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Stability Analysis of Infectious Diseases with Media Coverage and Poverty

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Abstract

In this paper ,the effect of poverty along with media coverage on the stability dynamics of infectious disease has been checked . The incorporation of the factor, poverty along with media coverage makes our model more closer to the real life situations. Using stability theory, the analysis of the model has been done by finding out all the equilibrium points of the system. Numerical simulation of the model is also performed to check the effect of key parameters on the spread of the disease.

Keywords : Poverty, Equilibrium points, Media coverage, Stability.

1. Introduction

In the recent years, researchers have made several attempts to formulate and develop realistic mathematical models for the infectious diseases. Infectious diseases are the root cause of mortality in most of the developing countries. It is the sole responsibility of department for prevention and disease control of any of the state to put their hundred percent efforts in its spread. Media coverage both electronic as well as print media acts as a valuable tool to spread knowledge and to make the people alert towards the spread of infectious disease once it broke down. The recent study on the dynamics of media coverage shows that when more people are infected ,the susceptible will take many measures to prevent themselves from being infected. It has been observed that media alert and education reduce the chances of further more infection of infectious disease among the individuals. Study [2] showed that media and education play an important role to impart knowledge about all the parameters responsible for the spread of AIDS epidemic among married couples. Research findings [1,3,4,5,6,7,8,9,10,11] also emphasized on the fact that media and education acts as an indigenous tools to combat the risk of infectious diseases menace. None of these authors consider poverty and studied its effect for the spread and control of infectious diseases with media coverage. These results inspired us to incorporate an important factor poverty along with media coverage to study its impact on the stability and transmission dynamics of infectious diseases. It also make our mathematical model more closer to the real life situations. It is a widely accepted fact that poverty lead to illiteracy and deaths due to starvation in most of the underdeveloped and developing countries.

The article is organized as follows. The model is presented in Section 2. In section 3, we evaluate the basic reproduction number and determine the existence and stability of both the disease free and disease endemic equilibrium of the model. In section 4, we consider a special case with no loss of constant immunity rate and check its stability towards epidemic equilibrium when $R_0 > 1$ and attains a disease free equilibrium if $R_0 < 1$. Section 5 comprises of a numerical simulation of the model taken under consideration. Conclusion is given in section 6.

2. The Mathematical Model

The disease model divides the population into classes of individuals that are susceptible, with density S(t), infected with density I(t) and those recovered , with density R(t).

$$\left.\begin{aligned} \dot{S} &= b - dS - \left(\beta_1 + (\alpha - \beta_2)\frac{I}{n+I}\right)SI + \mu R \\ \dot{I} &= \left(\beta_1 + (\alpha - \beta_2)\frac{I}{n+I}\right)SI - (d+\gamma+\rho)I \\ \dot{R} &= \gamma I - (d+\mu)R \end{aligned}\right\} \qquad (2.1)$$

- b is the recruitment rate of the susceptible population .
- γ is the constant recovery rate.
- d is the natural death rate of the population.
- ρ is the constant death rate related to disease .
- μ is the rate at which recovered individuals lose immunity and returned to the susceptible class.
- β_1 is the contact rate before media alert.
- α is the constant poverty rate.
- $\beta(I) = \beta_1 + (\alpha - \beta_2)\frac{I}{n+I}$ is the contact rate under the influence of poverty and media alert .The choice of this function to model the poverty and media alert is because of following reasons closer to real life situations. It is a quiet evident that the contact rate reduces as soon as the media starts doing its work by imparting information through both the modes electronic as well print but it affects minimum to the population living under poverty line due to their least access and sometimes non-availability of the above mentioned resources of information. We use$(\alpha - \beta_2)\frac{I}{n+I}$, a reduced transmission rate after media alert and poverty. It approaches its maximum value $(\alpha - \beta_2)$ as $I \to \infty$ and equals to $\frac{(\alpha-\beta_2)}{2}$ when number of infective reaches to n.

