

Analysis of Some Reliability Measures of a Deteriorating Reinforced Concrete Structure

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Abstract

In this paper, we study the reliability and availability characteristics of two-stage deterioration reinforce concrete structure. Failure and repair times are assumed exponential. The explicit expressions of reliability and availability characteristics such as mean time to system failure (MTSF), steady-state availability and busy period are derived using Kolmogorov forward equations method. Various cases are analyzed graphically to investigate the impact of system parameters on MTSF, availability and busy period

Keywords: Reliability, deterioration, reinforce concrete

1. Introduction

As the age of concrete structure increases, the structure slowly deteriorates correspondingly. Most of these structures are subjected to random deterioration which can result in unexpected failures and disastrous effect on safety and the economy it is therefore important to find a way to slow down the deterioration rate, and to prolong structure's service life. However in the real world some sections of a concrete structure are exposed to many of these environments hazard, once deterioration has begun it can occur in the embedded steel as well as the surface of the concrete. However, a concrete structure can be susceptible to a number of factors that can cause deterioration which could lead to a reduction in strength making for unsafe conditions. Therefore it is essential that a concrete structure is maintained correctly. The concept of deterioration and its impact on reliability measure of system effectiveness has been introduced by several authors (see for instance [3, 7, 8, 11]). Reliability is vital for proper utilization and maintenance of any reinforced concrete structure. It involves technique for increasing system effectiveness through reducing failure frequency and maintenance cost. For this reason, many researchers have studied reliability problem of different reinforced concrete structure (see, for instance, [1,5, 6, 9] and the references therein).

In this paper, we consider a three layer reinforced concrete structure exposed to slow and fast deterioration and derived its corresponding mathematical models using Kolmogorov's forward equation method. The contributions of this paper are twofold. First is to develop the explicit expressions for mean time to system failure (MTSF), steady-state availability and steady-state busy period. The second is to determine the impact of failure rate, repair rate, slow and fast deterioration rate, minor and major minimal maintenance rate on mean time to system failure, steady-state availability, and steady-state busy period.

The rest of the paper is organized as follows. Section 2 is the description and states of the reinforced concrete structure. Section 3 deals with models formulation. The results of our numerical simulations are presented and

discussed in Section 4. The paper is concluded in Section 5.

2. Description of the Reinforce Concrete Structure

We consider three layers reinforce concrete structure with three modes: normal, deterioration and failure. The layers are coating, concrete and reinforcement. The deterioration mode consists of two consecutive stages: slow and fast. It is assumed that the structure transits from normal to slow and later to fast deterioration with rate δ_1 and δ_2 respectively. It is also assumed that the two upper layers (coating and concrete) never fail simultaneously. Whenever the coating layer deteriorates with rate δ_1 , minor minimal maintenance is invoke with rate μ_1 to regain the structure to its early stage prior to deterioration or the deterioration will be faster with rate δ_2 where major minimal maintenance will be done with rate μ_2 . Concrete repair involves cleaning and preparing the defective area and the exposed reinforcing steel, after which rebuilding up the defective area using a suitable repair mortar. Coating layer fails with rate β_1 and is repaired with rate α_1 . If coating is not repaired, concrete will be exposed which later fails with rate β_2 and repaired with rate α_2 . Once any concrete has been repaired it is very important to protect the structure from future deterioration and failure by applying a protection coat. The structure failed when two consecutive layers (coating and concrete) have failed.

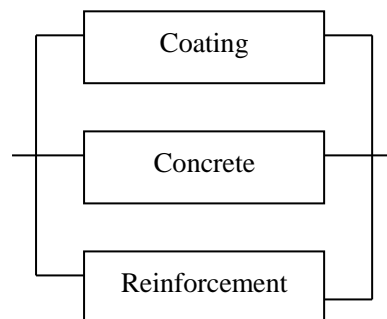


Fig. 1 Reliability block diagram of the reinforce concrete structure

Table 1 Transition rate table

	S_0	S_1	S_2	S_3	S_4	S_5
S_0		δ_1		β_1		
S_1	μ_1		δ_2	β_1		
S_2		μ_2			β_1	
S_3	α_1	α_1			δ_2	β_2
S_4			α_1	μ_2		β_2
S_5				α_2	α_2	

2.1. States of the System

State S_0 : The coating, concrete and reinforcement are normal

State S_1 : The structure is under slow deterioration and is receiving minor minimal maintenance

State S_2 : The structure is under fast deterioration and is receiving major minimal maintenance

State S_3 : Coating layer failed and is under repair, the concrete structure is in slow deterioration

State S_4 : Coating layer failed and is under repair, the concrete structure is in fast deterioration

State S_5 : Concrete layer failed and is under repair, the reinforcement is exposed and the structure failed.

