Mathematical Theory and Modeling www.iiste.org
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.4, No.6, 2014

MHD Stagnation-Point Flow of a Nanofluid with Heat and Mass Transfer in the Presence of Thermal Radiation

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Abstract
The steady two-dimensional magnetohydrodynamics stagnation point flow of a nanofluid with radiation effect is investigated. Using a similarity transformation, the governing partial differential equations are reduced to ordinary differential equations. The transformed equations are solved numerically for three types of nanoparticles, namely copper (Cu), alumina (Al_2O_3), and titania (TiO_2) by using shooting method. The features of the flow with heat and mass transfer characteristics for different values of governing parameters are analyzed and discussed. Comparison with published results is presented and it found to be in good agreement.
Key Words: Nanofluid, Heat and Mass Transfer, Thermal Radiation, Stagnation-Point Flow

1. Introduction

The study of stagnation-point flow with heat and mass transfer has received great interest due to its wide technical and industrial applications. Some of the applications are central receivers exposed to wind currents, electronic devices by fans, cooling of nuclear reactors during emergency shutdown and many hydrodynamic processes. The two dimensional flow near a stagnation-point was first discussed by Hiemenz (1911). Later, Schlichting & Bussmann (1943) extended the problem of stagnation-point flow numerically. Since then many researchers have extended the stagnation-point flow problem in many ways to include different physical effect. Layek et al. (2007) studied the stagnation point flow with heat generation/absorption and suction/injection. The stagnation point flow in the presence of chemical reaction was investigated by Bhattacharya et al. (2011). Recently, Muhammad et al. (2013) studied stagnation flow with convective boundary conditions.

In order to develop heat and mass transfer in viscous fluids, a small fraction of solid nanoparticles has to be added. The word nanofluid has been introduced by Choi (1995) and it refers to fluids in which nano-scale particles are suspended in the base fluid. The behavior of nanofluids provides a basis for heat and mass transfer intensification in industrial sectors. For example, power generation, transportation, thermal therapy for cancer treatment, ventilation, chemical sectors, air-conditioning, etc [Ding et al. (2007)]. More references on nanofluids can be seen in the book by Das et al. (2007) and in the review papers by Buongiorno (2006), Wang & Mujumdar (2007), Kakaç & Pramuanjaroenkij (2009), Eagen et al. (2010), Wong & Leon (2010) and Fan & Wang (2011). Recently, the thermophysical properties of nanofluids such as, diffusivity, thermal conductivity and viscosity have been studied by Kameswaran et al. (2013) and Mohammad et al. (2013).

Motivated by the above study, the purpose of this research is to investigate the combined effects of MHD and radiation on stagnation-point flow with heat and mass transfer in a nanofluid. Representative results for the velocity, temperature and concentration profiles as well as the skin friction, the Nusselt number and the Sherwood number are shown for various values of the physical parameters. To the best of author's knowledge no one yet consider this problem.