3. The Mathematical Analysis:

Firstly, we consider the disease free equilibrium for the model (2.1) at $E_0 = \left(\frac{b}{d}, 0, 0\right)$. Since the local stability of the disease free equilibrium is governed by the reproduction number of the model as mentioned in [3].The use of notation in [3] results

$$F = \begin{pmatrix} \left(\beta_1 + (\alpha - \beta_2)\frac{I}{n+I}\right)SI \\ 0 \end{pmatrix}, \qquad V = \begin{pmatrix} (d+\gamma+\rho)I \\ (\mu+d)R - \gamma I \end{pmatrix} \qquad (3.1)$$

For infected compartment I

$$F = \begin{pmatrix} \frac{b\beta_1}{d} & 0 \\ 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} d+\gamma+\rho & 0 \\ \gamma & \mu+d \end{pmatrix} \qquad (3.2)$$

and $FV^{-1} = \begin{pmatrix} \frac{b\beta_1}{d(d+\rho+\gamma)} & 0 \\ 0 & 0 \end{pmatrix}$ (3.3)

Define the Reproduction number as follows$R_0 = \frac{b\beta_1}{d(d+\rho+\gamma)}$ (3.4)

To find the positive equilibria , set

$$b - dS - \left(\beta_1 + (\alpha - \beta_2)\frac{I}{n+I}\right)SI + \mu R = 0$$

$$\left(\beta_1 + (\alpha - \beta_2)\frac{I}{n+I}\right)SI - (d+\gamma+\rho)I = 0$$
$$\gamma I - (d+\mu)R = 0$$

The endemic equilibrium (S, I, R) $satisfies$ $S > 0,\ I > 0,\ R > 0$ and

$S = \frac{d+\gamma+\rho}{\beta_1+(\alpha-\beta_2)\frac{I}{n+I}}$, $R = \frac{\gamma I}{\mu+d}$

Simplification of the system (2.1) after the substitution gives the quadratic in I

$k_0 I^2 + k_1 I + k_2 = 0$ (3.5)

Where $k_0 = -\left(\beta_1 + (\alpha - \beta_2)\right)\frac{\mu(d+\rho)+d(d+\gamma+\rho)}{\mu+d}$

$$k_1 = -\frac{d\beta_1 n\gamma}{\mu+d} - \beta_1 n(d+\rho) + b\beta_1\left(1 - \frac{1}{R_0}\right) + b(\alpha - \beta_2)$$
$$k_2 = dn(\rho + d + \gamma)(R_0 - 1)$$

From (3.5) , it is clear that

(i) if $R_0 \le 1, then\ we\ will\ have\ k_2 \le 0, k_0 < 0\ , k_1 < 0$ and there is no positive equilibrium.
(ii) if $R_0 > 1, we\ know\ if\ k_2 > 0, k_0 < 0$, then (2.1) has a positive equilibrium $E^*(S^*, I^*, R^*)$, called the endemic equilibrium.

By setting all the initial conditions non-negative i.e $S(0) \ge 0, I(0) \ge 0\ and\ R(0) \ge 0$, then to check for all the solutions to be non-negative as $t > 0$, denote $N = S + I + R$ such that $\frac{dN}{dt} = b - dN - \rho I \le b - dN$. Assume that $R_1^3 = \left\{(S, I, R) \middle| 0 < S + I + R \le \frac{b}{d}, S, I, R \ge 0\right\}$

Now , for $(S, I, R) \in R_1^3$, we have

$\frac{dS}{dt}\Big|_{S=0,\ R\ge 0} > 0,\ \frac{dI}{dt}\Big|_{I=0} = 0,\ \frac{dR}{dt}\Big|_{R=0,\ I\ge 0} \ \ge 0,\ \frac{dN}{dt}\Big|_{N=\frac{b}{d},\ S\ge 0,\ I\ge 0,\ R\ge 0} \le 0$.

Which shows that R_1^3 is positively invariant for the system (2.1) and thereby implies the conclusion for the stability of equilibrium.

Theorem 3.1. System (2.1) with all positive parameters has a unique disease free equilibrium $E_0(S_0, I_0, R_0)$ and is globally asymptotically stable if $R_0 < 1$.