3. Models Formulation

Let $P(t)$ be the probability row vector at time t , then the initial conditions for this problem are as follows:

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0)] = [1, 0, 0, 0, 0, 0]$$

we obtain the following system of differential equations:

$$P'_0(t) = -(\beta_1 + \delta_1)P_0(t) + \mu_1P_1(t) + \alpha_1P_3(t)$$

$$P'_1(t) = -(\beta_1 + \mu_1 + \delta_2)P_1(t) + \delta_1P_0(t) + \mu_2P_2(t) + \alpha_1P_3(t)$$

$$P'_2(t) = -(\beta_1 + \mu_2)P_2(t) + \delta_2P_1(t) + \alpha_1P_4(t)$$

$$P_3'(t) = -(\beta_2 + \delta_2 + 2\alpha_1)P_3(t) + \beta_1P_0(t) + \beta_1P_1(t) + \mu_2P_4(t) + \alpha_2P_5(t)$$

$$P_4'(t) = -(\beta_2 + \alpha_1 + \mu_2)P_4(t) + \beta_1P_2(t) + \delta_2P_3(t) + \alpha_2P_5(t)$$

$$P_5'(t) = -2\alpha_2P_5(t) + \beta_2P_3(t) + \beta_2P_4(t) \tag{1}$$

The differential equations in (1) above is transformed into matrix as $P' = TP$ (2)

where

$$T = \begin{bmatrix} -(\beta_1 + \delta_1) & \mu_1 & 0 & \alpha_1 & 0 & 0 \\ \delta_1 & -(\beta_1 + \mu_1 + \delta_2) & \mu_2 & \alpha_1 & 0 & 0 \\ 0 & \delta_2 & -(\beta_1 + \mu_2) & 0 & \alpha_1 & 0 \\ \beta_1 & \beta_1 & 0 & -(\beta_2 + \delta_2 + 2\alpha_1) & \mu_2 & \alpha_2 \\ 0 & 0 & \beta_1 & \delta_2 & -(\beta_2 + \alpha_1 + \mu_2) & \alpha_2 \\ 0 & 0 & 0 & \beta_2 & \beta_2 & -2\alpha_2 \end{bmatrix}$$

3.1 Mean Time to System Failure Analysis

It is difficult to evaluate the transient solutions, hence we follow [2,4,10], the procedure to develop the explicit expression for $MTSF$ is to delete the sixth row and column of matrix T and take the transpose to produce a new matrix, say Q . The expected time to reach an absorbing state is obtained from

$$E[T_{P(0) \rightarrow P(\text{absorbing})}] = MTSF = P(0)(-Q^{-1}) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \frac{N_1}{D_1} \tag{3}$$

Where

$$Q = \begin{bmatrix} -(\beta_1 + \delta_1) & \delta_1 & 0 & \beta_1 & 0 \\ \mu_1 & -(\beta_1 + \mu_1 + \delta_2) & \delta_2 & \beta_1 & 0 \\ 0 & \mu_2 & -(\beta_1 + \mu_2) & 0 & \beta_1 \\ \alpha_1 & \alpha_1 & 0 & -(\beta_2 + \delta_2 + 2\alpha_1) & \delta_2 \\ 0 & 0 & \alpha_1 & \mu_2 & -(\beta_2 + \mu_2 + \alpha_1) \end{bmatrix}$$

$$N_1 = (\beta_1\beta_2^2\delta_2 + 2\beta_1\beta_2\mu_2\delta_2 + \alpha_1\beta_1\mu_2\delta_2 + \beta_2\mu_1\mu_2\delta_2 + \alpha_1\mu_1\mu_2\delta_2 + \beta_1\beta_2\mu_1\delta_2 + \beta_1\beta_2\delta_2^2 + \beta_1^2\beta_2\delta_2 + 2\alpha_1^2\mu_1\mu_2 + \beta_2^2\mu_1\mu_2 + \beta_2\mu_1\mu_2^2 + 2\alpha_1\mu_1\mu_2^2 + \beta_1^2\beta_2\mu_2 + \alpha_1\beta_1^2\beta_2 + \alpha_1\beta_1^2\mu_2 + \beta_1\beta_2\mu_2^2 + \beta_1\beta_2^2\mu_2 + \alpha_1\beta_1\mu_2^2 + \beta_1^2\beta_2^2 + 3\alpha_1\beta_2\mu_1\mu_2 + \beta_1\beta_2\mu_1\mu_2 + 2\alpha_1\beta_1\beta_2\mu_2 + 2\alpha_1\beta_1\mu_1\mu_2 + 2\alpha_1\beta_1\beta_2\delta_2 + 2\alpha_1\beta_1\beta_2\mu_1 + \alpha_1^2\beta_1\mu_2 + \beta_1\beta_2^2\mu_1) + (2\alpha_1^2\mu_2\delta_1 + \alpha_1^2\beta_1\mu_2 + 2\alpha_1\mu_2^2\delta_1 + \alpha_1\beta_1\mu_2^2 + 2\alpha_1\beta_1\mu_2\delta_1 + \alpha_1\beta_1^2\mu_2 + \alpha_1\beta_1\mu_2\delta_2 + 3\alpha_1\beta_2\mu_2\delta_1 + \alpha_1\mu_2\delta_1\delta_2 + \alpha_1\beta_1\beta_2\mu_2 + \alpha_1\beta_1^2\beta_2 + 2\alpha_1\beta_1\beta_2\delta_1 + \beta_2\mu_2^2\delta_1 + \beta_2\mu_2\delta_1\delta_2 + \beta_1\beta_2\mu_2\delta_1 + \beta_2^2\mu_2\delta_1 + \beta_1\beta_2\delta_1\delta_2 + \beta_1\beta_2^2\delta_1) + \delta_2(\alpha_1\beta_1\mu_1 + 2\alpha_1^2\delta_1 + \alpha_1^2\beta_1 + 2\alpha_1\mu_2\delta_1 + \alpha_1\beta_1\mu_2 + \alpha_1\beta_1\delta_2 + 3\alpha_1\beta_2\delta_1 + \alpha_1\beta_1^2 +$$

