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Transient behavior of $M^{[X]}/G/1$ Retrial Queueing Model with Non Persistent Customers, Random break down, Delaying Repair and Bernoulli Vacation

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Abstract
In this paper we consider a single server batch arrival non-Markovian retrial queueing model with non persistent customers. In accordance with Poisson process, customers arrive in batches with arrival rate λ and are served one by one with first come first served basis. The server is being considered as unreliable that it may encounter break down at any time. In order to resume its service the server has to be sent for repair, but the repair does not start immediately so that there is a waiting time before the repair process. The customer, who finds the server busy upon arrival, can either join the orbit with probability p or he/she can leave the system with probability 1-p. Upon completion of a service the server may go for a vacation with probability θ or stay back in the system to serve a next customer with probability $1-\theta$, if any. We obtain the transient solution and steady solution of the model by using supplementary variable technique. Also we derive the system performance measures and reliability indices.

Key words: Batch size, break down, delay time, transient solution, steady solution, reliability indices.

1.Introduction

There is an extensive literature on retrial queues because of its wide applicability in telephone switching systems, telecommunication networks and computer networks. For excellent survey on retrial queues, the reader can refer Yang and Templeton (1987), Fallin (1990) and Kulkarni (1997). Artalejo (1999) and Gomez (2006) presented a bibliographical study on retrial queues. Also Artalejo and Falin (2002) have done a comparative analysis between standard and retrial queues. Farahmand(1990) analyzed single line queue with repeated attempts.

Retrial queues with vacation have also received remarkable attention during recent years. Artalejo(1997, 1999) discussed retrial queues with exhaustive vacation. Krishna kumar et al.(2002) studied multi server with general retrial times and Bernoulli schedule. Choi et al.(1990, 1993) studied M/G/1 retrial queue with vacation. Atencia (2005) also studied single server with general retrial retrial time and Bernoulli vacation. Zhou (2005) studied the same model with FCFS orbit policy. Choudhury(2007)discussed batch arrival retrial queue with single server having two stages of service and Bernoulli vacation. Retrial queues with unreliable server and repair have also been paid attention by numerous authors. Aissani(1988, 1993,1994)and Kulkarni(1990),Djellab(2002) studied retrial queueing system with repeated attempts for an unreliable server. Artalejo (1994) found new results for retrial queueing systems with break downs. Wang et al.(2008) incorporated reliability analysis on retrial queue with server breakdowns and repairs. Peishu Chen et al.(2010) discussed a retrial queue with modified vacation policy and server break downs. Choudhury (2008)studied M/G/1 model with two phase of service and break down. The same author(2012) extended his analysis by including delaying time before the repair of the server for batch arrivals. Ke (2009) studied the M/G/1 retrial queue with balking and feedback. Also the same author analyzed $M^{[x]}/(G1,G2)/1$ retrial queue under Bernoulli vacation schedules and starting failures. Jinting Wang et al.(2008) considered the transient analysis of M/G/1 retrial queue subject to disasters and server failures. The same author (2008) obtained steady state solution of the queue model with two-Phase Service. Many authors concentrated retrial queue models with all aspects for non- persistent customers. Krishnamoorthy et al (2005) studied retrial queue with non-persistent customer and orbital search. Kasthuri et al(2010) studied two phases of service of retial queue with non-persistent customers.

In this paper we consider a single server queueing system in which primary customers arrive according to compound Poisson stream with rate λ. Upon arrival, customer finds the sever busy or down or on vacation the customer may leave the service area as there is no place in front of the server, he/she may join the pool of customers called orbit with probability p or leave the system with probability 1-p. otherwise the server can get service immediately if the server is idle.There is a waiting time before the the server is getting to be repaired since the server is assumed to be unreliable. Also the server can opt for Bernoulli vacation. The rest of the paper is organized as follows: In Section 2, we give a brief description of the mathematical model. Section 3 deals with transient analysis of the model for which probability generating function of the distribution has been obtained.In section 4 steady state solution has been obtained for the model. Some important performance measures and reliability indices of this model are derived in Section 5. In section 6, numerical results related to the effect of various parameters on the system performance measures are analysed and conclusion for the model has been given in section7.

