 

# On Contra $S_S$-Continuous Functions

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**Abstract**

In this paper, we apply the notion of $S_S$-open set in topological spaces to introduce and investigate the concept of contra $S_S$-continuous which is a subclass of the class of contra semi continuous functions.

**Keywords:** $S_S$-closed, contra $S_S$-continuous, contra $S_S$ –closed and strongly contra $S_S$ –closed.

## 1. Introduction

In 1966, Dontchev [3], introduced the notion of contra continuity and established some results about S-closedness and strongly S-closedness. Subsequently, Dontchev and Noiri [4], introduced and studied contra semi continuity and gave several properties about these functions. Later Jafari and Noiri [5], investigated contra α- continuous and contra pre-continuous. Recently authors introduced $S_S$-open set for topological spaces where $S_S$-continuity had been investigated. For a subset A of X, $Cl(A)$ and $Int(A)$ represent the closure and interior respectively. A subset $A$ of X is called semi-open[9](α-open[11], pre-open [10], regular open [17]) set if $A \subseteq ClInt(A)$, $A \subseteq IntClInt(A)$,$A \subseteq IntCl(A), A = IntCl(A)$,The complement of semi-open(α-open, pre-open, regular open) set is called semi-closed (α- closed, pre- closed, regular closed)set. A subset $A$ of topological space $(X,\tau)$ is called θ-open set [16] if for each $x \in A$, there is an open set U such that $x \in U \subseteq Cl(U) \subseteq A$. A subset $A$ is called semi regular [2] if it is both semi-open and semi-closed. The main purpose of this paper is to introduce the notion of contra $S_S$-continuous functions and obtained some its properties. Also, we defined and studied the concept of contra $S_S$-closed and strongly $S_S$-closed.

## 2. Preliminaries

The following definitions and results are needed .

**Definition 2.1** . A topological space $X$ is called:

1) locally indiscrete [3], if every open set in $X$ is closed.
2) extremally disconnected [6], if the closure of every open subset of $X$ is open or the interior of every closed subset of $X$ is closed.
3) semi-$T_1$ [1], If for every two distinct points *x*, *y* in *X*, there exist two semi open sets, one containing *x* but not *y* and the another containing *y* but not *x*.

**Lemma 2.2**[1]. A space *X* is semi-$T_1$, if and only if, the singleton $\{x\}$ is semi-closed for any point $x \in X$.

**Definition 2.3.**A function $f: X \longrightarrow Y$ is called:

1) Contra continuous [3], if the inverse image of every open set in *Y* is closed set in *X*.
2) Semi-continuous [9] (resp., contra semi continuous [4]) if the inverse image of every open set in *Y* is semi-open (resp., semi-closed) set in *X*.

 

3) Perfectly continuous [15] (SR-continuous [12], RC-continuous [12]) if the pre image of every open set in *Y* is clopen (semi regular, regular closed) set in *X*.
4) pre-closed [5], if the image of every closed set in *X* is pre-closed set in *Y*.

**Theorem 2.4**[1]. For any spaces *X* and *Y*. if $A \subseteq X$ and $B \subseteq Y$, then

1) $sInt_{X\times Y}(A \times B) = sInt_X(A) \times sInt_Y(B)$
2) $sCl_{X\times Y}(A \times B) = sCl_X(A) \times sCl_Y(B)$

The following definitions and results are from [7].

**Definition 2.5** A semi-open subset *A* of a space *X* is called $S_S$-open if for each $x \in A$, there exist semi-closed *F* such that $x \in F \subseteq A$.

The complement of $S_S$-open set in *X* is called $S_S$-closed set in *X*.

**Proposition 2.6.** Let $\{A_\alpha : \alpha \in \Delta\}$ be collection of $S_S$-closed sets in topological space *X*, then $\bigcap\{A_\alpha : \alpha \in \Delta\}$ is $S_S$-closed.

