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On the Relative Efficiency of the Proposed Reparametized Randomized Response Model

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Abstract

In this paper, we proposed a new reparametized Randomized Response Model by incorporating a third answer option "Undecided" into the Randomized Response Model developed by Hussain-Shabbir[6]. The relative efficiency as well as the variance of the newly proposed reparametized Randomized Response Model over the existing Randomized Response Model was established when data were obtained through the randomized response model proposed by Hussain and Shabbir [6].

Keywords: Reparametized Randomized Response Model (RRRM), Relative Efficiency, Privacy Protection, Stigmatized Characters, Randomization Device

1. Introduction

The problem of determination of the total population of a stigmatized quantitative variable is famous in sampling theory. Warner [11] was the first to put forward a popular method to determine the proportion of stigmatized characters such as an induced abortions, use of drugs etc., through a randomization device like a deck of cards, spinners etc. such that the respondents' privacy should be protected. At present, Warner's Randomized Response Model (RRM) has been extended by many researchers. Greenberg et al.[5], Mangat and Singh [8], Mangat [9], Singh et al.[10], Christofides [4], Kim and Warde [7], Adebola and Adepetun [1], Adebola and Adepetun [2], Adebola and Adepetun [3] are some of the many to be referenced. In sections to follow, we present the derivation of the existing Hussain and Shabbir's Randomized Response Model, Proposed Reparametized Randomized Response Model and thereafter its relative efficiency over the existing one.

2. Derivation of Hussain and Shabbir's Randomized Response Model

Hussain and Shabbir [6] put forward a Randomized Response Model (RRM) based on the random use of one of the two randomization devices X_1 and X_2. In design, the two randomization devices X_1 and X_2 are the same as that of Warner's device but with different probabilities of choosing the stigmatize question. The basic idea behind this suggestion is to reduce considerably the suspicion among the respondents by providing them choice to randomly select the randomization device itself. Consequently, respondents may reveal their true status. A simple random sample with replacement (SRSWR) sampling is assumed to choose a sample of size n. Let $\propto$ and β be any two positive real numbers carefully selected such that $q = \frac{\alpha}{\propto+\beta}, (\alpha \neq \beta)$ is the probability of using X_1, where X_1 consists of the two statements of Warner's device but with pre-assigned probabilities P_1 and $1 - P_1$ and $1 - q = \frac{\beta}{\propto+\beta}$ is the probability of using X_2 ,where X_2 consists of the two statements of Warner's device also with pre-assigned probabilities P_2 and $1 - P_2$ respectively. For the i^{th} respondent, the probability of a "yes" response is given by

$$P(yes) = \emptyset = \frac{\alpha}{\alpha+\beta}[P_1\pi + (1 - P_1)(1 - \pi)] + \frac{\beta}{\alpha+\beta}[P_2\pi + (1 - P_2)(1 - \pi)] \qquad (2.1)$$

By expanding and simplifying equation (2.1), we have

$$\emptyset = \frac{\alpha[\mathrm{P_1}\pi + 1 - \pi - \mathrm{P_1} + \mathrm{P_1}\pi] + \beta[\mathrm{P_2}\pi + 1 - \pi - \mathrm{P_2} + \mathrm{P_2}\pi]}{\alpha + \beta}$$

$$= \frac{2\alpha \mathrm{P_1}\pi - \alpha\pi + \alpha - \mathrm{P_1}\alpha + 2\beta \mathrm{P_2}\pi - \beta\pi + \beta - \mathrm{P_2}\beta}{\alpha + \beta}$$

$$= \frac{\pi[2\alpha \mathrm{P_1} - \alpha + 2\beta \mathrm{P_2} - \beta] + \alpha + \beta - \mathrm{P_1}\alpha - \mathrm{P_2}\beta}{\alpha + \beta} \qquad (2.2)$$

Substituting $\mathrm{P_1} = 1 - \mathrm{P_2}$ into the equation (2.2) in line with Hussain-Shabbir's, we have

$$\emptyset = \frac{\pi[2\alpha(1 - \mathrm{P_2}) - \alpha - \beta + 2\beta \mathrm{P_2}] + \alpha + \beta - \alpha(1 - \mathrm{P_2}) - \mathrm{P_2}\beta}{\alpha + \beta}$$

$$= \frac{\pi[2\alpha - 2\mathrm{P_2}\alpha - \alpha - \beta + 2\beta \mathrm{P_2}] + \alpha + \beta - \alpha + \mathrm{P_2}\alpha - \mathrm{P_2}\beta}{\alpha + \beta}$$

$$= \frac{\pi[\alpha - \beta - 2\mathrm{P_2}\alpha + 2\beta \mathrm{P_2}] + \beta + \mathrm{P_2}\alpha - \mathrm{P_2}\beta}{\alpha + \beta} = \frac{\pi[1(\alpha - \beta) - 2\mathrm{P_2}(\alpha - \beta)] + \beta + \mathrm{P_2}\alpha - \mathrm{P_2}\beta}{\alpha + \beta}$$