2.Mathematical Description of the model

We consider an $M^{[x]}/G/1$ retrial queue with random break downs and Bernoulli vacation. Customers arrive at the system in batches of variable size in a compound Poisson process. Let $\lambda c_i \Delta t$ (i=1,2,3,....) be the first order probability that a batch of i customers arrives at the system during a short interval of time $(t, t+\Delta t)$, where $0 \le c_i \le 1$ and $\sum_{i=1}^{\infty} c_i = 1$ and $\lambda > 0$ is the mean arrival rate of batches and the customers are served one-by-one on a "first come-first served" basis. Upon arrival, if a customer finds the server idle, the customer gets service immediately. Otherwise, the server is found busy or down or on vacation, the customer is obliged to join a retrial orbit according to an FCFS discipline with probability p or leaves the system with probability 1- p.

The service times of the customers are identically independent random variables with probability distribution function B(x), density function b(x), k^{th} moment $b_k (k=1,2)$. When the server is serving the customers, it may encounter break down at random time so that the server will be down for a short span of time. The server's life times are generated by exogenous Poisson process with rate α. As soon as the server gets break down it is sent for repair so that the server stops providing service to the customers and waits for repair to start, which may refer to as waiting period of the server we define this waiting time as delay time. The delay times $Q_n; n \ge 0$ of the server are identically independent random variables with distribution function Q(y)and k^{th} finite moment $q_k; k \ge 1$. The repair times $R_n; n \ge 0$ of the server are identically independent random variables with distribution function R(y)and k^{th} finite moment $r_k; k \ge 1$. After the repair process is over the server is ready to resume its remaining service to the customers and in this case the service times are cumulative, which we may referred to as generalized service times. After each service completion the server may go for a vacation of random length V with probability θ or with probability $1-\theta$ he may serve the next unit; if any. The vacation times of the server are assumed to be identically independent random variables. All stochastic processes involved in the system are assumed to be independent of each other.

Now we obtain the probability generating function of the joint distribution of the state of the server and the number in the system by treating $I^0(t), B^0(t)$ are the elapsed retial time and service time of the customers at time t respectively also $D^0(t), R^0(t)$ and $V^0(t)$ are the elapsed delay time,elapsed repair time and elapsed vacation time of the server at time t,repectively as supplementary variables. Assuming that the system is empty initially. Let N(t) be the number of customers in the retrial queue at time t, and C(t) be the number of customer in service at time t. To make it a Markov process, Define the state probabilities at time t as follows: Y(t)= 0, if the server is idle at time t,

1, if the server is idle during retrial time at time t,
2, if the server is busy at time t,
3, if the server is on vacation at time t,
4, if the server is waiting for repair at time t,
5, if the server is under repair at time t.

Introducing the supplementary $I^0(t), B^0(t), D^0(t), R^0(t)$ and $V^0(t)$ to obtain a bivariate Markov process $Z(t) = N(t), X(t)$,

where $X(t)$ = 0 if Y(t)=0,

$X(t) = I^0(t)$ if Y(t)=1,

$X(t) = B^0(t)$ if Y(t)=2,

$X(t) = V^0(t)$ if Y(t)=3

$X(t) = D^0(t)$ if Y(t)=4,

$X(t) = R^0(t)$ if Y(t)=5.

