ &\quad \left. 2r_2y^*)a_1k_1)x^{*2} + 2(r_1(2r_2y^*-a_2k_2)A + a_1((\beta+2a_2)k_2-4r_2y^*)k_1)Ax^* - \right. \\ &\quad \left. A(r_2y^*-a_2k_2)(Aa_1-y^*\alpha)k_1 \right) > 0 \end{aligned}$$

6. Illustrative solution

Numerical examples and their graphical illustrations are as shown below in different cases. We use Runge – Kutta method of order four MAT Lab algorithms to validate the qualitative analysis results.

Example 1. In this example, we use the estimated parameter values in Table 1 with $x(0) = 4\,000$, and $y(0) = 4\,000$ where in a) the pollution parameters d_1 and d_2 are incorporated while in b) the pollution free parameters are considered, i.e $d_1 = d_2 = 0$.

Table 1 Parameters for Figure 1 a) and b)

Parameters	(a) With pollution	(b) Pollution free
r_1	0.80	0.80
r_2	0.65	0.65
k_1	600 000	600 000
k_2	500 000	500 000
q_1	0.05	0.05
q_2	0.08	0.08
E_1	1.20	1.20
E_2	1.50	1.50
d_1	0.30	0
d_2	0.20	0
α	0.0000005	0.0000005
β	0.0000002	0.0000002
A	60000	60000.00
(x_{msy}, y_{msy})	(165000,126920)	(277500,203850)
(x^*, y^*)	(329770,253850)	(554860,407690)

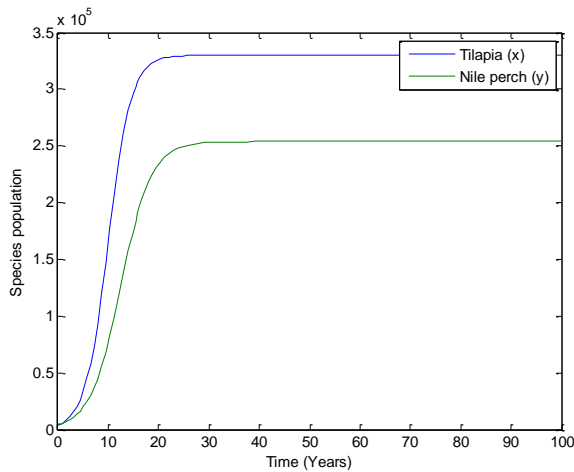


Figure 1 a) Graphical representation of parameters in Table 1 a)

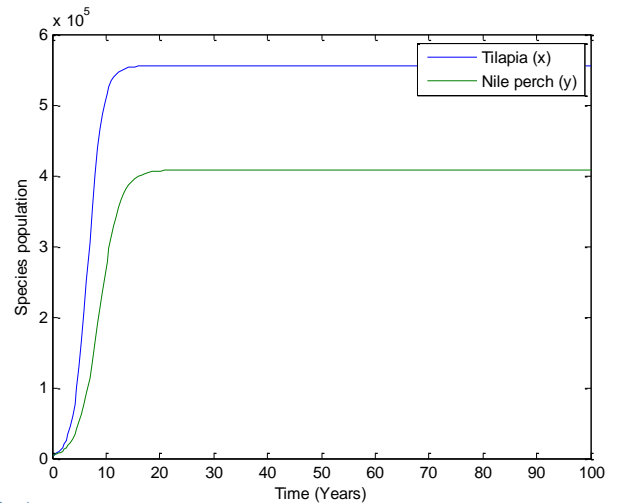


Figure 1 b) Graphical representation of parameters in Table 1 b)

Using these parameter values we see that in pollution free environment the maximum sustainable yield for both Tilapia and Nile perch increases rapidly. The same is true for the interior equilibrium point values.

Example 2. In this example, we use the estimated parameter values in Table 2 with $x(0) = 2\,000$, and $y(0) = 2\,000$ where in a) the pollution parameters d_1 and d_2 are incorporated while in b) the pollution free parameters are considered, i.e $d_1 = d_2 = 0$.