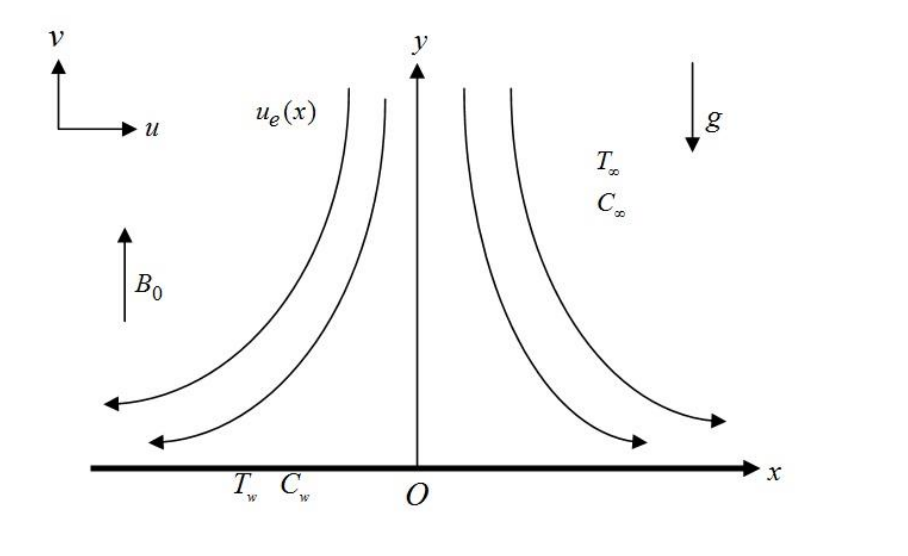

Figure 1 Physical model and coordinate system

2. Mathematical Formation

Consider a steady two-dimensional incompressible stagnation-point flow in nanofluid with heat and mass transfer in the presence of thermal radiation as shown in Fig 1. The surface located at $y = 0$ with a fixed stagnation point at $x = 0$. The magnetic field of strength B_o is applied in the positive direction of $y-$axis. It is assumed that the external flow velocity varies linearly in the form of $u_e(x) = ax$, where b is a positive constant. Under the assumptions along with boundary layer flow, the governing equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} + \frac{\sigma_{nf} B_o^2}{\rho_{nf}}(u_e - u) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho c_p)_{nf}}\frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial y} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

subjected to the boundary conditions

$$u = 0\ ,\ v = 0\ ,\ T = T_w\ ,\ C = C_w\ \text{ at }\ y = 0$$

$$u \to u_e(x) = ax\ ,\ T \to T_\infty\ ,\ C \to C_\infty\ \text{ as }\ y \to \infty \tag{5}$$

where u and v are the velocity components along the $x-$axis and $y-$axis, respectively. Further, T is the temperature of the nanofluid, σ_{nf} is the electrical conductivity of the nanofluid, q_r is the radiative heat flux, D is the diffusion coefficient, μ_{nf} is the viscosity of the nanofluid, α_{nf} is the thermal diffusivity of the nanofluid and ρ_{nf} is the density of the nanofluid, which is given by (Oztop and Abu-Nada 2008).

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}\ ,\ \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s\ ,\ \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \tag{6}$$

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s\ ,\ \frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)} \tag{7}$$

Here, ϕ is the nanoparticle volume fraction, $(\rho c_p)_{nf}$ is the heat capacity of the Nanofluid, k_{nf} is the thermal conductivity of the Nanofluid, k_f and k_s are the thermal conductivities of the fluid and of the solid fractions, respectively, ρ_f and ρ_s are the densities of the fluid and of the solid fractions, respectively. It is noted that the use of the above expression for $\frac{k_{nf}}{k_f}$ is restricted only to spherical nanoparticles.

Using the Rosseland approximation [Raptis (1998)], the radiation heat flux is given by

$$q_r = -\frac{4\sigma^*}{3k}\frac{\partial T^4}{\partial y} \tag{8}$$

where σ^* and k are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. As done by Raptis (1998), temperature differences within the flow are assumed to be sufficiently small so that T^4 may be expressed as a linear function of temperature T using a truncated Taylor series about the free stream temperature T_∞ i.e.,

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \tag{9}$$

From Eq. (3), (7) and (8), we have

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho c_p)_{nf}}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3k(\rho c_p)_{nf}}\frac{\partial^2 T}{\partial y^2} \tag{10}$$

The continuity equation (1) is satisfied by introducing a stream function ψ such that

$$u = \frac{\partial \psi}{\partial y} \quad , \quad v = -\frac{\partial \psi}{\partial x}$$

Introducing the following transformation

$$\eta = \sqrt{\frac{a}{v_f}}y \quad , \quad \psi = \sqrt{a v_f}\, x f(\eta) \quad , \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T\infty} \quad , \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{11}$$