Now We define following limiting probabilities:

$$I_0(t) = PN(t) = 0, X(t) = 0;$$
$$I_n(x,t)dx = PN(t) = n, X(t) = I^0(t); x < I^0(t) \le x + dx; x, t > 0, n \ge 1;$$
$$p_n(x,t)dx = PN(t) = n, X(t) = P^0(t); x < P_1^0(t) \le x + dx; x, t > 0, n \ge 0,$$
$$V_n(x)dx = PN(t) = n, X(t) = V^0(t); x < V^0(t) \le x + dx; x, t > 0, n \ge 0,$$

and for fixed values of x and $n \ge 1$

$$Q_n(x,y,t)dy = PN(t) = n, X(t) = Q^0(t); y < Q^0(t) \le y + dy; x, y, t > 0.$$
$$R_n(x,y,t)dy = PN(t) = n, X(t) = R^0(t); y < R^0(t) \le y + dy; x, y, t > 0.$$

Further it is assumed that I(x),B(x)and V(x) are continuous at x=0 and Q(y), R(y)are continuous at y=0 respectively, so that

$$\eta(x)dx = \frac{dI(x)}{1-I(x)}; \mu(x)dx = \frac{dB(x)}{1-B(x)}; \quad \nu(x)dx = \frac{dV(x)}{1-V(x)}; \beta(x)dy = \frac{dQ(y)}{1-Q(y)};$$
$$\gamma(x)dx = \frac{dR(y)}{1-R(Y)}$$

are the first order differential (hazard rate) functions of I(), B(),V(), Q()and R()respectively.

3.The Transient State Equations

we derive the following system of equations that govern the dynamics of the system behavior:

$$\frac{d}{dt} I_0(t) = -\lambda I_0(t) + (1-\theta)\int_0^\infty P_0(x,t)\mu(x)dx + \int_0^\infty V_0(x,t)\nu(x)dx \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda + \eta(x)\right) I_n(x,t) = 0; \tag{2}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + p\lambda + \alpha + \mu(x)\right) P_n(x,t) = \lambda p \Sigma_{i=1}^n c_i P_{n-i}(x,t) + \int_0^\infty Q_n(x,y,t)\beta(y)dy; n \ge 0 \tag{3}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + p\lambda + \beta(y)\right) Q_n(x,y,t) = \lambda p \Sigma_{i=1}^n c_i Q_{n-i}(x,y,t); n \ge 0 \tag{4}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + p\lambda + \gamma(y)\right) R_n(x,y,t) = \lambda p \Sigma_{i=1}^n c_i R_{n-i}(x,y,t); n \ge 0 \tag{5}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + p\lambda + \nu(x)\right) V_n(x,t) = \lambda p \Sigma_{i=1}^n c_i V_{n-i}(x,t); n \ge 0 \tag{6}$$

with boundary conditions

$$I_n(0,t) = (1-\theta)\int_0^\infty P_n(x,t)\mu(x)dx + \int_0^\infty V_n(x,t)\nu(x)dx \tag{7}$$

$$P_0(0,t) = \int_0^\infty I_1(x,t)\eta(x)dx + \lambda c_1 I_0(t) \tag{8}$$

$$P_n(0,t) = \int_0^\infty I_{n+1}(x,t)\eta(x)dx + \lambda c_{n+1} I_n(t) + \lambda \int_0^\infty \Sigma_{i=1}^n c_i I_{n-i}(x,t)dx; n \geq 1 \tag{9}$$

$$Q_n(x,0,t) = \alpha P_n(x,t); n \geq 0 \tag{10}$$

$$R_n(x,0,t) = \int_0^\infty Q_n(x,y,t)\beta(y)dy \tag{11}$$

$$V_n(0,t) = \theta \int_0^\infty P_n(x,t)\mu(x)dx \tag{12}$$

Now we define the probability generating function:

$$I_q(x,z,t) = \sum_{n=1}^\infty z^n I_n(x,t); I_q(z,t) = \sum_{n=1}^\infty z^n I_n(t); \tag{13}$$

$$P_q(x,z,t) = \sum_{n=0}^\infty z^n P_n(x,t); P_q(z,t) = \sum_{n=1}^\infty z^n P_n(t); \tag{14}$$

$$V_q(x,z,t) = \sum_{n=0}^\infty z^n V_n(x,t); V_q(z,t) = \sum_{n=0}^\infty z^n V_n(t) \tag{15}$$

$$Q_q(x,y,z,t) = \sum_{n=0}^\infty z^n Q_n(x,y,t); Q_q(x,z,t) = \sum_{n=0}^\infty z^n Q_n(x,t); \tag{16}$$