**Proposition 2.7**. Let X be topological space and $A, B \subseteq X$. If $A \in S_SO(X)$ and *B* is both $\alpha-$open and semi-closed, then $A \cap B \in S_SO(X)$

**Proposition 2.8**. Let $(Y, \tau_Y)$ be an subspace of $(X, \tau)$and $A \subseteq Y$, then the following properties are true:

1) If $A \in S_SO(Y, \tau_Y)$ and *Y* is semi-regular, then $A \in S_SO(X, \tau)$.
2) If $A \in S_SO(X, \tau)$ and *Y*is $\alpha$-open, then $A \in S_SO(Y, \tau_Y)$.

**Proposition 2.9**. If $(X, \tau)$ is a semi-$T_1$ space, then $S_SO(X, \tau) = SO(X, \tau)$.

**Definition 2.10**. A function $f: X \longrightarrow Y$ is called $S_S$-continuous at a point $x \in X$, if for each an open set V of Y containing f(x), there exist an $S_S$-open set U of X containing x such that $f(U) \subseteq V$.

**Proposition 2.11**. For a function $f: X \longrightarrow Y$, the following statements are equivalent:

1) $f$ is $S_S$-continuous,
2) The inverse image of every open set in *Y* is an $S_S$-open set in *X*,
3) The inverse image of every closed set in *Y* is an $S_S$-closed set in *X*.

**Definition 2.12 [8]**. A function $f: X \to Y$ is called weakly $S_S$-continuous. If for each $x \in X$ and each open set *H* in *Y* containing $f(x)$, there is an $S_S$-open set G containing x such that $f(G) \subseteq Cl(H)$.

## 3. Contra $S_S$-continuous function

**Definition 3.1.** A function $f: X \longrightarrow Y$ is called contra $S_S$-continuous if $f^{-1}(U)$ is $S_S$-closed in *X* for each open set *U* in *Y*.

**Theorem 3.2** for a function $f: X \longrightarrow Y$ the following conditions are equivalent.

1) $f$ is contra $S_S$-continuous.
2) The inverse image of every closed set in Y is $S_S$-open set in *X*.
3) For each $x \in X$, and each closed subset *F* of *Y* containing $f(x)$, there is $S_S$-open *U* containing *x* such that $f(U) \subseteq F$.

**Proof.(1)⟹(2)**. Let *F* be closed subset of *Y*, then $Y - F$ is an open set in *Y*. since $f$ is contra $S_S$-continuous, then $f^{-1}(Y - F) = X - f^{-1}(F)$ is $S_S$-closed set in *X*. Hence $f^{-1}(F)$ is $S_S$-open set in *X*.

**(2)⟹(3).** Let $F$ be closed subset of $Y$ containing $f(x)$, then by (2)$f^{-1}(F)$ is $S_S$-open set in $X$ containing $x$. since$f(f^{-1}(F)) \subseteq F$. Take $U$=$f^{-1}(F)$. Hence$f(U) \subseteq F$.

**(3)⟹(1).** Let $x \in X$ and let $H$ be an open set in Y, therefore $Y - H$ is closed subset of $Y$ containing$f(x)$. Then by (3) there exist $S_S$-open set $U$ containing x such that $f(U) \subseteq Y - H$ implies that $U \subseteq f^{-1}(Y - H) = X - f^{-1}(H)$. Hence $f^{-1}(H)$ is $S_S$-closed set in $X$.

**Remark 3.3.** Every contra $S_S$-continuous is contra semi continuous.

But the converse is not true as showing in the following example,

**Example 3.4.**Let $X = \{a, b, c\}$with the topologies$\tau$={$\phi$,$X$,{$c$}} and $\sigma$={ $\phi$,$X$,{$a$},{$b$},{$a$,$b$}}. If $f: (X, \tau) \longrightarrow (X, \sigma)$ defined by $f(a) = f(b) = b$ and $f(c) = c$. Then f is contra semi continuous but it is not contra $S_S$-continuous because $f^{-1}(\{b\}) = \{a, b\}$ which is not $S_S$-closed set in$(X, \tau)$.

**Proposition 3.5.** Let $f: X \longrightarrow Y$ be a semi continuous function, then$f$ is contra semi continuous if and only if it is contra $S_S$-continuous.

Proof: sufficiently, obvious.