Proof: The Jacobian matrix of the model (2.1) at E_0 is

$$A = \begin{pmatrix} -d & -\frac{b\beta_1}{d} & \mu \\ 0 & (\gamma+d+\rho)(R_0-1) & 0 \\ 0 & \gamma & -(\mu+d) \end{pmatrix} \quad (3.6)$$

The characteristic equation about E_0 is

$(d+\lambda)[(\gamma+d+\rho)(R_0-1) - \lambda](\mu+d+\lambda) = 0$ (3.7)

We have $(\gamma+d+\rho)(R_0-1) < 0$ as $R_0 < 1$, thus all the Eigen values of (3.7) are negative and thereby proves the local asymptotically stability of the point E_0 of the system (2.1). Now by selecting the Lypunov function $V = I$ and finding out the derivative of V with respect to the solution of the considered system (2.1), we get

$$\dot{V} = \dot{I} = \left(\beta_1 + (\alpha - \beta_2)\frac{I}{n+I}\right)SI - (d+\gamma+\rho)I$$

$$\leq \beta_1 SI - (d+\gamma+\rho)I \leq (d+\gamma+\rho)(R_0 - 1)I < 0.$$

Thus, E_0 is globally asymptotically stable.

Theorem 3.2. Suppose $R_0 > 1$, then there is a unique endemic equilibrium $E^*(S^*, I^*, R^*)$ of (2.1) which is locally asymptotically stable.

Proof : The Jacobian matrix of system (2.1) at the equilibrium point $E^*(S^*, I^*, R^*)$ is

$$A\,(S^*, I^*, R^*) = \begin{pmatrix} -d - \left(\beta_1 + (\alpha-\beta_2)\frac{I^*}{n+I^*}\right)I^* & -(d+\gamma+\rho) + \frac{(\beta_2-\alpha)(d+\gamma+\rho)nI^*}{[\beta_1 n + (\beta_1+\alpha-\beta_2)I^*](n+I^*)} & \mu \\ \left(\beta_1 + (\alpha-\beta_2)\frac{I^*}{n+I^*}\right)I^* & -\frac{(\beta_2-\alpha)(d+\gamma+\rho)nI^*}{[\beta_1 n + (\beta_1+\alpha-\beta_2)I^*](n+I^*)} & 0 \\ 0 & \gamma & -(\mu+d) \end{pmatrix} \quad (3.8)$$

Suppose
$$A_1 = \left(\beta_1 + (\alpha-\beta_2)\frac{I^*}{n+I^*}\right)I^*$$

$$A_2 = -\frac{(\beta_2-\alpha)(d+\gamma+\rho)nI^*}{[\beta_1 n + (\beta_1+\alpha-\beta_2)I^*](n+I^*)}$$

$$A_3 = -(d+\gamma+\rho) + \frac{(\beta_2-\alpha)(d+\gamma+\rho)nI^*}{[\beta_1 n + (\beta_1+(\alpha-\beta_2)I^*](n+I^*)}$$

Which shows that

$A_1 > 0, A_2 < 0, A_3 < 0$, *now the transformed characteristic equation about* E^* is

$$\lambda^3 + k_1\lambda^2 + k_2\lambda + k_3 = 0\,, \quad (3.9)$$

Where $k_1 = -(A_2 - 2d - A_1 - \mu) > 0$,

$$k_2 = -[(\mu+d)(A_2 - d - A_1) + (d + A_1)A_2 + A_1A_3] > 0\,,$$

$$k_3 = A_1[\mu(d+\rho) + d(d+\gamma+\rho) - dA_2(\mu+d)] > 0\,.$$

Which is stable by Routh –Hurwitz criterion, because it has only positive coefficients and also $k_1k_2 - k_3 > 0$ which proves the local asymptotically stability of E^* as it follows from the Routh-Hurwitz criterion that all the eigen values of (3.9) have the negative parts if and only if $R_0 > 1$.

4. Special Case of (2.1) if $\mu = 0$

Suppose $\mu = 0$ in (2.1) then the transformed SIR model is

$$\left.\begin{aligned} \dot{S} &= b - dS - \left(\beta_1 + (\alpha-\beta_2)\frac{I}{n+I}\right)SI \\ \dot{I} &= \left(\beta_1 + (\alpha-\beta_2)\frac{I}{n+I}\right)SI - (d+\gamma+\rho)I \\ \dot{R} &= \gamma I - dR \end{aligned}\right\} \quad (4.1)$$