$$R_q(x,y,z,t) = \sum_{n=0}^\infty z^n R_n(x,y,t); R_q(x,z,t) = \sum_{n=0}^\infty z^n R_n(x,t); \tag{17}$$

$$C(z) = \sum_{n=1}^\infty c_n z^n \tag{18}$$

which are convergent inside the circle given by $|z| \leq 1$ and define the Laplace transform of a function f(t) as

$$\bar{f}(s) = \int_0^\infty f(t)e^{-st}dt \tag{19}$$

Taking Laplace transform for equations (1) - (12)

$$(s+\lambda)\bar{I}_0(s) = 1 + (1-\theta)\int_0^\infty \overline{P_0}(x,s)\mu(x)dx + \int_0^\infty \overline{V_0}(x,s)\nu(x)dx \tag{20}$$

$$\left(\frac{d}{dx} + s + \lambda + \eta(x)\right)\bar{I}_n(x,s) = 0 \tag{21}$$

$$\left(\frac{d}{dx} + s + \lambda p + \alpha + \mu(x)\right)\overline{P_n}(x,s) = \lambda p \Sigma_{i=1}^n c_i \overline{P}_{n-i}(x,s) + \int_0^\infty \overline{Q_n}(x,y,s)\beta(y)dy; n \geq 0 \tag{22}$$

$$\left(\frac{d}{dx} + s + \lambda p + \beta(y)\right)\overline{Q_n}(x,y,s) = \lambda p \Sigma_{i=1}^n c_i \overline{Q}_{n-i}(x,y,s); n \geq 0 \tag{23}$$

$$\left(\frac{d}{dx}+s+\lambda p+\gamma(y)\right)\overline{R}_n(x,y,s)=\lambda p\Sigma_{i=1}^{n}c_i\overline{R}_{n-i}(x,y,s);n\geq 0 \tag{24}$$

$$\left(\frac{d}{dx}+s+\lambda p+\nu(x)\right)\overline{V}_n(x,s)=\lambda p\Sigma_{i=1}^{n}c_i\overline{V}_{n-i}(x,s);n\geq 0 \tag{25}$$

$$\bar{I}_n(0,s)=(1-\theta)\int_0^\infty\overline{P}_n(x,s)\mu(x)dx+\int_0^\infty\overline{V}_n(x,s)\nu(x)dx \tag{26}$$

$$\overline{P}_0(0,s)=\int_0^\infty\bar{I}_1(x,s)\eta(x)dx+\lambda c_1\bar{I}_0(s) \tag{27}$$

$$\overline{P}_n(0,s)=\int_0^\infty\bar{I}_{n+1}(x,s)\eta(x)dx+\lambda c_{n+1}\bar{I}_n(s)+\lambda\int_0^\infty\Sigma_{i=1}^{n}c_i\bar{I}_{n-i}(x,s)dx;n\geq 1 \tag{28}$$

$$\overline{Q}_n(x,0,s)=\alpha\overline{P}_n(x,s);n\geq 0 \tag{29}$$

$$\overline{R}_n(x,0,s)=\int_0^\infty\overline{Q}_n(x,y,s)\beta(y)dy \tag{30}$$

$$\overline{V}_n(0,s)=\theta\int_0^\infty\overline{P}_n(x,s)\mu(x)dx \tag{31}$$

Applying probability generating function for the equations(20)-(31)

$$\left(\frac{d}{dx}+s+\lambda+\eta(x)\right)\bar{I}_q(x,z,s)=0 \tag{32}$$

$$\left(\frac{d}{dx}+s+\lambda p(1-C(z))+\alpha+\mu(x)\right)\overline{P}_q(x,z,s)=\int_0^\infty\overline{Q}_q(x,y,z,s)\beta(y)dy \tag{33}$$

$$\left(\frac{d}{dx}+s+\lambda p(1-C(z))+\beta(y)\right)\overline{Q}_q(x,y,z,s)=0 \tag{34}$$