Employing (11), the nonlinear partial differential equation (2), (4) and (10) are transformed into the following ordinary differential equations

$$\frac{1}{(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)} f''' + ff'' - f'^2 + 1 + M(1-f') = 0 \tag{12}$$

$$\left(1+\frac{4R}{3}\right)\frac{\frac{k_{nf}}{k_f}}{\left(1-\phi+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)}\theta'' + \Pr f\theta' = 0 \tag{13}$$

$$\phi'' + Sc\,\phi' = 0 \tag{14}$$

subjected to the boundary condition (5) which then becomes

$$f(0) = 0 \ , \ f'(0) = 0 \ , \ \theta(0) = 1 \ , \ \phi(0) = 1$$
$$f'(\infty) \to 1 \ , \ \theta(\infty) \to 0 \ , \ \phi(\infty) \to 0 \tag{15}$$

where prime denote differentiation with respect to η, $\Pr = \frac{v_f}{\alpha_f}$ is the Prandtl number, $M = \frac{\sigma_{nf} B_o}{a\rho_{nf}}$ is the magnetic parameter, $R = \frac{4\sigma^* T_\infty^3}{k_{nf} k}$ is the radiation parameter and $Sc = \frac{v_f}{D}$ is the Schmidt number. The physical quantities of interest are the skin friction coefficient C_f, the local Nusselt number Nu_x and the local Sherwood number Sh_x, which can be expressed as

$$C_f = \frac{\tau_w}{\rho_{nf} u_e^2} \quad , \quad Nu_x = \frac{x}{k_f} \frac{q_w}{T_f - T_\infty} \quad , \quad Sh_x = \frac{x}{D} \frac{J_w}{C_w - C_\infty} \tag{16}$$

where τ_w is the surface shear stress, q_w is the heat flux and J_w is the mass flux, which are defined as

$$\tau_w = \mu_{nf} \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad , \quad q_w = -\left[\left(k_{nf} + \frac{16\sigma^* T_\infty^3}{3k} \right) \frac{\partial T}{\partial y} \right]_{y=0} \quad , \quad J_w = -D \left(\frac{\partial C}{\partial y} \right)_{y=0} \tag{17}$$

Substituting (11) into (17) and using (16), we obtain

$$C_f \, Re_x^{1/2} = \frac{1}{(1-\phi)^{2.5}} f''(0) \; , \; \frac{Nu_x}{Re_x^{1/2}} = -\left(1 + \frac{4R}{3} \right) \frac{k_{nf}}{k_f} \theta'(0) \; , \; \frac{Sh_x}{Re_x^{1/2}} = -\phi'(0) \tag{18}$$

where $Re_x = \frac{u_e x}{v_f}$ is the local Reynolds number.

3. Results and Discussion

The transformed non-linear ordinary differential equations (11) to (13) with boundary conditions (14) are solved numerically by using shooting method. The effects of MHD, radiation parameter and the solid volume fraction of the nanofluid for three types of nanofluids, namely Cu-water, alumina-water, and titania-water, as working fluids. Following Oztop and Abu-Nada (2008), Prandtl number is taken as 6.2 (water) and the volume fraction of nanoparticles is from 0 to 0.2 $(0 \le \phi \le 0.2)$, where $\phi = 0$ represents the regular fluid. In order to check the accuracy of the current method, our results are compared with results reported by Mohammad (2013) in Table 2 and found to be in good agreement.

Figures 2-4 respectively present the velocity, temperature and concentration of the fluid for different values of nanoparticle volume fraction. Figure 2 and 4 show that the momentum and concentration boundary layer thickness decrease with increase of nanoparticle volume fraction while, the thermal boundary layer thickness increase as the nanoparticle volume fraction increase as shown in Figure 3.

Figure 5 shows that the temperature of the fluid in the boundary layer increases with increasing thermal radiation parameter. This is due to the fact that, the divergence of the radiative heat flux $\frac{\partial q_r}{\partial y}$ increases as the Rosseland radiative absorptivity k decreases (see expression for R) which in turn increase the rate of radiative heat transfer to the fluid, which causes the fluid temperature to increase. In view of this fact, the effect of radiation becomes more significant as $R \to \infty$ and the radiation effect can be neglected when $R = 0$.

The effect of Schmidt number on the concentration profiles is shown in Figure 6. It is observed from this figure that the concentration decreases with increasing Sc. The Schmidt number represents the relative ease of molecular momentum and mass transfer and is very important in calculations of binary mass transfer in multiphase flows. The effect of an increase in the Schmidt number values is to reduce the boundary layer and this leads to a thin diffusion layer.