$$\alpha_1\delta_1\delta_2 + \alpha_1\beta_1\beta_2 + \alpha_1\beta_1\delta_1 + \beta_2\mu_2\delta_1 + \beta_2^2\delta_1 + \beta_2\delta_1\delta_2) + \beta_1(\mu_1\mu_2^2 + \beta_2\mu_1\mu_2 + \alpha_1\mu_1\mu_2 + \beta_1\mu_1\mu_2 + \beta_1\beta_2\mu_1 + \alpha_1\beta_1\mu_2 + \beta_1\mu_2^2 + \mu_2^2\delta_1 + \beta_1\beta_2\mu_2 + \beta_1^2\mu_2 + \beta_2\mu_2\delta_1 + \beta_1\mu_2\delta_2 + \beta_1\mu_2\delta_1 + \mu_2\delta_1\delta_2 + \beta_1\beta_2\delta_1 + \beta_1^2\beta_2 + \beta_1\beta_2\delta_2) + \beta_1\delta_2(\mu_1\mu_2 + \beta_1\mu_1 + \alpha_1\beta_1 + 2\alpha_1\delta_1 + \beta_1\mu_2 + \mu_2\delta_1 + \delta_1\delta_2 + \beta_1\delta_1 + \beta_1^2 + \beta_1\delta_2 + \beta_2\delta_1)$$

$$D_1 = \alpha_1\beta_1\mu_2 + \beta_1\beta_2\mu_2 + \beta_1\beta_2\mu_1 + \beta_1\mu_1\mu_2 + 2\beta_1\mu_2\delta_2 + \beta_1\beta_2\delta_2 + \beta_2\mu_1\mu_2 + \beta_1\mu_1\delta_2 + 2\mu_2\delta_1\delta_2 + \beta_1\delta_2^2 + \alpha_1\beta_1\delta_2 + \beta_1^2\delta_2 + \mu_1\mu_2\delta_2 + \alpha_1\mu_1\mu_2 + \beta_2\mu_2\delta_1 + \beta_1\mu_2\delta_1 + \beta_1\beta_2\delta_1 + \beta_2\delta_1\delta_2 + \beta_1\delta_1\delta_2 + \delta_1\delta_2^2 + \beta_1\mu_2^2 + \beta_1^2\mu_2 + \beta_1^2\beta_2 + \mu_1\mu_2^2 + \mu_2^2\delta_1 + 2\alpha_1\delta_1\delta_2 + \alpha_1\mu_2\delta_1$$

3.2 System Availability Analysis

For the availability case of table 1 using the initial condition in section 3 for this system,

$$P(0) = [P_1(0), P_2(0), P_3(0), P_4(0), P_5(0)] = [1, 0, 0, 0, 0]$$

The system of differential equations in (1) for the system above can be expressed in matrix form as:

$$\begin{bmatrix} P_0' \\ P_1' \\ P_2' \\ P_3' \\ P_4' \\ P_5' \end{bmatrix} = \begin{bmatrix} -(\beta_1 + \delta_1) & \mu_1 & 0 & \alpha_1 & 0 & 0 \\ \delta_1 & -(\beta_1 + \mu_1 + \delta_2) & \mu_2 & \alpha_1 & 0 & 0 \\ 0 & \delta_2 & -(\beta_1 + \mu_2) & 0 & \alpha_1 & 0 \\ \beta_1 & \beta_1 & 0 & -(\beta_2 + \delta_2 + 2\alpha_1) & \mu_2 & \alpha_2 \\ 0 & 0 & \beta_1 & \delta_2 & -(\beta_2 + \alpha_1 + \mu_2) & \alpha_2 \\ 0 & 0 & 0 & \beta_2 & \beta_2 & -2\alpha_2 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix}$$

Let V be the time to failure of the system. Following [2,4,10], the steady-state availability is given by

$$A_V = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) \tag{4}$$

In steady state, the derivatives of state probabilities become zero, thus (2) becomes

$$TP(\infty) = 0 \tag{5}$$

which in matrix form is

$$\begin{bmatrix} -(\beta_1 + \delta_1) & \mu_1 & 0 & \alpha_1 & 0 & 0 \\ \delta_1 & -(\beta_1 + \mu_1 + \delta_2) & \mu_2 & \alpha_1 & 0 & 0 \\ 0 & \delta_2 & -(\beta_1 + \mu_2) & 0 & \alpha_1 & 0 \\ \beta_1 & \beta_1 & 0 & -(\beta_2 + \delta_2 + 2\alpha_1) & \mu_2 & \alpha_2 \\ 0 & 0 & \beta_1 & \delta_2 & -(\beta_2 + \alpha_1 + \mu_2) & \alpha_2 \\ 0 & 0 & 0 & \beta_2 & \beta_2 & -2\alpha_2 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

using the normalizing condition

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) = 1 \quad (6)$$

we substitute (6) in the last row of (5) following [2,3,5]. The resulting matrix is

$$\begin{bmatrix} -(\beta_1 + \delta_1) & \mu_1 & 0 & \alpha_1 & 0 & 0 \\ \delta_1 & -(\beta_1 + \mu_1 + \delta_2) & \mu_2 & \alpha_1 & 0 & 0 \\ 0 & \delta_2 & -(\beta_1 + \mu_2) & 0 & \alpha_1 & 0 \\ \beta_1 & \beta_1 & 0 & -(\beta_2 + \delta_2 + 2\alpha_1) & \mu_2 & \alpha_2 \\ 0 & 0 & \beta_1 & \delta_2 & -(\beta_2 + \alpha_1 + \mu_2) & \alpha_2 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Expression for A_v thus is:

$$A_v = \frac{N_2}{D_2}$$

$$\begin{aligned} N_2 = & \alpha_1 \alpha_2 (4\alpha_1 \mu_1 \mu_2 + 4\mu_1 \mu_2^2 + 4\beta_1 \mu_1 \mu_2 + 3\beta_2 \mu_1 \mu_2 + 2\mu_1 \mu_2 \delta_2 + 2\beta_1 \beta_2 \mu_1 + 2\alpha_1 \beta_1 \mu_2 + 2\beta_1 \mu_2^2 + 2\beta_1 \mu_2 \delta_2 + \beta_1 \beta_2 \mu_2 + \\ & 2\beta_1^2 \mu_2 + \beta_1 \beta_2 \delta_2 + \beta_1^2 \beta_2) + \alpha_1 \alpha_2 (4\alpha_1 \mu_2 \delta_1 + 2\alpha_1 \beta_1 \mu_2 + 4\mu_2^2 \delta_1 + 2\beta_1 \mu_2^2 + 4\beta_1 \mu_2 \delta_1 + 2\beta_1^2 \mu_2 + 2\mu_2 \delta_1 \delta_2 + 2\beta_1 \beta_2 \mu_2 + \\ & 3\beta_2 \mu_2 \delta_1 + 2\beta_1 \mu_2 \delta_2 + 2\beta_1 \beta_2 \delta_1 + \beta_1^2 \beta_2) + \alpha_1 \alpha_2 (\beta_1 \beta_2 \mu_1 + 2\beta_1 \mu_1 \delta_2 + 2\alpha_1 \beta_1 \delta_2 + 4\alpha_1 \delta_1 \delta_2 + 2\beta_1 \mu_2 \delta_2 + 4\mu_2 \delta_1 \delta_2 + 2\beta_1^2 \delta_2 + \\ & 2\beta_1 \beta_2 \delta_2 + 2\delta_1 \delta_2^2 + 2\beta_1 \delta_2^2 + 3\beta_2 \delta_1 \delta_2 + 2\beta_1 \delta_1 \delta_2 + \beta_1^2 \beta_2 + \beta_1 \beta_2 \delta_1) + \alpha_2 \beta_1 (2\alpha_1 \mu_1 \mu_2 + 2\mu_1 \mu_2^2 + \beta_2 \mu_1 \mu_2 + 2\beta_1 \mu_1 \mu_2 + \beta_1 \beta_2 \mu_1 + \\ & 2\alpha_1 \mu_2 \delta_1 + 2\alpha_1 \beta_1 \mu_2 + 2\mu_2^2 \delta_1 + 2\beta_1 \mu_2^2 + 2\beta_1 \mu_2 \delta_1 + 2\beta_1^2 \mu_2 + 2\beta_1 \mu_2 \delta_2 + 2\mu_2 \delta_1 \delta_2 + \beta_2 \mu_2 \delta_1 + \beta_1 \beta_2 \mu_2 + \beta_1 \beta_2 \delta_1 + \beta_1 \beta_2 \delta_2 + \\ & \beta_2 \delta_1 \delta_2 + \beta_1^2 \beta_2) + \alpha_2 \beta_1 (\beta_2 \mu_1 \mu_2 + 2\mu_1 \mu_2 \delta_2 + 2\beta_1 \mu_1 \delta_2 + \beta_1 \beta_2 \mu_1 + 2\alpha_1 \beta_1 \delta_2 + 4\alpha_1 \delta_1 \delta_2 + 2\beta_1 \mu_2 \delta_2 + \beta_2 \mu_2 \delta_1 + 2\mu_2 \delta_1 \delta_2 + \\ & \beta_1 \beta_2 \mu_2 + 2\beta_1^2 \delta_2 + \beta_2 \delta_1 \delta_2 + 2\beta_1 \delta_1 \delta_2 + \beta_1 \beta_2 \delta_2 + 2\beta_1 \delta_2^2 + \beta_1^2 \beta_2 + 2\delta_1 \delta_2^2 + \beta_1 \beta_2 \delta_1) \end{aligned}$$

$$\begin{aligned} D_2 = & 2\beta_1^2 \beta_2 \mu_2 \delta_2 + \alpha_1 \beta_1^2 \beta_2 \delta_2 + 2\alpha_2 \beta_1^2 \beta_2 \delta_2 + 2\alpha_1 \alpha_2 \beta_1 \delta_2^2 + 4\alpha_1 \alpha_2 \beta_1^2 \delta_2 + 4\alpha_2 \beta_1^2 \mu_2 \delta_2 + \beta_1^2 \beta_2 \delta_2^2 + \beta_1^2 \beta_2^2 \delta_2 + \beta_1^3 \beta_2 \delta_2 + \\ & 2\alpha_2 \beta_1^2 \delta_2^2 + 2\alpha_2 \beta_1^3 \delta_2 + 6\alpha_1 \alpha_2 \beta_1 \mu_2 \delta_2 + 2\alpha_1 \alpha_2 \mu_1 \mu_2 \delta_2 + \beta_1 \beta_2 \delta_1 \delta_2^2 + \beta_1 \beta_2 \mu_1 \mu_2 \delta_2 + 2\beta_1 \beta_2 \mu_2 \delta_1 \delta_2 + 2\alpha_2 \beta_1^2 \mu_1 \delta_2 + \\ & 2\alpha_2 \beta_1 \mu_1 \mu_2 \delta_2 + 2\alpha_2 \beta_1^2 \delta_1 \delta_2 + 2\alpha_2 \beta_1 \delta_1 \delta_2^2 + 2\alpha_2 \beta_1 \beta_2 \delta_1 \delta_2 + 4\alpha_2 \beta_1 \mu_2 \delta_1 \delta_2 + \beta_1 \beta_2^2 \delta_1 \delta_2 + \beta_1^2 \beta_2 \mu_1 \mu_2 + \beta_1^2 \beta_2 \mu_2 \delta_1 + 2\alpha_2 \beta_1^2 \mu_2 \delta_1 + \end{aligned}$$