$$\left(\frac{d}{dx}+s+\lambda p(1-C(z))+\gamma(y)\right)\overline{R}_q(x,y,z,s)=0 \tag{35}$$

$$\left(\frac{d}{dx}+s+\lambda p(1-C(z))+\nu(x)\right)\overline{V}_q(x,z,s)=0 \tag{36}$$

$$\bar{I}_q(0,z,s)=(1-\theta)\int_0^\infty\overline{P}_q(x,z,s)\mu(x)dx+\int_0^\infty\overline{V}_q(x,z,s)\nu(x)dx+1-(s+\lambda)\bar{I}_0(s) \tag{37}$$

$$z\overline{P}_q(0,z,s)=\int_0^\infty\bar{I}_q(x,z,s)\eta(x)dx+\lambda C(z)\bar{I}_0(s)+\lambda C(z)\int_0^\infty\bar{I}_q(x,z,s)dx \tag{38}$$

$$\overline{Q}_q(x,0,z,s)=\alpha\overline{P}_q(x,z,s) \tag{39}$$

$$\overline{R}_q(x,0,z,s) = \int_0^\infty \overline{Q}_q(x,y,z,s)\beta(y)dy \tag{40}$$

$$\overline{V}_q(0,z,s) = \theta\int_0^\infty \overline{P}_q(x,z,s)\mu(x)dx \tag{41}$$

solving for equations(32)-(36)

$$\overline{I}_q(x,z,s) = \overline{I}_q(0,z,s)e^{-(s+\lambda)x-\int_0^x \eta(t)dt} \tag{42}$$

$$\overline{P}_q(x,z,s) = \overline{P}_q(0,z,s)e^{-\phi(z,s)x-\int_0^x \mu(t)dt} \tag{43}$$

where

$$\phi(z,s) = (s+\lambda p(1-C(z))) + \alpha[1-\overline{\phi}(s+\lambda p(1-C(z)))\overline{R}(s+\lambda p(1-C(z)))]$$

$$\overline{Q}_q(x,y,z,s) = \overline{Q}_q(x,0,z,s)e^{-(s+\lambda p(1-C(z))x)-\int_0^y \beta(t)dt} \tag{44}$$

$$\overline{R}_q(x,y,z,s) = \overline{R}_q(x,0,z,s)e^{-(s+\lambda p(1-C(z))x)-\int_0^y \gamma(t)dt} \tag{45}$$

$$\overline{V}_q(x,z,s) = \overline{V}_q(0,z,s)e^{-(s+\lambda p(1-C(z))x)-\int_0^x \nu(t)dt} \tag{46}$$

Integrate equations (42)-(46)by parts with respect to x

$$\overline{I}_q(z,s) = \overline{I}_q(0,z,s)\left[\frac{1-\overline{I}(s+\lambda)}{(s+\lambda)}\right] \tag{47}$$

$$\overline{P}_q(z,s) = \overline{P}_q(0,z,s)\left[\frac{1-\overline{B}(\phi(z,s))}{(\phi(z,s))}\right] \tag{48}$$

$$\overline{Q}_q(x,z,s) = \overline{Q}_q(x,0,z,s)\left[\frac{1-\overline{Q}(s+\lambda p(1-C(z)))}{(s+\lambda p(1-C(z)))}\right] \tag{49}$$

$$\overline{R}_q(x,z,s) = \overline{R}_q(x,0,z,s)\left[\frac{1-\overline{R}(s+\lambda p(1-C(z)))}{(s+\lambda p(1-C(z)))}\right] \tag{50}$$

$$\overline{V}_q(z,s) = \overline{V}_q(0,z,s)\left[\frac{1-\overline{V}(s+\lambda p(1-C(z)))}{(s+\lambda p(1-C(z)))}\right] \tag{51}$$

where $\overline{I}(s+\lambda), \overline{B}(\phi(z,s)), \overline{Q}(s+\lambda p(1-C(z))), \overline{R}(s+\lambda p(1-C(z)))$ and $\overline{V}(s+\lambda p(1-C(z)))$ are the Laplace-Stieltjes transform of the retrial time, service time, delay time, repair time and vacation completion time of the server respectively.