From the above model it is clear that the first two equations are the independent of the third so the reduced model taken under consideration for further calculation takes the form

$$\left.\begin{aligned} \dot{S} &= b - dS - \left(\beta_1 + (\alpha-\beta_2)\frac{I}{n+I}\right)SI \\ \dot{I} &= \left(\beta_1 + (\alpha-\beta_2)\frac{I}{n+I}\right)SI - (d+\gamma+\rho)I \end{aligned}\right\} \quad (4.2)$$

By assuming the left hand side of (4.2) be zero we got a disease free equilibrium point $E_0\left(\frac{b}{d}, 0\right)$ whose local stability can be determined by calculating the Eigen values. Also we have $S = \frac{d+\gamma+\rho}{\beta_1 + (\alpha-\beta_2)\frac{I}{n+I}}$ and $AI^2 + BI + C = 0$, *where*

$$A = (\beta_1 + \alpha - \beta_2)(d + \gamma + \rho),$$
$$B = (\beta_1 n + d)(d + \gamma + \rho) - b(\beta_1 + \alpha - \beta_2)$$
$$C = dn(d + \gamma + \rho) - b\beta_1 n$$

After substitution of $b = \frac{d(d+\gamma+\rho)}{\beta_1}$ in B, we have

$$B = (d + \gamma + \rho)[\beta_1 n + d(1 - R_0)] + \frac{d(d + \gamma + \rho)(\alpha - \beta_2)}{\beta_1} R_0$$

Which exhibits an epidemic equilibrium as $C < 0, A > 0 \; if \; R_0 > 1$ and a disease free equilibrium if $R_0 < 1 \; as \; C > 0, A > 0, B > 0$.

Theorem 4.1. System (4.2) with all positive parameters has a unique disease free equilibrium $E_0\left(\frac{b}{d}, 0\right)$ and is globally asymptotically stable if $R_0 < 1$. If $R_0 > 1$ then it has a unique endemic equilibrium $E^*(S^*, I^*)$ having global asymptotical stability.

Proof: The Jacobian matrix of the model (4.2) at E_0 is

$$A = \begin{pmatrix} -d & -\frac{b\beta_1}{d} \\ 0 & \frac{b\beta_1}{d} - (d + \gamma + \rho) \end{pmatrix} \tag{4.3}$$

As $R_0 < 1$ implies that $\frac{b\beta_1}{d} - (d + \gamma + \rho) = (d + \gamma + \rho)(R_0 - 1) < 0$ thereby establish the stability of the disease free equilibrium and unstable otherwise.

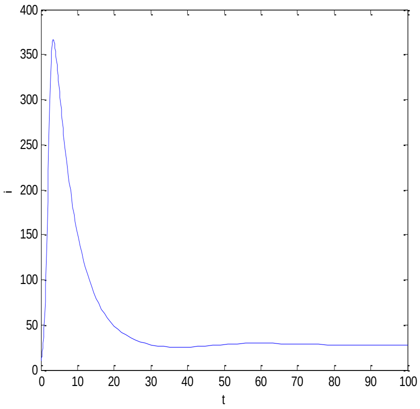
Now the evaluation of the determinant of the Jacobian matrix for the equilibrium point $E^*(S^*, I^*)$ gives

$$(d + \gamma + \rho)\left(\beta_1 + (\alpha - \beta_2)\frac{I^*}{n + I^*}\right) + \frac{d\beta_2 S^* I^*}{(n + I^*)^2} > 0$$

Which shows that that $E^*(S^*, I^*)$ is locally asymptotically stable if $R_0 > 1$. We will now show that disease free and endemic equilibrium points are globally asymptotically stable with respect to the conditions $R_0 < 1 \; and \; R_0 > 1$. The plane $\omega = \left\{(S, I) \in R^2 | 0 \leq S + I \leq \frac{b}{d}, S, I \geq 0\right\}$ is positively invariant for system (4.2). Now, set $B = \frac{1}{SI}$,then $\frac{\partial(BF)}{\partial S} + \frac{\partial(BG)}{\partial I} = -\frac{b}{IS^2} - \frac{(\alpha - \beta_2)}{n + I^2} < 0$ .Thus Bendixon-Dulac Criterion shows that the region ω does not have non trivial periodic orbit thereby established the global asymptotically stability for $E^* \; if \; R_0 > 1$, and $E_0 \; if \; R_O < 1$.