$$\begin{aligned}
 &2\alpha_2\beta_1^2\beta_2\delta_1 + 2\alpha_2\beta_1^2\beta_2\mu_1 + 2\alpha_2\beta_1^2\mu_1\mu_2 + 2\alpha_2\beta_1^2\beta_2\mu_2 + 2\alpha_2\beta_1\mu_1\mu_2^2 + \beta_1\beta_2^2\mu_2\delta_1 + 2\alpha_2\beta_1\mu_2^2\delta_1 + \beta_1\beta_2\mu_1\mu_2^2 + \beta_1\beta_2\mu_2^2\delta_1 + \\
 &\beta_1\beta_2^2\mu_1\mu_2 + \beta_1^3\beta_2^2 + \beta_1^3\beta_2\mu_2 + 2\alpha_2\beta_1^3\beta_2 + 2\alpha_2\beta_1^3\mu_2 + 2\alpha_2\beta_1^2\mu_2^2 + \beta_1^2\beta_2^2\mu_2 + \beta_1^2\beta_2\mu_2^2 + \beta_1^2\beta_2^2\delta_1 + \beta_1^2\beta_2^2\mu_1 + \\
 &2\alpha_2\beta_1\beta_2\mu_2\delta_1 + 2\alpha_2\beta_1\beta_2\mu_1\mu_2 + \alpha_1\beta_1^2\beta_2\mu_2 + 6\alpha_1\alpha_2\beta_1^2\mu_2 + 3\alpha_1\alpha_2\beta_1^2\beta_2 + 4\alpha_1\alpha_2\mu_1\mu_2^2 + 4\alpha_1\alpha_2\beta_1\mu_2^2 + 4\alpha_1\alpha_2\mu_2^2\delta_1 + \\
 &2\alpha_1^2\alpha_2\beta_1\delta_2 + 4\alpha_1^2\alpha_2\beta_1\mu_2 + 4\alpha_1^2\alpha_2\delta_1\delta_2 + 4\alpha_1^2\alpha_2\mu_1\mu_2 + 4\alpha_1^2\alpha_2\mu_2\delta_1 + 2\alpha_1\beta_1\beta_2\delta_1\delta_2 + 2\alpha_1\alpha_2\beta_1\mu_1\delta_2 + 6\alpha_1\alpha_2\beta_1\delta_1\delta_2 + \\
 &\alpha_1\beta_1\beta_2\mu_1\mu_2 + \alpha_1\beta_1\beta_2\mu_2\delta_1 + 3\alpha_1\alpha_2\beta_2\delta_1\delta_2 + 3\alpha_1\alpha_2\beta_2\mu_2\delta_1 + \alpha_1\alpha_2\beta_2\mu_1\mu_2 + 3\alpha_1\alpha_2\beta_1\beta_2\delta_2 + 3\alpha_1\alpha_2\beta_1\beta_2\mu_1 + 6\alpha_1\alpha_2\beta_1\mu_1\mu_2 + \\
 &3\alpha_1\alpha_2\beta_1\beta_2\delta_1 + 3\alpha_1\alpha_2\beta_1\beta_2\mu_2 + 6\alpha_1\alpha_2\beta_1\mu_2\delta_1
 \end{aligned}$$

3.3 Busy Period Analysis

Using the same initial condition in section 3 above as for the reliability case

$$P(0) = [P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0)] = [1, 0, 0, 0, 0, 0] \text{ and (5) and (6) the busy period is}$$

obtained as follows:

In the steady state, the derivatives of the state probabilities become zero and this will enable us to compute steady state busy period :

The system of differential equations in (1) for the system above can be expressed in matrix form as:

$$\begin{bmatrix} P_0' \\ P_1' \\ P_2' \\ P_3' \\ P_4' \\ P_5' \end{bmatrix} = \begin{bmatrix} -(\beta_1 + \delta_1) & \mu_1 & 0 & \alpha_1 & 0 & 0 \\ \delta_1 & -(\beta_1 + \mu_1 + \delta_2) & \mu_2 & \alpha_1 & 0 & 0 \\ 0 & \delta_2 & -(\beta_1 + \mu_2) & 0 & \alpha_1 & 0 \\ \beta_1 & \beta_1 & 0 & -(\beta_2 + \delta_2 + 2\alpha_1) & \mu_2 & \alpha_2 \\ 0 & 0 & \beta_1 & \delta_2 & -(\beta_2 + \alpha_1 + \mu_2) & \alpha_2 \\ 0 & 0 & 0 & \beta_2 & \beta_2 & -2\alpha_2 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix}$$