The variations of the skin friction, Nusselt number and Sherwood number with ϕ for different types of nanoparticle are illustrated in Figures 7-9. These figures show that the skin friction, Nusselt number and Sherwood number increase with increasing ϕ. It is also observed that the skin friction coefficient, the Nusselt number and the Sherwood number have the highest values for Cu compared to TiO_2 and Al_2O_3.

4. Conclusion

We have studied MHD stagnation point flow of a nanofluid with radiation effect. Using the suitable similarity transformation, the resulting equations were solved numerically for three types of nanoparticles, namely Cu-water, alumina-water, and titania-water. The obtained numerical results were compared with those from previous studies to verify their validity. In the light of the present investigation, we found that due to increase in the value of nanoparticle volume fraction, the velocity and temperature of the fluid increase while opposite trend is observed for concentration of the fluid.

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Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.4, No.6, 2014

Physical Properties	Fluid phase (water)	Cu	Al_2O_3	TiO_2
$C_p(J/kgK)$	4179	385	765	686.2
$\rho(kg/m^3)$	997.1	8933	3970	4250
$k(W/mK)$	0.613	400	40	8.9538
$\alpha\times10^{-7}(m^2/s)$	1.47	1163.1	131.7	30.7

Table 1 Thermophysical properties of fluid and nanoparticles.

ϕ	Cu		Al_2O_3		TiO_2	
	$C_f\,Re_x^{1/2}$	$Nu_x/Re_x^{1/2}$	$C_f\,Re_x^{1/2}$	$Nu_x/Re_x^{1/2}$	$C_f\,Re_x^{1/2}$	$Nu_x/Re_x^{1/2}$
0.0	1.5851 [1.58400]	1.6507 [1.650833]	1.5851 [1.585400]	1.6507 [1.650833]	1.5851 [1.585400]	1.6507 [1.650833]
0.05	1.9979 [1.997958]	1.8470 [1.847150]	1.8112 [1.811292]	1.7943 [1.794396]	1.8228 [1.822888]	1.7741 [1.774156]
0.10	2.4059 [2.405934]	2.0315 [2.031578]	2.0600 [2.060035]	1.9382 [1.938243]	2.0825 [2.082549]	1.8965 [1.896534]
0.15	2.8757 [2.875792]	2.2188 [2.218840]	2.3368 [2.336844]	2.0831 [2.083274]	2.3704 [2.370472]	2.0147 [2.014743]
0.20	3.3691 [3.369125]	2.4026 [2.402679]	2.6478 [2.647829]	2.2297 [2.229723]	2.6937 [2.693763]	2.1247 [2.134747]

Table 2 Comparison of the skin friction and local Nusselt number for the different nanoparticles when $M=1$ and $R=0.5$.
[] Mohammad et al. [12].

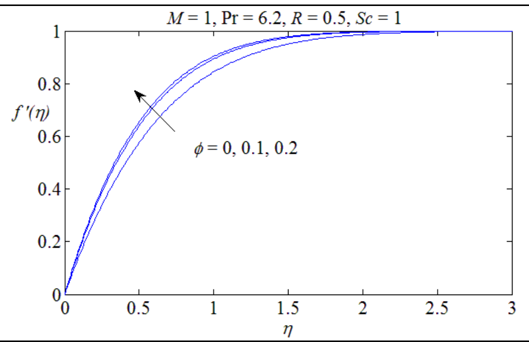

Fig 2: Velocity profiles for different values of ϕ for *Cu*-water nanofluid.

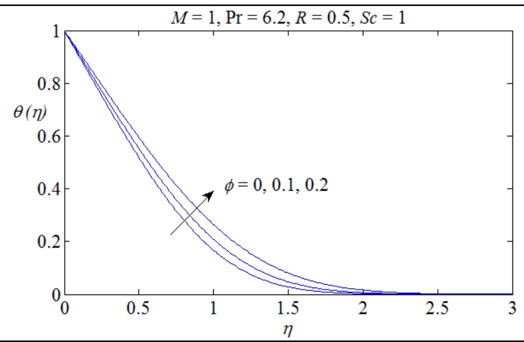

Fig 3: Temperature profiles for different values of ϕ for *Cu*-water nanofluid.