$$= \frac{\pi[(\alpha - \beta)(1 - 2\mathrm{P_2})] + \beta + \mathrm{P_2}\alpha - \mathrm{P_2}\beta}{\alpha + \beta} = \frac{\pi[(\alpha - \beta)(1 - 2\mathrm{P_2})] + \beta + \mathrm{P_2}\alpha - \beta(1 - \mathrm{P_1})}{\alpha + \beta}$$

$$= \frac{\pi[(\alpha - \beta)(1 - 2\mathrm{P_2})] + \mathrm{P_2}\alpha + \mathrm{P_1}\beta}{\alpha + \beta} \qquad (2.3)$$

To provide the equal privacy protection in both the randomization devices $\mathrm{X_1}$ and $\mathrm{X_2}$, we put $\mathrm{P_2} = 1 - \mathrm{P_1}$ into the equation (2.3), obtained:

$$= \frac{\pi[(\alpha - \beta)(1 - 2(1 - \mathrm{P_1})] + \mathrm{P_2}\alpha + \mathrm{P_1}\beta}{\alpha + \beta} = \frac{\pi[(\alpha - \beta)(2\mathrm{P_1} - 1)] + \mathrm{P_1}\beta + \mathrm{P_2}\alpha}{\alpha + \beta}$$

$$\emptyset = \frac{\pi[(\alpha - \beta)(2\mathrm{P_1} - 1)] + \mathrm{P_1}\beta + \mathrm{P_2}\alpha}{\alpha + \beta} \qquad (2.4)$$

Hence,

$$\pi = \frac{\emptyset(\alpha + \beta) - \mathrm{P_1}\beta - \mathrm{P_2}\alpha}{(2\mathrm{P_1} - 1)(\alpha - \beta)}, \mathrm{P_1} \neq {}^{1}/_{2}, \alpha \neq \beta \qquad (2.5)$$

The unbiased moment estimator of true probability of yes response (response rate) π is given by

$$\hat{\pi} = \frac{\hat{\emptyset}(\alpha + \beta) - \mathrm{P_1}\beta - \mathrm{P_2}\alpha}{(2\mathrm{P_1} - 1)(\alpha - \beta)} \qquad (2.6)$$

where $\widehat{\emptyset} = \frac{y}{n}$ and y is the number of respondents reporting a "yes" answer.

When $P_1 = 1 - P_2$, the variance of the estimator is given then by

$$\mathrm{Var}\,(\widehat{\pi}) = V\left[\frac{\widehat{\emptyset}(\alpha+\beta) - P_1\beta - P_2\alpha}{(2P_1-1)(\alpha-\beta)}\right] = \frac{(\alpha+\beta)^2 V(\widehat{\emptyset})}{(2P_1-1)^2(\alpha-\beta)^2} = \frac{(\alpha+\beta)^2\emptyset(1-\emptyset)}{n(2P_1-1)^2(\alpha-\beta)^2} \qquad (2.7)$$

$$\emptyset^2 = \left[\frac{\begin{array}{c}2\alpha P_1\pi - \alpha\pi + \alpha - P_1\alpha\\ +2\beta P_2\pi - \beta\pi + \beta - P_2\beta\end{array}}{\alpha+\beta}\right]\left[\frac{\begin{array}{c}2\alpha P_1\pi - \alpha\pi + \alpha - P_1\alpha\\ +2\beta P_2\pi - \beta\pi + \beta - P_2\beta\end{array}}{\alpha+\beta}\right]$$

$$= \frac{\left[\begin{array}{c}
4\alpha^2P_1^2\pi^2 - 2\alpha^2P_1\pi^2 + 2\alpha^2P_1\pi - 2\alpha^2P_1^2\pi + 4\alpha\beta P_1P_2\pi^2 - 2\alpha\beta P_1\pi^2\\
+2\alpha\beta P_1\pi - 2\alpha\beta P_1P_2\pi - 2\alpha^2P_1\pi^2 + \alpha^2\pi^2 - \alpha^2\pi + \alpha^2P_1\pi - 2\alpha\beta P_2\pi^2\\
+\alpha\beta\pi^2 - \alpha\beta\pi + \alpha\beta P_2\pi + 2\alpha^2P_1\pi - \alpha^2\pi + \alpha^2 - \alpha^2P_1 + 2\alpha\beta P_2\pi - \alpha\beta\pi\\
+\alpha\beta - \alpha\beta P_2 - 2\alpha^2P_1^2\pi + \alpha^2P_1\pi - \alpha^2P_1 + \alpha^2P_1^2 - 2\alpha\beta P_1P_2\pi + \alpha\beta P_1\pi\\
-\alpha\beta P_1 + \alpha\beta P_1P_2 + 4\alpha\beta P_1P_2\pi^2 - 2\alpha\beta P_2\pi^2 + 2\alpha\beta P_2\pi - 2\alpha\beta P_1P_2\pi\\
+4\beta^2P_2^2\pi^2 - 2\beta^2P_2\pi^2 + 2\beta^2P_2\pi - 2\beta^2P_2^2\pi - 2\alpha\beta P_1\pi^2 + \alpha\beta\pi^2 - \alpha\beta\pi\\
+\alpha\beta P_1\pi - 2\beta^2P_2\pi^2 + \beta^2\pi^2 - \beta^2\pi + \beta^2P_2\pi + 2\alpha\beta P_1\pi - \alpha\beta\pi + \alpha\beta\\
-\alpha\beta P_1 + 2\beta^2P_2\pi - \beta^2\pi + \beta^2 - \beta^2P_2 - 2\alpha\beta P_1P_2\pi + \alpha\beta P_2\pi - \alpha\beta P_2\\
+\alpha\beta P_1P_2 - 2\beta^2P_2^2\pi + \beta^2P_2\pi - \beta^2P_2 + \beta^2P_2^2
\end{array}\right]}{(\alpha+\beta)^2} \qquad (2.8)$$