Multiply equation (42) by $\eta(x)$ and integrate w.r.t x

$$\int_0^\infty \bar{I}_q(x,z,s)\eta(x)dx = \bar{I}_q(0,z,s)\bar{I}(s+\lambda) \quad (52)$$

Multiply equation (43) by $\mu(x)$ and integrate w.r.t x

$$\int_0^\infty \bar{P}_q(x,z,s)\mu(x)dx = \bar{P}_q(0,z,s)\bar{B}(\phi(z,s)) \quad (53)$$

Multiply equation (44) by $\beta(x)$ and integrate w.r.t x

$$\int_0^\infty \bar{Q}_q(x,y,z,s)\beta(y)dy = \bar{Q}_q(x,0,z,s)\bar{Q}(s+\lambda p(1-C(z))) \quad (54)$$

Multiply equation (45) by $\gamma(y)$ and integrate w.r.t y

$$\int_0^\infty \bar{R}_q(x,y,z,s)\gamma(y)dy = \bar{R}_q(x,0,z,s)\bar{R}(s+\lambda p(1-C(z))) \quad (55)$$

Multiply equation (46) by $\nu(y)$ and integrate w.r.t y

$$\int_0^\infty \bar{V}_q(x,z,s)\nu(x)dx = \bar{V}_q(0,z,s)\bar{V}(s+\lambda p(1-C(z))) \quad (56)$$

from equation(53), equation (37) becomes

$$\bar{I}_q(0,z,s) = [1-(s+\lambda)\bar{I}_0(s)] + [(1-\theta)+\theta\bar{V}(s+\lambda p(1-C(z)))]\bar{B}(\phi(z,s))\bar{P}_q(0,z,s) \quad (57)$$

$$\bar{I}_q(z,s) = [1-(s+\lambda)\bar{I}_0(s)] + [(1-\theta)+\theta\bar{V}(s+\lambda p(1-C(z)))]\bar{B}(\phi(z,s))\bar{P}_q(0,z,s)\left[\frac{1-\bar{I}(s+\lambda)}{(s+\lambda)}\right] \quad (58)$$

using equations (52),(56) equation(38) becomes

$$\bar{P}_q(0,z,s) = \frac{\lambda C(z)\bar{I}_0(s) + \left[1-(s+\lambda)\bar{I}_0(s)\right]\left[\lambda C(z)\left(\frac{(1-\bar{I}(s+\lambda))}{(s+\lambda)}\right)+\bar{I}(s+\lambda)\right]}{D(z,s)} \quad (59)$$

where

$$D(z,s) = z - [(1-\theta)+\theta\bar{V}(s+\lambda p(1-C(z)))]\bar{B}(\phi(z,s))\left[\lambda C(z)\left(\frac{(1-\bar{I}(s+\lambda))}{(s+\lambda)}\right)+\bar{I}(s+\lambda)\right] \quad (60)$$

substitute the value for $\bar{P}_q(0,z,s)$ we can obtain the probability generating function of various states of the system $I_q(z,s), P_q(z,s), Q_q(x,z,s), R_q(x,z,s), V_q(z,s),$ in the transient state.

4.Steady State Distribution

In this section we shall derive the steady state probability distribution for our queueing model. To define the steady state probabilities, suppress the argument 't' where ever it appears in the time dependent analysis. By using well known Tauberian property as follows:

$$Lt_{s\to 0} s\bar{f}(s) = Lt_{t\to\infty} f(t) \quad (61)$$

$$\bar{I}_q(z) = \frac{I_0[[1-\bar{I}(\lambda)](C(z)-1)[(1-\theta)+\theta\bar{V}(\lambda p(1-C(z)))]\bar{B}(\phi(z))-D(z)]}{D(z)} \quad (62)$$

$$\bar{P}_q(z) = \frac{I_0\bar{I}(\lambda)\lambda(C(z)-1)[1-\bar{B}(\phi(z))]}{D(z)\phi(z)} \quad (63)$$