Necessity, let $F$ be a closed subset of $Y$. Since$f$ is both contra semi continuous and semi continuous, then $f^{-1}(H)$ is semi clopen subset of $X$, so it is $S_S$-open. Therefore $f$ is contra $S_S$-continuous.

**Proposition 3.6.** If $f: X \longrightarrow Y$ is contra semi continuous and $X$ is semi $T_1$-space, then $f$ is contra $S_S$-continuous.

**Proof.** Let $F$ be an open subset of $Y$. Since$f$ is contra semi continuous, then $f^{-1}(F)$ is a semi-closed subset of X. Thus $X - f^{-1}(F)$ is semi-open in X and since X is semi $T_1$-space, then by Proposition 2.9, $X - f^{-1}(F)$ is $S_S$-open. Therefore,$f^{-1}(F)$is $S_S$-closed and hence$f$ is contra $S_S$-continuous.

**Corollary 3.7.** A function $f: X \longrightarrow Y$ is contra $S_S$-continuous if it is one of the following:

1) $f$ is strongly continuous
2) $f$ is perfectly continuous
3) $f$ is RC-continuous
4) $f$ is SR-continuous

**Proof.** Straightforward.

**Proposition 3.8.** If a function $f: X \longrightarrow Y$ is contra $S_S$-continuous, then $f$ is weakly $S_S$-continuous.

Proof: let $V$ be an open subset of $Y$, then $Cl(V)$ is closed set in $Y$. Since $f$ is contra $S_S$-continuous, then by Theorem 3.2,$f^{-1}(Cl(V))$ is $S_S$-open in X and since$f(f^{-1}(Cl(V))) \subseteq Cl(V)$. Take $U$= $f^{-1}(Cl(V))$, therefore $f(U) \subseteq Cl(V)$. Hence $f$ is weakly $S_S$-continuous.

The converse of the above proposition is not true as it is shown in the next example.

**Example 3.9.** Let$X = \{a, b, c\}$and let= $\{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$,$\sigma = \{\phi, X, \{a\}\}$ be two topologies on $X$. Then the function $f: (X, \tau) \rightarrow (X, \sigma)$ defined by $f(a) = a$, $f(b) = f(c) = c$ is weakly $S_S$-continuousbut it is not contra $S_S$-continuous because $f^{-1}(\{a\})$ is not $S_S$-closed.

**Proposition 3.10.** Let$f: X \rightarrow Y$ be any function and $Y$ be extremally disconnected, then $f$ is contra $S_S$-continuous if and only if the inverse image of each clopen subset of $Y$ is $S_S$-open subset of $X$.

Proof: sufficiently, straightforward.

Necessity, suppose the inverse image of clopen subset in $Y$ is $S_S$-open. Let $F$ be a closed subset of $Y$

 

containing $f(x)$. Since $X$ is extremally disconnected then $Int(F)$ is clopen set in $Y$. So by hypothesis, $f^{-1}(Int(H)$ is $S_S$-open set in $X$. Since $f(f^{-1}(Int(F))) \subseteq Int(F)$. Take $U = f^{-1}(Int(F))$ then $f(U) \subseteq Int(F) \subseteq F$ .Therefore by Theorem 3.2., $f$ is contra $S_S$-continuous.

Clearly that contra $S_S$-continuity and $S_S$-continuity are independent

**Proposition 3.11.** If a function $f: X \to Y$ is contra $S_S$-continuous and $Y$ is regular space then $f$ is $S_S$-continuous.

Proof: let $V$ be an open set in $Y$ containing $f(x)$ for $\in X$. Since $Y$ is regular, then there is an open set $W$ in $Y$ such that $f(x) \in W \subseteq Cl(W) \subseteq V$. Since $f$ is contra $S_S$-continuous then by Theorem 3.2, there is an $S_S$-open set U in X containing x such that $f(U) \subseteq Cl(W) \subseteq V$. Hence $f$ is $S_S$-continuous.

**Corollary 3.12.** If a function $f: (X, \tau) \to (R, \tau_U)$ is contra $S_S$-continuous, then $f$ is $S_S$-continuous.

**Proposition 3.13.** If a function $f: X \to Y$ is $S_S$-continuous and $Y$ is locally indiscrete, then $f$ is contra $S_S$-continuous.