$$\emptyset - \emptyset^2 = \frac{\left[\begin{array}{c}
-4\alpha^2P_1^2\pi^2 + 2\alpha^2P_1\pi^2 + 2\alpha^2P_1^2\pi - 4\alpha\beta P_1P_2\pi^2 + 2\alpha\beta P_1\pi^2\\
+2\alpha\beta P_1P_2\pi + 2\alpha^2P_1\pi^2 - \alpha^2\pi^2 - \alpha^2P_1\pi + 2\alpha\beta P_2\pi^2 - \alpha\beta\pi^2\\
-4\alpha\beta P_2\pi - 2\alpha^2P_1\pi + \alpha^2\pi + 2\alpha^2P_1^2\pi - \alpha^2P_1\pi - \alpha^2P_1^2 + \alpha^2P_1\\
+2\alpha\beta P_1P_2\pi - \alpha\beta P_1\pi - \alpha\beta P_1P_2 - 4\alpha\beta P_1P_2\pi^2 + 2\alpha\beta P_2\pi^2\\
+2\alpha\beta P_1P_2\pi - 4\beta^2P_2^2\pi^2 + 2\beta^2P_2\pi^2 + 2\beta^2P_2^2\pi + 2\alpha\beta P_1\pi^2\\
-\alpha\beta\pi^2 + \alpha\beta\pi - \alpha\beta P_1\pi + 2\beta^2P_2\pi^2 - \beta^2\pi^2 - \beta^2P_2\pi - 2\alpha\beta P_1\pi\\
+\alpha\beta\pi + \alpha\beta P_1 - 2\beta^2P_2\pi + \beta^2\pi + 2\alpha\beta P_1P_2\pi + \alpha\beta P_2 - \alpha\beta P_1P_2\\
+2\beta^2P_2^2\pi - \beta^2P_2\pi + \beta^2P_2 - \beta^2P_2^2
\end{array}\right]}{(\alpha+\beta)^2}$$

Further simplification of the above equation gives

$$\emptyset - \emptyset^2 = \frac{\left[\begin{array}{c}
-4\alpha^2P_1^2\pi^2 + 4\alpha^2P_1\pi^2 + 4\alpha^2P_1^2\pi - 8\alpha\beta P_1P_2\pi^2 + 4\alpha\beta P_1\pi^2\\
+8\alpha\beta P_1P_2\pi - \alpha^2\pi^2 - 4\alpha^2P_1\pi + 4\alpha\beta P_2\pi^2 - 2\alpha\beta\pi^2\\
-4\alpha\beta P_2\pi + \alpha^2\pi + \alpha^2P_1 - \alpha^2P_1^2 - 4\alpha\beta P_1\pi - 2\alpha\beta P_1P_2\\
-4\beta^2P_2^2\pi^2 + 4\beta^2P_2\pi^2 + 4\beta^2P_2^2\pi + 2\alpha\beta\pi - \beta^2\pi^2\\
-4\beta^2P_2\pi + \alpha\beta P_1 + \beta^2\pi + \alpha\beta P_2 + \beta^2P_2 - \beta^2P_2^2
\end{array}\right]}{(\alpha+\beta)^2} \qquad (2.9)$$