Table 2 Parameters for Figure 2 a) and b)

Parameters	(a) With pollution	(b) Pollution free
r_1	0.80	0.80
r_2	0.65	0.65
k_1	600000	600000
k_2	500000	500000
q_1	0.10	0.10
q_2	0.12	0.12
E_1	1.20	1.20
E_2	1.50	1.50
d_1	0.34	0
d_2	0.15	0
α	0.000001	0.000001
β	0.00000025	0.00000025
A	60000	60000
(x_{msy}, y_{msy})	(127500, 123080)	(255000, 180770)
(x^*, y^*)	(254980, 246150)	(509950, 361540)

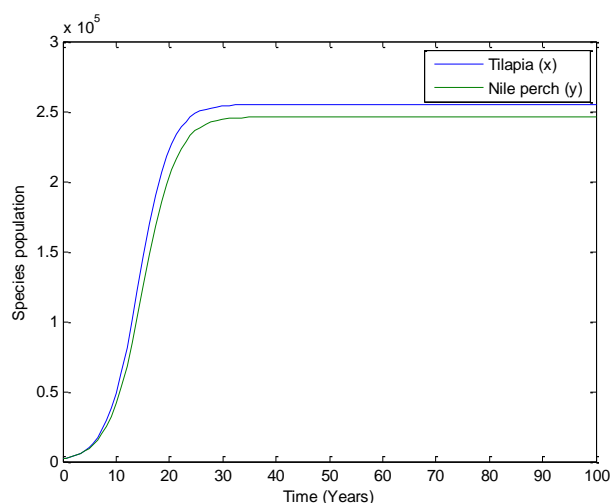


Figure 2a) Graphical representation of parameters in Table 2 a)

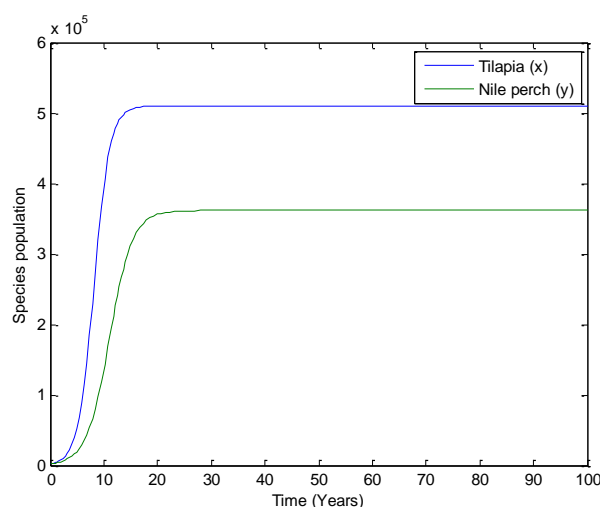


Figure 2b) Graphical representation of parameters in Table 2 b)

Using these parameter values it can also be seen that in pollution free environment the maximum sustainable yield for Tilapia raised by 100% and that of Nile perch raised by at least 46% of its value with pollution constraints. The same is true for the interior equilibrium point values, the value of Tilapia population at the point of coexistence raised by 100% in pollution free environment while the Nile perch population rose to nearly 50% of its value with pollution constraints.

5. Conclusion

This paper analyzes the bioeconomic model for Tilapia – Nile perch fishery in polluted environment with the constant harvesting efforts. The model has been analyzed to get the maximum sustainable yield (MSY) points and the corresponding conditions for their existence have been established. The equilibrium points of the model were found together with establishment of the conditions for their existence. The stability analysis of the interior equilibrium point has been investigated by using Jacobian matrix method. Later, numerical simulations and their corresponding graphs are presented. It was revealed that the water pollution has significant effects to the maximum sustainable yields of both Tilapia and Nile perch produces. This effect also manifests the rapid changes of species population at the interior equilibrium point.

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