**Proof.** Straightforward.

**Definition 3.14.** A topological space $(X, \tau)$ is called SCC-space if every $S_S$-closed subset of $X$ is closed.

**Example 3.15.** Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b, c\}\}$ then clearly $(X, \tau)$ is SCC-space.

**Proposition 3.16.** Let $f: X \to Y$ be surjective, pre-closed and contra $S_S$-continuous. If $X$ is SCC-space, then $Y$ is extremally disconnected.

**Proof.** let $V$ be an open set in $Y$, then $U = f^{-1}(V)$ is $S_S$-closed subset of X. But $X$ is SCC-space, then $U$ is closed in X. f is pre-closed, then $f(U) = V$ is pre-closed in Y which implies that $Cl(V) = Cl(Int(V)) \subseteq V$. And so $cl(V)$ is an open set in Y. Hence $Y$ is extremally disconnected .

**Proposition 3.17.** If a function $f: X \to Y$ is contra $S_S$-continuous, then for any subset $A$ of $X$, $f(S_S - int(A)) \subseteq Cl(f(A))$

**Proof.** let $A \subseteq X$, then $Cl(f(A))$ is closed subset of $Y$. since $f$ is contra $S_S$-continuous, then $f^{-1}(Cl(f(A)))$ is $S_S$-open set in $X$. Therefore, $S_S - Intf^{-1}(Cl(f(A))) = f^{-1}(Cl(f(A)))$ . Since $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(Cl(f(A)))$ implies that $S_S - int(A) \subseteq S_S - Intf^{-1}(Cl(f(A))) = f^{-1}(Cl(f(A)))$ . Hence $f(S_S - Int(A)) \subseteq Cl(f(A))$.

**Definition 3.18.** A topological space $(X, \tau)$ is called $S_S$-locally indiscrete if every $S_S$-open subset of X is closed.

**Example 3.19.** let $X$={a,b,c} with topology $\tau = \{\phi, X, \{a\}, \{b,c\}\}$, then $(X, \tau)$ is $S_S$-locally indiscrete.

**Proposition 3.20.** If a function $f: X \to Y$ is contra $S_S$-continuous and $X$ is $S_S$-locally indiscrete space, then $f$ is continuous.

**Proof.** let $F$ be any closed subset of $Y$. Since $f$ is contra $S_S$-continuous, then $f^{-1}(F)$ is an $S_S$-open subset of X. But X is $S_S$-locally indiscrete, then $f^{-1}(F)$ is closed. Hence $f$ is continuous.

**Proposition 3.21.** If $f: X \to Y$ is contra $S_S$-continuous, then $S_S - Cl(f^{-1}(V)) \subseteq f^{-1}(Cl_\theta(V))$.

**Proof.** if $x \notin f^{-1}(Cl_\theta(V))$ implies that $f(x) \notin Cl_\theta(V)$, then there is an open set $G$ containing $f(x)$ such that $Cl(G) \cap V = \phi$. Since $f$ is contra $S_S$-continuous, then there is an $S_S$-open set $U$ such that $f(U) \subseteq Cl(G)$ and hence $U \cap f^{-1}(V) = \phi$. This shows that $x \notin S_S - Cl(f^{-1}(V))$.

**Proposition 3.22.** If a function $f: X \to Y$ is contra $S_S$-continuous and $U$ is α-open and semi closed subset of $X$ then $f|U: U \longrightarrow Y$ is contra $S_S$-continuous.

**Proof.** let $H$ be a closed set in Y. Since $f$ is contra $S_S$-continuous, then by Theorem 3.2. $f^{-1}(H)$ is $S_S$-open set in $X$ and since $U$ is α-open and semi-closed subset of $X$, then by Proposition2.7, $(f|U)^{-1}(H) = f^{-1}(H) \cap U$ is $S_S$-open in X. by Proposition2.8(2), $(f|U)^{-1}(H)$ is $S_S$-open set in $U$. This shows that $f|U$ is contra

 

$S_S$-continuous.