Hence, we have

$$V(\hat{\pi})_{conv} = \frac{(\alpha+\beta)^2\begin{bmatrix}\pi(4\alpha^2P_1^2 + 8\alpha\beta P_1P_2 - 4\alpha^2P_1 - 4\alpha\beta P_2 + \alpha^2 \\ -4\alpha\beta P_1 + 4\beta^2P_2^2 + 2\alpha\beta - 4\beta^2P_2 + \beta^2) \\ -\pi^2(4\alpha^2P_1^2 + 8\alpha\beta P_1P_2 - 4\alpha^2P_1 - 4\alpha\beta P_2 + \alpha^2 \\ -4\alpha\beta P_1 + 4\beta^2P_2^2 + 2\alpha\beta - 4\beta^2P_2 + \beta^2)\end{bmatrix} + [\alpha^2P_1 - \alpha^2P_1^2 - 2\alpha\beta P_1P_2 + \alpha\beta P_1 + \alpha\beta P_2 + \beta^2P_2 - \beta^2P_2^2]}{n(2P_1-1)^2(\alpha-\beta)^2(\alpha+\beta)^2}$$

$$= \frac{\pi(1-\pi)}{n} + \frac{\begin{bmatrix}\alpha^2P_1 - \alpha^2P_1^2 - 2\alpha\beta P_1P_2 + \alpha\beta P_1 + \alpha\beta P_2 + \beta^2P_2 \\ -\beta^2P_2^2\end{bmatrix}}{n(2P_1-1)^2(\alpha-\beta)^2(\alpha+\beta)^2}$$

Factoring and setting $P_2 = 1 - P_1$ in the numerator of the second term of the above equation, we have

$$(P_2\alpha + P_1\beta)(P_1\alpha + P_2\beta) = \alpha^2P_1 - \alpha^2P_1^2 - 2\alpha\beta P_1P_2 + \alpha\beta P_1 + \alpha\beta P_2 + \beta^2P_2 - \beta^2P_2^2 \quad (*)$$

Hence, we have

$$V(\hat{\pi})_{conv} = \frac{\pi(1-\pi)}{n} + \frac{(P_2\alpha + P_1\beta)(P_1\alpha + P_2\beta)}{n(2P_1-1)^2(\alpha-\beta)^2(\alpha+\beta)^2} \quad (2.10)$$

3. *The Proposed Reparametized Randomized Response Model*

Despite the success achieved by many authors in developing an efficient Randomized Response Models (RRMs), the developed Models only considered a dichotomous option of "yes" and "no" response. In view of this, we propose a new Reparametized Randomized Response Model (RRRM) that will be based on the random use of one of the three randomization devices, X_1, X_2 and X_3. In design, the three randomization devices X_1, X_2 and X_3 are identical to that of Warner's device but with different probabilities of selection. In addition to α and β proposed earlier by Hussain and Shabbir, we introduce δ, a positive real number such that $q = \frac{\alpha}{\alpha+\beta+\delta}$, $\alpha \neq \beta \neq \delta$ is the probability of using X_1, where X_1 consists of the two statements of Warner's device and the new introduce device also with preset probabilities P_1, P_2 and P_3 respectively. By using Hussain and Shabbir's probability of a "yes" response for the i[th] respondent, the probability of a "yes" response when the third option "undecided" is included is given by

$$Q(yes) = \varphi = \frac{\alpha}{\alpha+\beta+\delta}[P_1\pi + (1-P_1)(1-\pi)] + \frac{\beta}{\alpha+\beta+\delta}[P_2\pi + (1-P_2)(1-\pi)] + \frac{\delta}{\alpha+\beta+\delta}[P_3\pi + (1-P_3)(1-\pi)] \quad (3.1)$$

In order to ensure equal privacy protection in the three randomization devices X_1, X_2, and X_3, we put $P_1 = 1 - P_2 - P_3$ into equation (3.1), to get

$$\pi = \frac{\varphi(\alpha+\beta+\delta) - [(\alpha+\beta+\delta) - P_1\alpha - P_2\beta - P_3\delta]}{2P_1\alpha + 2P_2\beta + 2P_3\delta - \alpha - \beta - \delta} \quad (3.2)$$

Hence, the unbiased sample estimate of π is given as

$$\hat{\pi} = \frac{\hat{\varphi}(\alpha+\beta+\delta) - [(\alpha+\beta+\delta) - P_1\alpha - P_2\beta - P_3\delta]}{2P_1\alpha + 2P_2\beta + 2P_3\delta - \alpha - \beta - \delta} \quad (3.3)$$

Where $\hat{\varphi} = \frac{x}{n}$ and x is the number of respondents reporting a "yes" answer when $P_1 = 1 - P_2 - P_3$. The variance of the estimator is given then by