$$\bar{Q}_q(x,z) = \frac{\alpha I_0\bar{I}(\lambda)[1-\bar{B}(\phi(z))][\bar{Q}(s+\lambda p(1-C(z)))-1]}{D(z)\phi(z)} \quad (64)$$

$$\bar{R}_q(x,z) = \frac{\alpha I_0\bar{I}(\lambda)[1-\bar{B}(\phi(z))]\bar{Q}(s+\lambda p(1-C(z)))[\bar{R}(s+\lambda p(1-C(z)))-1]}{D(z)\phi(z)} \quad (65)$$

$$V_q(z) = \frac{\theta I_0\bar{I}(\lambda)\bar{B}(\phi(z))[\bar{V}(s+\lambda p(1-C(z)))-1]}{D(z)} \quad (66)$$

using the normalization condition I_0 can be obtained

$$I_0 + Lt_{z\to 1}(I_q(z)+Q_q(z)+Q_q(z)+R_q(z)+V_q(z)) = 1 \quad (67)$$

$$I_0 = \frac{[1-C_{[1]}(1-\bar{I}(\lambda))-p\rho]}{\bar{I}(\lambda)(1+\rho(1-p))} \quad (68)$$

$$\rho = \lambda C_{[1]}[\mu_1(1+\alpha(r_1+d_1)+\theta v_1)] \quad (69)$$

In addition, various system state probabilities also be given from equations (62)-(66) by putting z=1.

Prob [the server is idle in non-empty queue]= $I_q(1)$

$$= \frac{C_{[1]}(1-\bar{I}(\lambda))-p\rho}{\bar{I}(\lambda)(1+\rho(1-p))} \quad (70)$$

Prob [the server is busy]= $P_q(1)$

$$= \frac{\lambda C_{[1]}\mu_1}{(1+\rho(1-p))} \quad (71)$$

Prob [the server is under waiting to be repaired]= $D_q(1)$

$$= \frac{\alpha\lambda C_{[1]}\mu_1 d_1}{(1+\rho(1-p))} \quad (72)$$

Prob [the server is on repair]= $R_q(1)$

$$= \frac{\alpha\lambda C_{[1]}\mu_1 d_1}{(1+\rho(1-p))} \quad (73)$$

Prob [the server is on vacation] = $V_q(1)$

$$=\frac{\theta\lambda C_{[1]}v_1}{(1+\rho(1-p))} \tag{74}$$

Blocking probability

$$=\frac{\rho}{(1+\rho(1-p))} \tag{75}$$

The necessary and sufficient condition for stability condition is given by the following

$$C_{[1]}(1-\bar{I}(\lambda))+p\rho<1 \tag{76}$$

The expected number of customers in the orbit

$$E[N_0]=\frac{p\rho(1-\bar{I}(\lambda))C_{[1]}}{1-C_{[1]}(1-\bar{I}(\lambda))-p\rho}+\frac{[\lambda C_{[1]}][\mu_2(1+\alpha(d_1+r\beta_1))^2+\alpha\mu_1(d_2+2d_1r_1+r_2)]}{2[1-C_{[1]}(1-\bar{I}(\lambda))-p\rho]}$$

$$+\frac{\theta[\lambda C_{[1]}]^2[v_2+2v_1\mu_1(1+\alpha(d_1+r_1))]}{2[1-C_{[1]}(1-\bar{I}(\lambda))-p\rho]}\frac{C_{[1]}(1-\bar{I}(\lambda))+p\rho}{[1-C_{[1]}(1-\bar{I}(\lambda))-p\rho]}C_{[R]} \tag{77}$$

where $C_{[R]}=\dfrac{C_{[2]}}{2C_{[1]}}$ is the residual batch size.