Let V be the time to failure of the system. The steady-state busy period is given by

$$B_V = P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(t) \tag{4}$$

In steady state, the derivatives of state probabilities become zero, thus (2) becomes

$$TP(\infty) = 0 \tag{5}$$

which in matrix form is

$$\begin{bmatrix} -(\beta_1 + \delta_1) & \mu_1 & 0 & \alpha_1 & 0 & 0 \\ \delta_1 & -(\beta_1 + \mu_1 + \delta_2) & \mu_2 & \alpha_1 & 0 & 0 \\ 0 & \delta_2 & -(\beta_1 + \mu_2) & 0 & \alpha_1 & 0 \\ \beta_1 & \beta_1 & 0 & -(\beta_2 + \delta_2 + 2\alpha_1) & \mu_2 & \alpha_2 \\ 0 & 0 & \beta_1 & \delta_2 & -(\beta_2 + \alpha_1 + \mu_2) & \alpha_2 \\ 0 & 0 & 0 & \beta_2 & \beta_2 & -2\alpha_2 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

using the normalizing condition

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) = 1 \quad (6)$$

we substitute (6) in the last row of (5) following [2,3,5]. The resulting matrix is

$$\begin{bmatrix} -(\beta_1 + \delta_1) & \mu_1 & 0 & \alpha_1 & 0 & 0 \\ \delta_1 & -(\beta_1 + \mu_1 + \delta_2) & \mu_2 & \alpha_1 & 0 & 0 \\ 0 & \delta_2 & -(\beta_1 + \mu_2) & 0 & \alpha_1 & 0 \\ \beta_1 & \beta_1 & 0 & -(\beta_2 + \delta_2 + 2\alpha_1) & \mu_2 & \alpha_2 \\ 0 & 0 & \beta_1 & \delta_2 & -(\beta_2 + \alpha_1 + \mu_2) & \alpha_2 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

In the steady state, the derivatives of the state probabilities become zero and this will enable us to compute steady state busy period :

$$B_V(\infty) = 1 - P_0(\infty) \quad (7)$$

The steady state busy period $B_V(\infty)$ is therefore:

$$B_V(\infty) = \frac{N_3}{D_2}$$

$$\begin{aligned} N_3 = & \alpha_1 \alpha_2 (4\alpha_1 \mu_2 \delta_1 + 2\alpha_1 \beta_1 \mu_2 + 4\mu_2^2 \delta_1 + 2\beta_1 \mu_2^2 + 4\beta_1 \mu_2 \delta_1 + 2\beta_1^2 \mu_2 + 2\mu_2 \delta_1 \delta_2 + 2\beta_1 \beta_2 \mu_2 + \\ & 3\beta_2 \mu_2 \delta_1 + 2\beta_1 \mu_2 \delta_2 + 2\beta_1 \beta_2 \delta_1 + \beta_1^2 \beta_2) + \alpha_1 \alpha_2 (\beta_1 \beta_2 \mu_1 + 2\beta_1 \mu_1 \delta_2 + 2\alpha_1 \beta_1 \delta_2 + 4\alpha_1 \delta_1 \delta_2 + 2\beta_1 \mu_2 \delta_2 + 4\mu_2 \delta_1 \delta_2 + 2\beta_1^2 \delta_2 + \\ & 2\beta_1 \beta_2 \delta_2 + 2\delta_1 \delta_2^2 + 2\beta_1 \delta_2^2 + 3\beta_2 \delta_1 \delta_2 + 2\beta_1 \delta_1 \delta_2 + \beta_1^2 \beta_2 + \beta_1 \beta_2 \delta_1) + \alpha_2 \beta_1 (2\alpha_1 \mu_1 \mu_2 + 2\mu_1 \mu_2^2 + \beta_2 \mu_1 \mu_2 + 2\beta_1 \mu_1 \mu_2 + \beta_1 \beta_2 \mu_1 + \\ & 2\alpha_1 \mu_2 \delta_1 + 2\alpha_1 \beta_1 \mu_2 + 2\mu_2^2 \delta_1 + 2\beta_1 \mu_2^2 + 2\beta_1 \mu_2 \delta_1 + 2\beta_1^2 \mu_2 + 2\beta_1 \mu_2 \delta_2 + 2\mu_2 \delta_1 \delta_2 + \beta_2 \mu_2 \delta_1 + \beta_1 \beta_2 \mu_2 + \beta_1 \beta_2 \delta_1 + \beta_1 \beta_2 \delta_2 + \\ & \beta_2 \delta_1 \delta_2 + \beta_1^2 \beta_2) + \alpha_2 \beta_1 (\beta_2 \mu_1 \mu_2 + 2\mu_1 \mu_2 \delta_2 + 2\beta_1 \mu_1 \delta_2 + \beta_1 \beta_2 \mu_1 + 2\alpha_1 \beta_1 \delta_2 + 4\alpha_1 \delta_1 \delta_2 + 2\beta_1 \mu_2 \delta_2 + \beta_2 \mu_2 \delta_1 + 2\mu_2 \delta_1 \delta_2 + \\ & \beta_1 \beta_2 \mu_2 + 2\beta_1^2 \delta_2 + \beta_2 \delta_1 \delta_2 + 2\beta_1 \delta_1 \delta_2 + \beta_1 \beta_2 \delta_2 + 2\beta_1 \delta_2^2 + \beta_1^2 \beta_2 + 2\delta_1 \delta_2^2 + \beta_1 \beta_2 \delta_1) + \beta_1 \beta_2 (\alpha_1 \beta_1 \mu_2 + \beta_1 \beta_2 \mu_2 + \beta_1 \beta_2 \mu_1 + \\ & \beta_1 \mu_1 \mu_2 + 2\beta_1 \mu_2 \delta_2 + \beta_1 \beta_2 \delta_2 + \beta_2 \mu_1 \mu_2 + \beta_1 \mu_1 \delta_2 + 2\mu_2 \delta_1 \delta_2 + \beta_1 \delta_2^2 + \alpha_1 \beta_1 \delta_2 + \beta_1^2 \delta_2 + \mu_1 \mu_2 \delta_2 + \alpha_1 \mu_1 \mu_2 + \beta_2 \mu_2 \delta_1 + \\ & \beta_1 \mu_2 \delta_1 + \beta_1 \beta_2 \delta_1 + \beta_2 \delta_1 \delta_2 + \beta_1 \delta_1 \delta_2 + \delta_1 \delta_2^2 + \beta_1 \mu_2^2 + \beta_1^2 \mu_2 + \beta_1^2 \beta_2 + \mu_1 \mu_2^2 + \mu_2^2 \delta_1 + 2\alpha_1 \delta_1 \delta_2 + \alpha_1 \mu_2 \delta_1) \end{aligned}$$