$$V(\hat{\pi}) = \frac{(\alpha+\beta+\delta)^2 \begin{bmatrix} \pi(4\alpha^2P_1^2 - 4\alpha^2P_1 - 4\alpha\beta P_1 + 8\alpha\beta P_1P_2 - 4\alpha\delta P_1 \\ +8\alpha\delta P_1P_3 + \alpha^2 + 2\alpha\beta - 4\alpha\beta P_2 + 2\alpha\delta \\ -4\alpha\delta P_3 - 4\beta^2P_2 + 4\beta^2P_2^2 - 4\beta\delta P_2 + 8\beta\delta P_2P_3 \\ +\beta^2 + 2\beta\delta - 4\beta\delta P_3 - 4\delta^2P_3 + 4\delta^2P_3^2 + \delta^2) \\ -\pi^2(4\alpha^2P_1^2 - 4\alpha^2P_1 - 4\alpha\beta P_1 + 8\alpha\beta P_1P_2 \\ -4\alpha\delta P_1 + 8\alpha\delta P_1P_3 + \alpha^2 + 2\alpha\beta - 4\alpha\beta P_2 \\ +2\alpha\delta - 4\alpha\delta P_3 - 4\beta^2P_2 + 4\beta^2P_2^2 - 4\beta\delta P_2 \\ +8\beta\delta P_2P_3 + \beta^2 + 2\beta\delta - 4\beta\delta P_3 - 4\delta^2P_3 \\ +4\delta^2P_3^2 + \delta^2) \end{bmatrix} \begin{matrix} +[\alpha^2P_1 + \alpha\beta P_2 + \alpha\delta P_3 - \alpha^2P_1^2 + \alpha\beta P_1 - 2\alpha\beta P_1P_2 \\ +\alpha\delta P_1 - 2\alpha\delta P_1P_3 + \beta^2P_2 + \beta\delta P_3 - \beta^2P_2^2 + \\ \beta\delta P_2 - 2\beta\delta P_2P_3 + \delta^2P_3 - \delta^2P_3^2] \end{matrix}}{n[2P_1(\alpha-\delta) + 2P_2(\beta-\delta) - (\alpha+\beta-\delta)]^2(\alpha+\beta+\delta)^2} \quad (3.4)$$

On simplification of equation (3.4), we have

$$V(\hat{\pi})_{prop} = \frac{\pi(1-\pi)}{n} + \frac{(P_1\alpha + P_2\beta + P_3\delta)(P_3\alpha + P_2\beta + P_1\delta)}{n[2P_1(\alpha-\delta) + 2P_2(\beta-\delta) - (\alpha+\beta-\delta)]^2(\alpha+\beta+\delta)^2} \quad (3.5)$$

4. *Relative Efficiency Comparison*

Here, we show that the new Reparametized Randomized Response Model(RRRM) is more efficient than the existing one via both relative efficiency and variance approach by adopting the data used by Hussain and Shabbir [6]. In what follows, the proposed Reparametized Randomized Response Model (RRRM) is more efficient than Hussain and Shabbir [6] dichotomous Randomized Response Model (RRM) if we have

$$\text{Relative Efficiency(RE)} = \frac{\text{Variance of the proposed (RRRM)}}{\text{Variance of the existing (RRM)}} < 1 \quad (4.1)$$

Or if

$$RE = \frac{\frac{\pi(1-\pi)}{n} + \frac{(P_1\alpha + P_2\beta + P_3\delta)(P_3\alpha + P_2\beta + P_1\delta)}{n[2P_1(\alpha-\delta) + 2P_2(\beta-\delta) - (\alpha+\beta-\delta)]^2(\alpha+\beta+\delta)^2}}{\frac{\pi(1-\pi)}{n} + \frac{(P_2\alpha + P_1\beta)(P_1\alpha + P_2\beta)}{n(2P_1-1)^2(\alpha-\beta)^2(\alpha+\beta)^2}} < 1 \quad (4.2)$$

The condition given in (4.2) is true, for P_1, P_2, P_3, and π ranging from 0.1 to 0.9 if α and β, α and δ, β and δ differ from each other by at least 9 where α, β, and δ are any suitable real numbers.

Table 4.1:Comparison between existing RRM and proposed RRRM when $P_1 = 0.6$, $P_2 = 0.3, P_3 = 0.1$, $\pi = 0.5, \alpha = 20, \beta = 11, \delta = 2$, for varying sample sizes (n)

n	P_1	P_2	P_3	π	α	β	δ	Conventional Variance (RRM) (eqn. 2.10)	Proposed Variance (RRRM) (eqn.3.5)	Relative Efficiency (RE) (eqn.4.1)
50	0.6	0.3	0.1	0.5	20	11	2	0.00624	0.00546	0.87500
100	0.6	0.3	0.1	0.5	20	11	2	0.00312	0.00273	0.87500
150	0.6	0.3	0.1	0.5	20	11	2	0.00208	0.00182	0.87500
200	0.6	0.3	0.1	0.5	20	11	2	0.00156	0.00136	0.87179
500	0.6	0.3	0.1	0.5	20	11	2	0.000624	0.000546	0.87500

Table 4.2 :Comparison between existing RRM and proposed RRRM when $P_1 = 0.4$, $P_2 = 0.4, P_3 = 0.2$, $\pi = 0.4$, $\alpha = 20, \beta = 11, \delta = 2$, for varying sample sizes (n)

n	P_1	P_2	P_3	π	α	β	δ	Conventional Variance (RRM) (eqn.2.10)	Proposed Variance (RRRM) (eqn.3.5)	Relative Efficiency (RE) (eqn.4.1)
50	0.4	0.4	0.2	0.4	20	11	2	0.00579	0.00484	0.83592
100	0.4	0.4	0.2	0.4	20	11	2	0.00289	0.00242	0.83737
150	0.4	0.4	0.2	0.4	20	11	2	0.00193	0.00161	0.83420
200	0.4	0.4	0.2	0.4	20	11	2	0.00145	0.00121	0.83448
500	0.4	0.4	0.2	0.4	20	11	2	0.000579	0.000484	0.83592