After finding the expected number of units in the orbit,we can obtain the related performance measures viz mean number of units in the system, mean waiting time in the queue and mean waiting time in the system by using Little's formula

$$E[N_s]=E[N_0]+\rho \tag{78}$$

$$E[W_s]=\frac{E[N_s]}{\rho\lambda C_{[1]}} \tag{79}$$

$$E[W_0]=\frac{E[N_0]}{\rho\lambda C_{[1]}} \tag{80}$$

5.Reliability Indices

Let $A_v(t)$ be the system availability at time 't' i.e the probability that the server is either working for a customer or in an idle period such that the steady state availability of the server is given by

$$A_v=Lt_{t\to\infty}A_v(t) \tag{81}$$

$$A_v=P_{00}+Lt_{z\to1}P_q(1)=1-\frac{\lambda C_{[1]}[\mu_1(d_1+r_1)+\theta v_1]}{1+\rho(1-p)} \tag{82}$$

The steady state failure frequency of the server

$$F=\alpha P_q(1)=\frac{\alpha\lambda C_{[1]}\mu_1}{1+\rho(1-p)} \tag{83}$$

6. Numerical Analysis

Some numerical results have been presented in order to in order to illustrate the effect of various parameters on the performance measures and reliability analysis of our system. For the effect of parameters α and θ on system performance measures. Table 1 and table 2 show the effect of parameters on system's idle time, traffic intensity, reliability indices and performance measures of our model.

Table 1 : ρ , Q_0 and Reliability indices for various vales of α and θ

α	Θ	ρ	Availability	Q_0	Failure frequency	Blocking probabability
1	0.25	0.3314	0.8686	0.7383	0.1188	0.2181
1	0.5	0.4029	0.7971	0.6724	0.1151	0.2730
1	0.75	0.4743	0.7257	0.6104	0.1117	0.3247
2	0.25	0.3914	0.8086	0.7009	0.2334	0.2493
2	0.5	0.4629	0.7371	0.6372	0.2264	0.3023
2	0.75	0.5343	0.6657	0.5772	0.2197	0.3523
3	0.25	0.4514	0.7486	0.6647	0.3441	0.2794
3	0.5	0.5229	0.6771	0.6032	0.3339	0.3307
3	0.75	0.5943	0.6057	0.5452	0.3242	0.3790
4	0.25	0.5114	0.6886	0.6298	0.4511	0.3085
4	0.5	0.5829	0.6171	0.5703	0.4378	0.3581
4	0.75	0.6543	0.5457	0.5142	0.4254	0.4049

Table 2 : Performance measures for for various vales of α and θ

α	Θ	$I_q(1)$	$P_q(1)$	$Q_q(1)$	$R_q(1)$	$V_q(1)$	L_q	L_s
1	0.25	0.0436	0.1188	0.0356	0.0178	0.0636	02074	0.4521
1	0.5	0.0546	0.1151	0.0345	0.0173	0.1234	0.3091	0.6253
1	0.75	0.0649	01117	0.0335	0.0168	0.1795	0.4248	0.8124
2	0.25	0.0499	0.1167	0.0700	0.0350	0.0625	0.2888	0.5735
2	0.5	0.0605	0.1132	0.0679	0.0340	0.1213	0.4077	0.7639
2	0.75	0.0705	0.1098	0.0659	0.0330	0.1765	0.5437	0.9713
3	0.25	0.0559	0.1147	0.1032	0.0516	0.0615	0.3826	0.7073
3	0.5	0.0661	0.1113	0.1002	0.0501	0.1192	0.5209	0.9171
3	0.75	0.0758	0.1081	0.0973	0.0486	0.1737	0.6799	0.1475
4	0.25	0.0617	0.1128	0.1353	0.0677	0.0604	0.4902	0.8550
4	0.5	0.0716	0.1095	0.1314	0.0657	0.1173	0.6505	1.0867
4	0.75	0.0810	0.1063	0.1276	0.0638	0.1709	0.8357	1.3433

7.Conclusion

In this paper, we have obtained the probability generating function of various states of the system in transient state and also discussed the steady state solution with performance measures of the system and the reliability indices like availability of the server and failure frequency of the server. The prescribed model can be modeled in the design of computer networks. As a future work we can try to incorporate the effect of balking/reneging on this service system.

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