4. Results and Discussions

In this section, we numerically obtained the results for mean time to system failure, system availability, busy period and profit function for all the developed models. For the model analysis, the following set of parameters values are fixed

throughout the simulations for consistency:

$$\beta_1 = 0.1, \beta_2 = 0.2, \alpha_1 = 0.4, \alpha_2 = 0.1, \delta_1 = 0.1, \delta_2 = 0.1, \mu_1 = 0.3, \mu_2 = 0.4$$

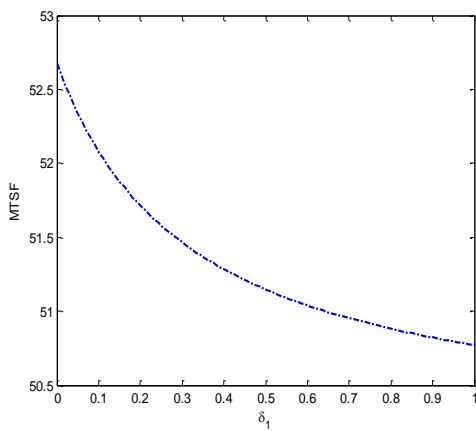


Fig. 2 Effect of δ_1 on MTSF

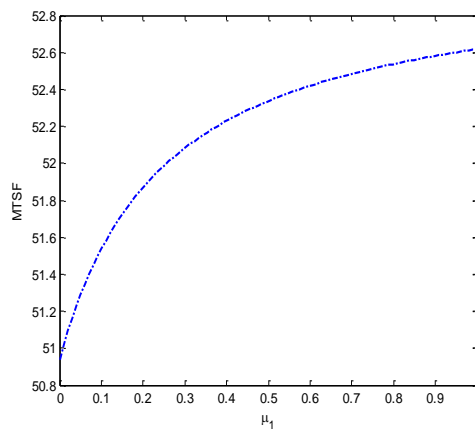


Fig. 3 Effect of μ_1 on MTSF

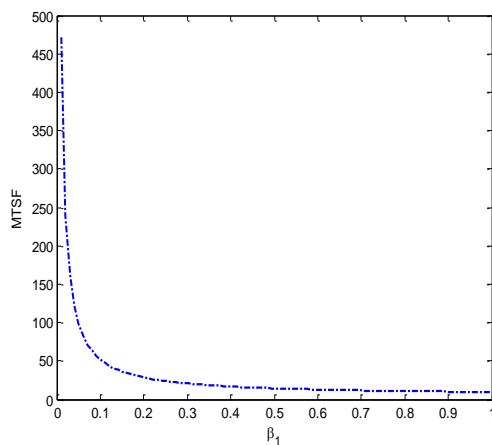


Fig. 4 Effect of β_1 on MTSF

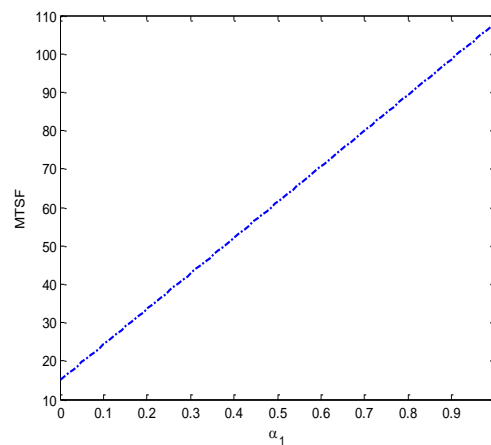