Table 4.3: Comparison between existing RRM and proposed RRRM when $P_1 = 0.2$, $P_2 = 0.5, P_3 = 0.3,\ \pi = 0.7, \alpha = 20, \beta = 11, \delta = 2$, for varying sample sizes (n)

n	P_1	P_2	P_3	π	α	β	δ	conventional variance (RRM) (eqn.2.10)	Proposed Variance (RRRM) (eqn.3.5)	Relative Efficiency (RE) (eqn.4.1)
50	0.2	0.5	0.3	0.7	20	11	2	0.00428	0.00421	0.98364
100	0.2	0.5	0.3	0.7	20	11	2	0.00214	0.00211	0.98598
150	0.2	0.5	0.3	0.7	20	11	2	0.00143	0.00140	0.97902
200	0.2	0.5	0.3	0.7	20	11	2	0.00107	0.00105	0.98131
500	0.2	0.5	0.3	0.7	20	11	2	0.000428	0.000421	0.98364

Table 4.4: Comparison between existing RRM and proposed RRRM when $P_1 = 0.15$, $P_2 = 0.6, P_3 = 0.25,\ \pi = 0.8\ , \alpha = 20, \beta = 11, \delta = 2$, for varying sample sizes (n)

n	P_1	P_2	P_3	π	α	β	δ	conventional variance (RRM) (eqn.2.10)	Proposed Variance (RRRM) (eqn.3.5)	Relative Efficiency (RE) (eqn.4.1)
50	0.15	0.6	0.25	0.8	20	11	2	0.00327	0.00321	0.98165
100	0.15	0.6	0.25	0.8	20	11	2	0.00163	0.00161	0.98773
150	0.15	0.6	0.25	0.8	20	11	2	0.00109	0.00107	0.98165
200	0.15	0.6	0.25	0.8	20	11	2	0.00082	0.000803	0.97927
500	0.15	0.6	0.25	0.8	20	11	2	0.000327	0.000321	0.98165

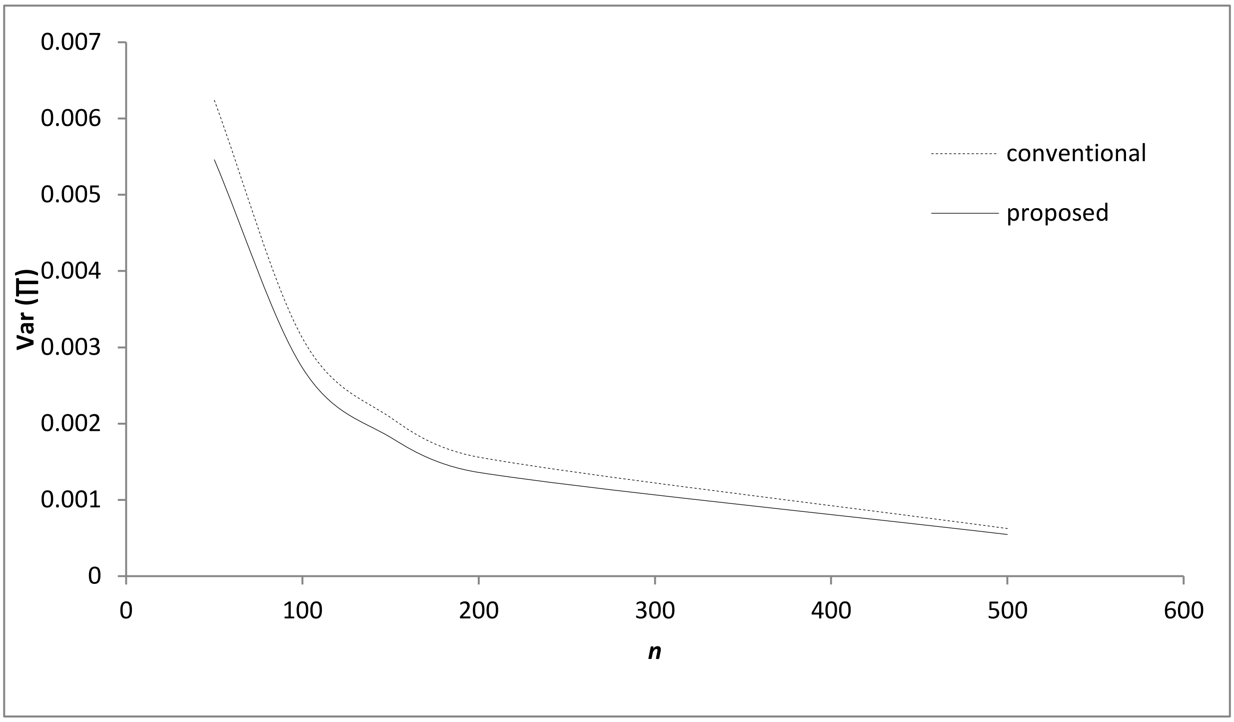

Figure 4.1:Comparison between existing RRM and proposed RRRM when $P_1 = 0.6$,

$P_2 = 0.3, P_3 = 0.1$, $\pi = 0.5, \alpha = 20, \beta = 11, \delta = 2$, for varying sample sizes (n)