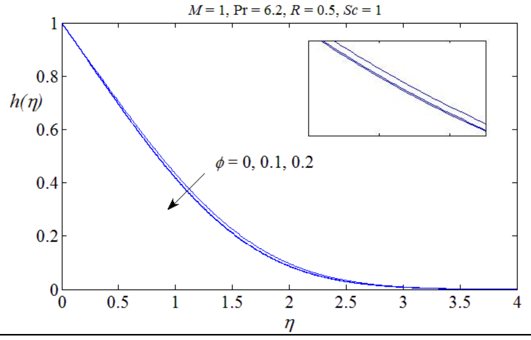

Fig 4: Concentration profiles for different values of ϕ for *Cu*-water nanofluid.

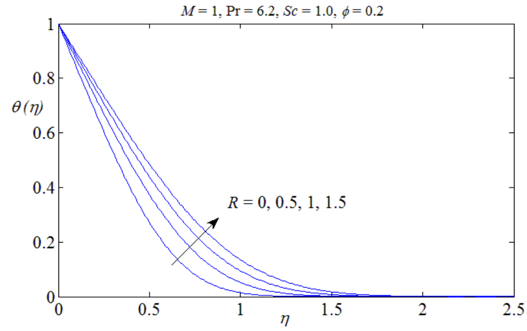

Fig 5: Temperature profiles for different values of R for *Cu*-water nanofluid.

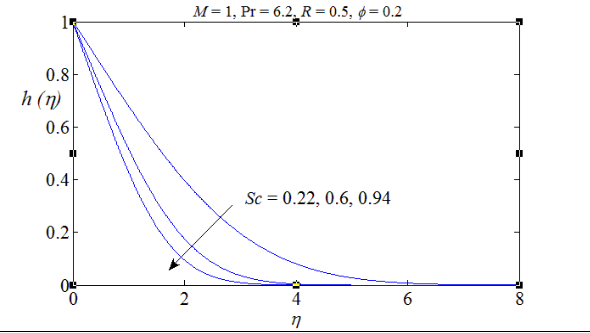

Fig 6: Concentration profiles for different values of Sc for *Cu*-water nanofluid.

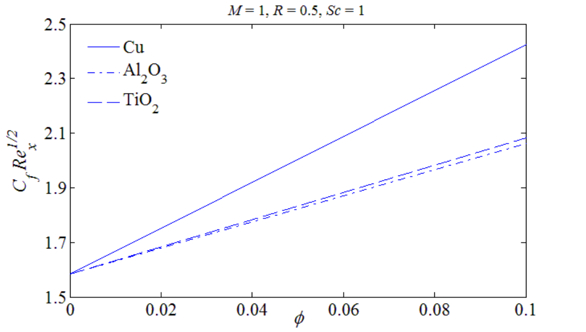

Fig 7: Variation of skin friction with ϕ for different types of nanoparticles.

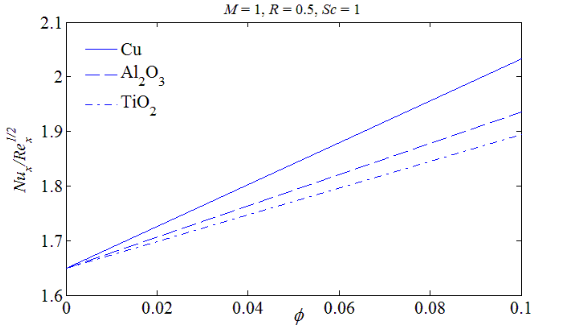

Fig 8: Variation of Nusselt number with ϕ for different types of nanoparticles.

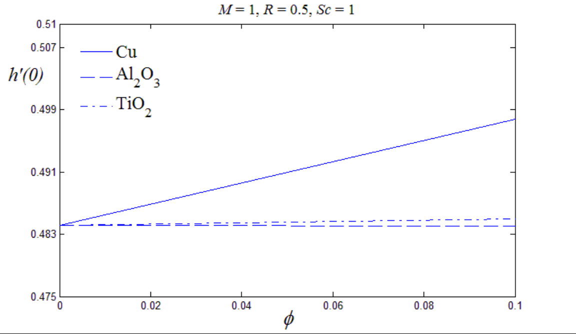

Fig 9: Variation of Sherwood number with ϕ for different types of nanoparticles.