**Proposition 3.23.** A function $f: X \to Y$ is contra $S_S$-continuous if for each $x \in X$, there exist semi regular set $A$ of $X$ containing x such that $f|A: A \longrightarrow Y$ is contra $S_S$-continuous

**Proof.** let $x \in X$, then there exist semi regular set $A$ of $X$ containing x. let $F$ be closed subset of $Y$ containing $f(x)$, then by Theorem 3.2, there exist $S_S$-open set $U$ in $X$ containing x such that $(f|A)(U) \subseteq F$. Since $A$ is semi regular set in $X$, by Proposition2.8(1),$U$ is an $S_S$-open set in X and hence $f(U) \subseteq F$. Thus $f$ is contra $S_S$-continuous.

**Proposition 3.24.** let $f: X \to Y$ and $g: Y \to Z$ be any two functions then

1) $g \circ f: X \longrightarrow Z$ is contra $S_S$-continuous if $f$ is $S_S$-continuous and $g$ is contra continuous.
2) $g \circ f: X \longrightarrow Z$ is contra $S_S$-continuous if $f$ is contra $S_S$-continuous and $g$ is continuous

**Proof.**(**1**)Let $H$ be an open set in $Z$. since $g$ is contra continuous, then $g^{-1}(H)$ is closed subset of $Y$ and since $f$ is $S_S$-continuous, then by Proposition 2.11, $f^{-1}(g^{-1}(H)) = (g \circ f)^{-1}(H)$ is $S_S$-closed. Hence $g \circ f$ is contra $S_S$-continuous.

**(2)** Similar to (1).

**Definition 3.25.** A function $f: X \longrightarrow Y$ is called $S_S$-irresolute if $f^{-1}(U)$ is $S_S$-open in $X$ for each $S_S$-open set $U$ in $Y$.

**Example 3.26.** Let $X=\{a,b,c\}$ with topology $\tau=\{\phi,X,\{c\}\}$ and $\sigma=\{\phi,X,\{a\}\}$ then clearly $f: (X,\tau) \longrightarrow (X,\sigma)$ defined by $f(a) = a, f(b) = c$ and $f(c) = b$is $S_S$-irresolute function.

**Proposition 3.27**. If$f: X \to Y$is $S_S$-irresolute and $g: Y \to Z$is contra $S_S$-continuous, then $g \circ f: X \longrightarrow Z$ is contra $S_S$-continuous.

**Proof.** Let $F$ be closed subset of $Z$. since $g$ is contra $S_S$-continuous, then $g^{-1}(F)$ is an $S_S$-open set in $Y$ and since $f$is $S_S$-irresolute, thus$f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is an $S_S$-open subset of X. Hence $g \circ f$ is contra $S_S$-continuous.

**Proposition 3.28.** If a function $f: X \to \prod_{\alpha\in\Delta} Y_\alpha$ is contra $S_S$-continuous then $P_\alpha \circ f: X \to Y_\alpha$ is contra $S_S$-continuous for each $\alpha$ in the index set $\Delta$ where $P_\alpha : \prod_{\alpha\in\Delta} Y_\alpha \to Y_\alpha$ is projection map from $\prod_{\alpha\in\Delta} Y_\alpha$ onto$Y_\alpha$.

**Proof.** let $H_\alpha$ be an closed set in $Y_\alpha$for each $\alpha \in \Delta$ since $P_\alpha$ is continuous function then $P_\alpha^{-1}(H_\alpha)$ is an closed set in $\prod_{\alpha\in\Delta} Y_\alpha$ for each $\alpha$ but $f$ is contra $S_S$-continuous, then we have$( P_\alpha \circ f)^{-1}(H_\alpha) = f^{-1}(P_\alpha^{-1}(H_\alpha))$ is $S_S$-open for each $\alpha \in \Delta$ . therefore by Theorem 3.2, we have $P_\alpha \circ f$ is contra $S_S$-continuous function.

**Proposition 3.29.** If $f_i: X_i \longrightarrow Y_i$ is contra $S_S$ –continuous for *i=1,2*. Let $f: X_1 \times X_2 \longrightarrow Y_1 \times Y_2$ be a function defined as follows $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$then $f$ is contra $S_S$ –continuous.