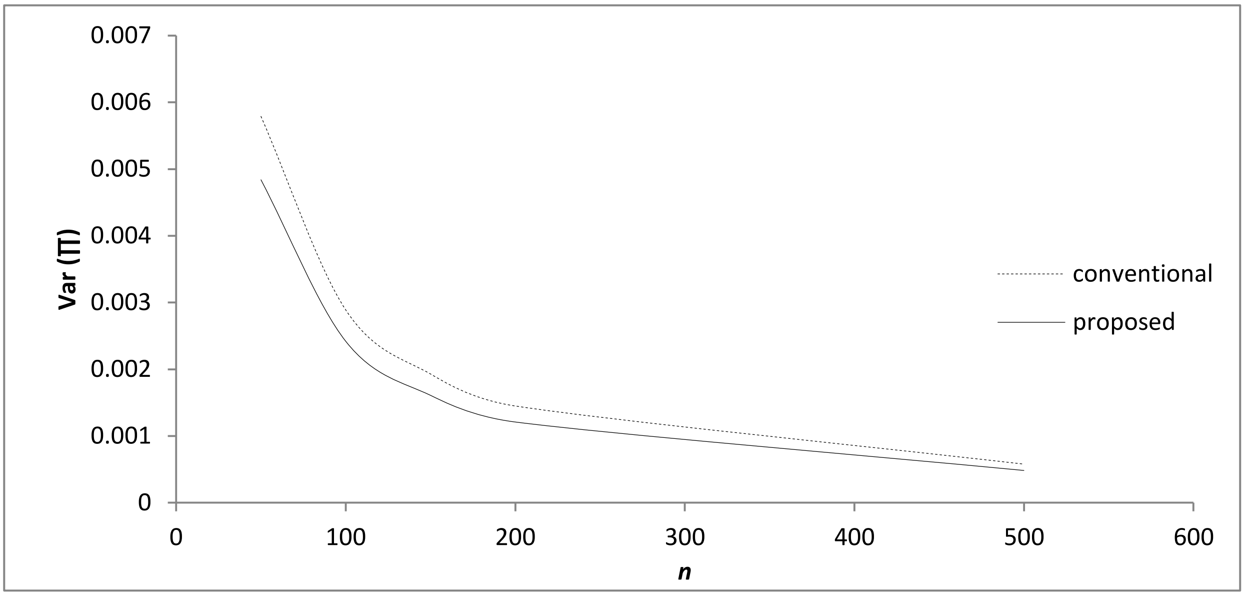

Figure 4.2:Comparison between existing RRM and proposed RRRM when $P_1 = 0.4$,

$P_2 = 0.4, P_3 = 0.2,\ \pi = 0.4,\ \alpha = 20, \beta = 11, \delta = 2$, for varying sample sizes (n)