5. Numerical Analysis and Discussion

In this section, we present a numerical simulation of the model to investigate the effect of various parameters involved in the model. Working on the same pattern as in [1], let $\beta_1 = 0.002, b = 5, d = 0.02$, $\mu = 0.01, \gamma = 0.05, \rho = 0.1$ in Fig. 1, 2, 3 and 4. For $\alpha = 0$, it is clear from the numerical analysis given in [1] that the effective media coverage delays the arrival of the infection peak, and decrease the number of infected individuals. The incorporation of the poverty factor makes our model more realistic and also shows the same trend as in [1] which is clear from the following figures with different values of $\beta_2 \; and \; \alpha$. Fig 1 and 2 shows the case when there is no media alert i.e $\beta_2 = 0$, $\alpha = 0.0024$, $n = 6 \; and \; n = 30$, the rate of infection reaches its peak in presence of poverty. Fig 3 and 4 where $\beta_2 = 0.0018$, $\alpha = 0.0008$, $n = 6 \; and \; n = 30$ shows decay in number of infected persons but not as much as in [1] because of the presence of poverty which advocates the fact that poverty is the main hurdle in eradication of infectious diseases completely from a population. Comparison of our results with [1] shows that it due to poverty that media alert alone fails to control the full spread of the disease.



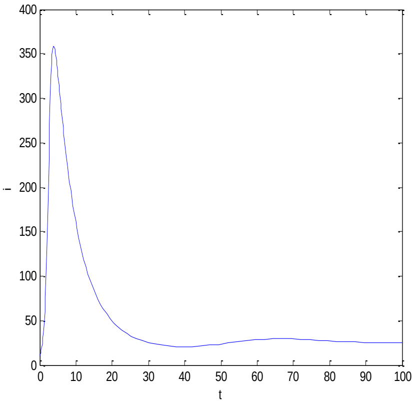
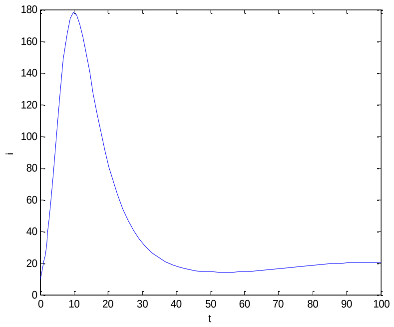


Fig1 $\beta_2 = 0\,, \alpha = 0.0024,\ n = 6$ Fig2 $\beta_2 = 0\,, \alpha = 0.0024,\ n = 30$



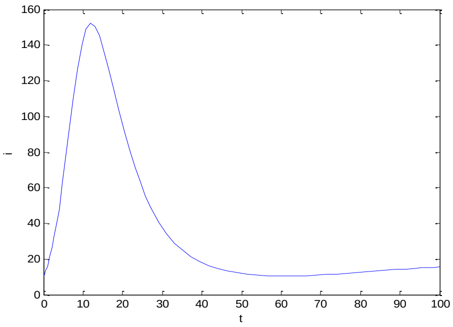


Fig3 $\beta_2 = 0.0018\,, \alpha = 0.0008,\ n = 30$ Fig4 $\beta_2 = 0.0018\,, \alpha = 0.0008,\ n = 6$

6.Conclusion

In this paper some major factors that surround infectious disease(s) modeling are discussed. The article presents specific aspects that mathematical modelers should consider while designing mathematical models on infectious disease(s). Three major factors highlighted in the mathematical model are transmission, media coverage and poverty. Our results from this particular cohort show that in real life every one take preventive measures to protect themselves from infection as soon as the infected individuals are reported by the media coverage ,which will reduce the transmission rate more or less but it affects little to the population living under poverty line due to their least access to the media tools. It is found that unique disease free equilibrium follows global asymptotically stability if the conditions in theorem 3.1 is satisfied. The endemic equilibrium is always locally asymptotically stable iff $R_0 > 1$ and satisfies Routh-Hurwitz criterion as shown in theorem 3.2. Global asymptotically stability for the endemic equilibrium is also given by taking special case for $\boldsymbol{\mu} = \mathbf{0}$**.** Inclusion of the factor poverty makes our model more realistic.

Remark: If $\alpha = 0$ then our model reduces to [1] .

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