**Proof.** let $U_1 \times U_2 \subseteq Y_1 \times Y_2$where $U_i$ is an open set in $Y$ for *i=1,2*. Then $f^{-1}(U_i)$is $S_S$ –closed subset of $X_i$, since $f_i$ is contra $S_S$ –continuous for i=1,2. Therefore $f^{-1}(U_1 \times U_2) = f^{-1}(U_1) \times f^{-1}(U_2)$ is $S_S$ –closed subset of $X_1 \times X_2$. Hence $f$ contra $S_S$ –continuous.

**Proposition 3.30.**Let $h: X \to X_1 \times X_2$ be a contra $S_S$ –continuous function defined as follows: $h(x) = (h_1(x), h_2(x))$ then $h_i: X \longrightarrow X_i$ is contra $S_S$ –continuous for *i=1,2*.

**Proof.** let $U_1$ be an open set in $X_1$. Then $U_1 \times X_2$ is an open set in $X_1 \times X_2$ and then $h_1^{-1}(U_1) = h^{-1}(U_1 \times X_2)$ is $S_S$ –closed set in X. hence $h_1: X \longrightarrow X_1$is contra $S_S$ –continuous. Similarly for $h_i$ for *i*=2.

**Proposition 3.31**. Let $f: X \to Y$be any function. If $g: X \to X \times Y$ defined by $g(x) = (x, f(x))$ is a contra $S_S$ –continuous, then $f$ is contra $S_S$ –continuous.

**Proof.** Let $F$ be closed subset of $Y$, then $X \times F$ is closed subset of$X \times Y$. Since $g$ is a contra $S_S$ –continuous, then $g^{-1}(X \times F) = f^{-1}(F)$ is an $S_S$ –open subset of X. Hence $f$ is contra $S_S$ –continuous.

 

## 4. Functions with contra $S_S$ –closed and strongly contra $S_S$ –closed graphs

**Definition 4.1.** The graph $G(f)$of the function $f: X \to Y$ is said to be contra $S_S$ –closed if for each$(x, y) \in (X \times Y) - G(f)$, there exist an $S_S$ –open set $U$ containing $x$ and a closed set $V$ in $Y$ containing $y$ such that$(U \times V) \cap G(f) = \phi$.

**Proposition 4.2**. The graph $G(f)$of the function $f: X \to Y$ is contra $S_S$–closed if for each$(x, y) \in (X \times Y) - G(f)$, there exist an $S_S$ –open set $U$ containing $x$ and a closed set $V$ in $Y$ containing $y$ such that$f(U) \cap V = \phi$.

**Proof.** Follows from the definition.

**Theorem 4.3.** If a function $f: X \to Y$ is contra $S_S$ –continuous and $Y$ is Urysohn then $G(f)$ is contra $S_S$ –closed.

**Proof.** Let$(x, y) \in (X \times Y) - G(f)$. Then$y \neq f(x)$ and since $Y$ is Urysohn, there exist two open sets $A$ and $B$ in $Y$ such that $Cl(A) \cap Cl(B) = \phi$. Since $f$ is contra $S_S$ –continuous, then there exist an $S_S$ –open set $U$ containing $x$ such that $f(U) \subseteq Cl(A)$ implies that $f(U) \cap Cl(B) = \phi$. Therefore by Proposition 4.2,$G(f)$ is contra $S_S$ –closed.

**Theorem 4.4.** If a function $f: X \to Y$ is $S_S$ –continuous and $Y$ is $T_1$-space, then $G(f)$ is contra $S_S$ –closed.

**Proof.** Let $(x, y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$ and since $Y$ is $T_1$-space, there exists an open set $H$ in $Y$ such that $f(x) \in H$, $y \notin H$. Since $f$ is $S_S$ –continuous, then there exists an $S_S$ –open set $U$ containing $x$ such that $f(U) \subseteq H$ which implies that $f(U) \cap (Y - H) = \phi$ where $Y - H$ is a closed set in $Y$ containing $y$. Hence by Proposition 4.2, we obtain that $G(f)$ is contra $S_S$ –closed.