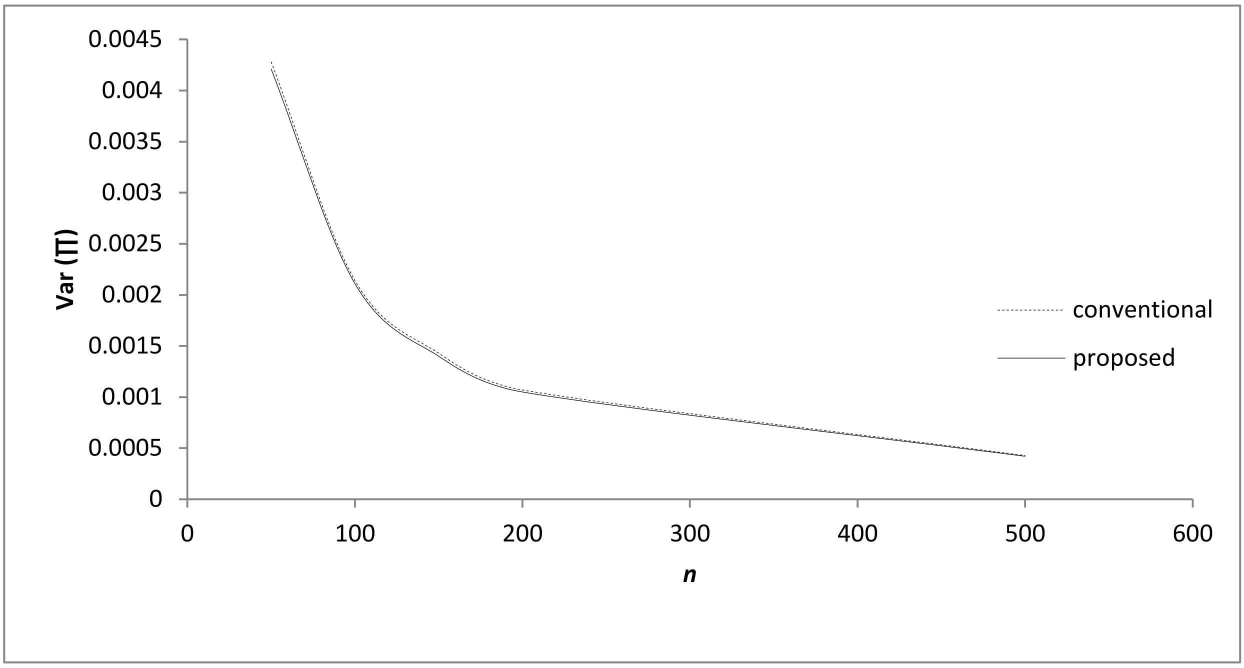

Figure 4.3:Comparison between existing RRM and proposed RRRM when $P_1 = 0.2,$

$P_2 = 0.5, P_3 = 0.3,\ \pi = 0.7, \alpha = 20, \beta = 11, \delta = 2,$ for varying sample sizes (n)

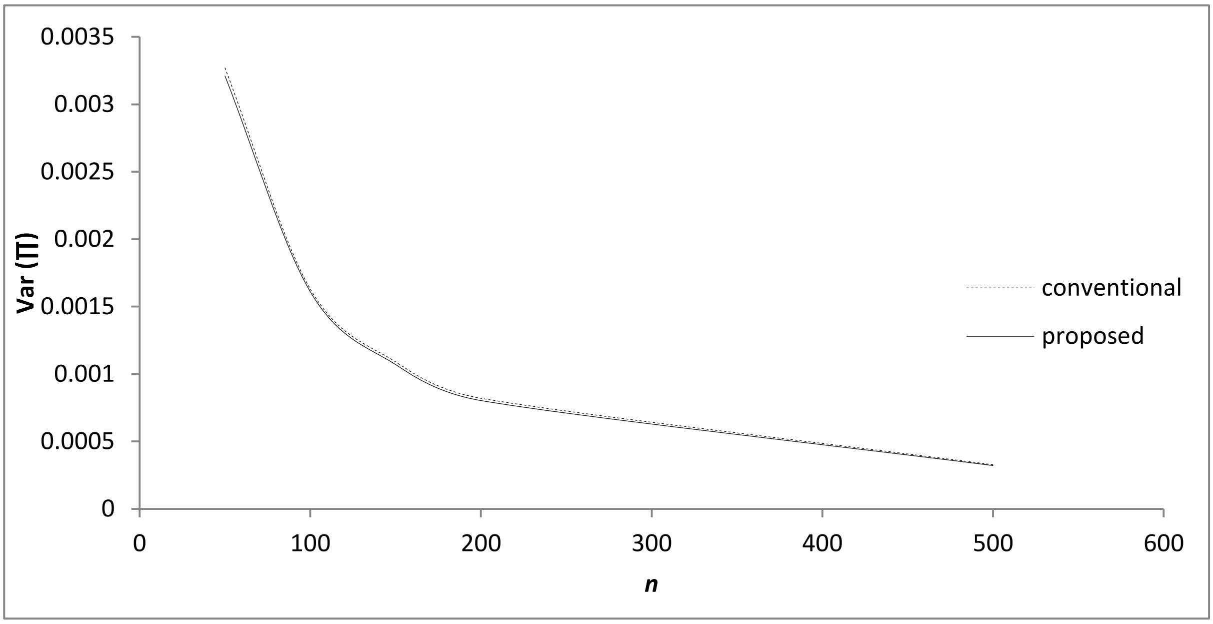

Figure 4.4:Comparison between existing RRM and proposed RRRM when $P_1 = 0.15,$

$P_2 = 0.6, P_3 = 0.25,\ \pi = 0.8$, $\alpha = 20, \beta = 11, \delta = 2,$ for varying sample sizes (n)

Note:

Var(π) in the figures above represents variance for both existing and proposed Models as obtained in equations 2.10 and 3.5 respectively.

…….conventional variance (equation 2.10)

_____proposed variance (equation 3.5)

5. Conclusion

In this study, the derivation of Hussain and Shabbir's Randomized Response Model was presented. The relative efficiency as well as the variance of our proposed Reparametized Randomized Response Model (RRRM) over that of the existing one was established by adopting the data used by Hussain and Shabbir [6] . It was obvious in the results on Tables and Figures that the proposed Model is indeed more efficient than the existing one.

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