**Definition 4.5.** The graph $G(f)$of the function $f: X \to Y$ is strongly contra $S_S$–continuous if for each$(x, y) \in (X \times Y) - G(f)$, there exist an $S_S$ –open set $U$ containing x and a regular closed set $V$ in $Y$ containing $y$ such that$(U \times V) \cap G(f) = \phi$.

**Proposition 4.6.**The graph $G(f)$of the function $f: X \to Y$ is strongly contra $S_S$ –continuous if for each$(x, y) \in (X \times Y) - G(f)$, there exist an $S_S$ –open set $U$ containing x and a regular closed set $V$ in $Y$ containing y such that$f(U) \cap V = \phi$.

**Proof.** Follows from the definition.

**Theorem 4.7.** If a function $f: X \to Y$ is contra $S_S$ –continuous and $Y$ is Urysohn, then $G(f)$ is strongly $S_S$ –closed in $X \times Y$.

**Proof.** Let$(x, y) \in (X \times Y) - G(f)$. Then$y \neq f(x)$ and since $Y$ is Urysohn, there exist two open sets $A$ and $B$ in $Y$ such that $Cl(A) \cap Cl(B) = \phi$. Since $f$ is contra $S_S$ –continuous, then there exist an $S_S$ –open set $U$ containing $x$ such that $f(U) \subseteq Cl(A)$ implies that $f(U) \cap Cl(int(B)) = f(U) \cap Cl(B) = \phi$ where $Cl(int(B))$is regular closed in Y. Hence by Proposition 4.6, $G(f)$ strongly $S_S$ –closed in $X \times Y$.

### References

**[1]** N. K. Ahmed, On some types of separation axioms, M.Sc. Thesis, *College of Science, Salahaddin Univ.*,1990.

**[2]** K. Dlaska, N. Ergun and M. Ganster, On the topology generated by semi-regular sets, *Indian J. Pure Appl. Math.*, 25(11) (1994), 1163-1170.

**[3]** J. Dontchev, Contra-continuous functions and strongly S-closed spaces, *Internat. J. Math.& Math. Sci.*, 19(2) (1996), 303-310.

**[4]** J. Dontchev and T. Noiri, contra semi- continuous functions, *Math. Pannon*, 10(2) (1999), 159-168.

**[5]** S. Jafari and T. Noiri, contra $\alpha$- continuous functions between topological spaces. *Iranian Int. J. Sci.*, 2 (2) (2001),153-167.

**[6]** D. S. Jankovic, A note on mapping of extremally disconnected spaces, *Acta Math. Hungar.*, 46(1-2) (1985), 83-92.

**[7]** A. B. Khalaf, A. H. Majeed and J. M. Jamil, $S_S$ –open sets and $S_S$ –continuous functions, *Inter. J. Adv. Ath. Sci.* , 2(1)(2014), 71-80.

**[8]** A. B. Khalaf, A. H. Majeed and J. M. Jamil, Almost and weakly $S_S$-continuous functions, *Submitted.*

**[9]** N. Levine, semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly*,70(1) (1963), 36-41.

**[10]** A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb, on pre-continuous and weak pre-continuous mapping, *Proc. Math. Phys. Soc.,* [Egypt], 53(1982), 47-53.

**[11]** O. Njastad, On some classes of nearly open sets, *Pacific J. Math.,* 15(3) (1965), 961-970.

**[12]** T. Noiri, Supercontinuity and some strong forms of continuity, *Indian J. Pure Appl. Math.* (15) (3) (1984), 241-250.

**[13]** J. N. Sharma, Topology (General and Algebraic), *Krishana Parkashan Mandir*, $4^{th}$ Edition (2011).

**[14]** T. Shyla Isac Mary and P. Thangavelu, contra RPS-continuous functions, *Asian Journal of current Engineering and Maths*,1 (2012),219-221.

**[15]** J. Tong, On decomposition of continuity in topological spaces, *Acta Math. Hungar.,* 54(1-2) (1989), 51-55.

**[16]** N. V. Velicko, H-closed topological spaces, *Amer. Math. Transl.*,78(2) (1968), 103-118.f

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