Fig. 5 Effect of α_1 on MTSF

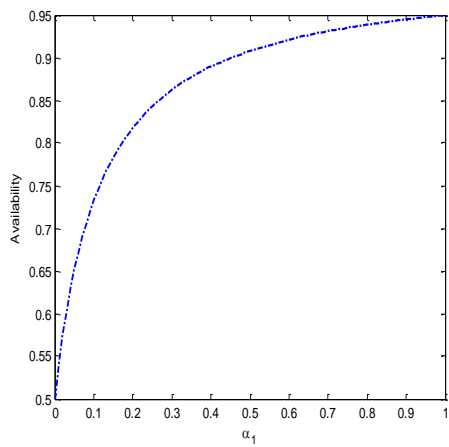


Fig. 6 Effect of α_1 on Availability

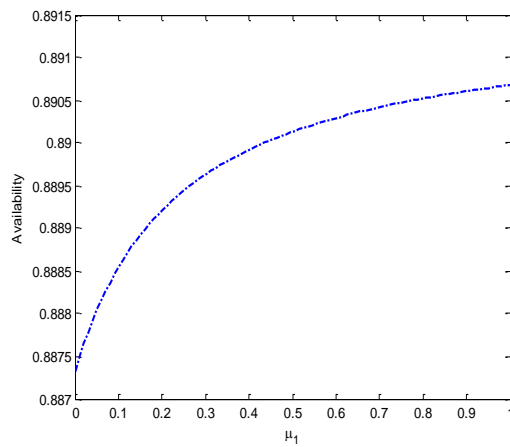


Fig. 7 Effect of μ_1 on Availability

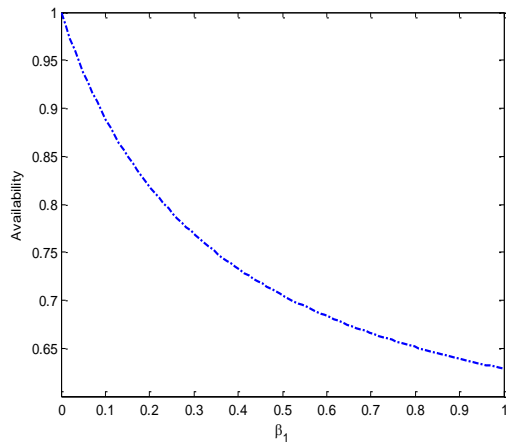


Fig. 8 Effect of β_1 on Availability

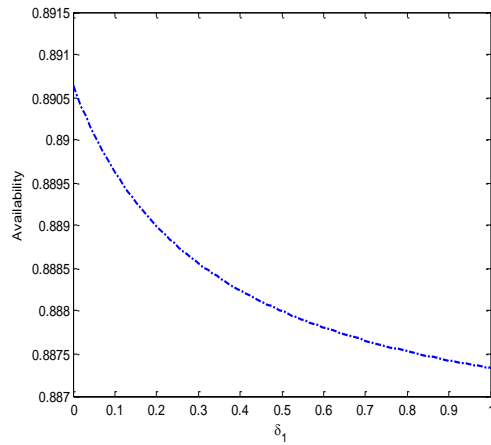


Fig. 9 Effect of δ_1 on Availability

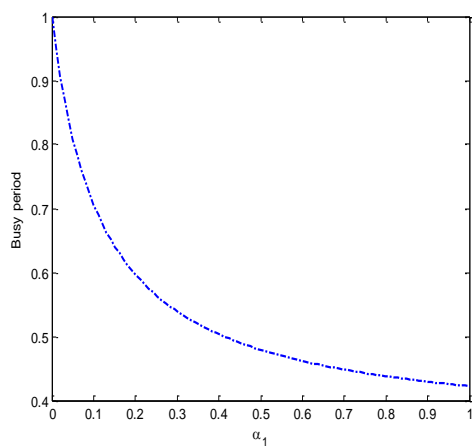


Fig. 10 Effect of α_1 on Busy period

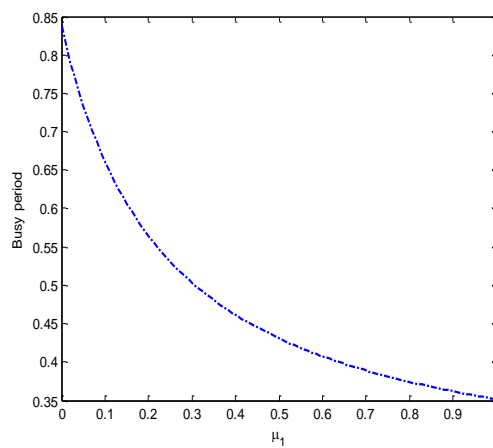


Fig. 11 Effect of μ_1 on Busy period

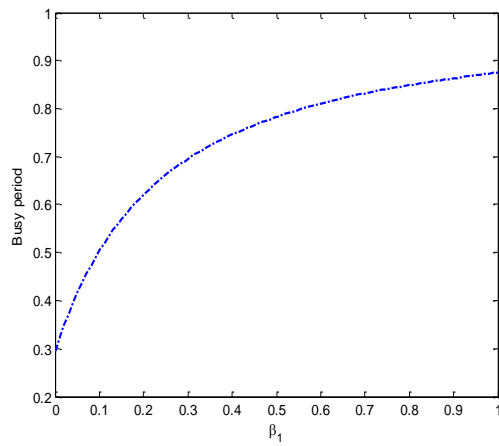


Fig. 12 Effect of β_1 on Busy period

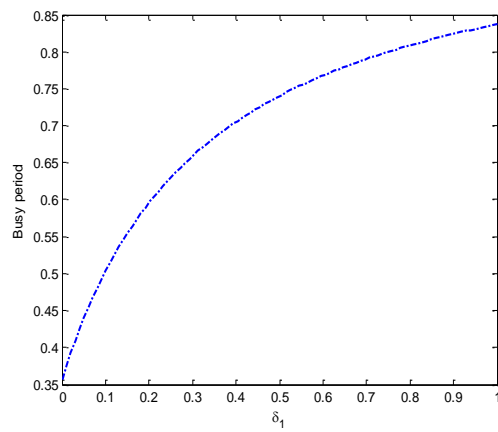


Fig. 13 Effect of δ_1 on Busy period

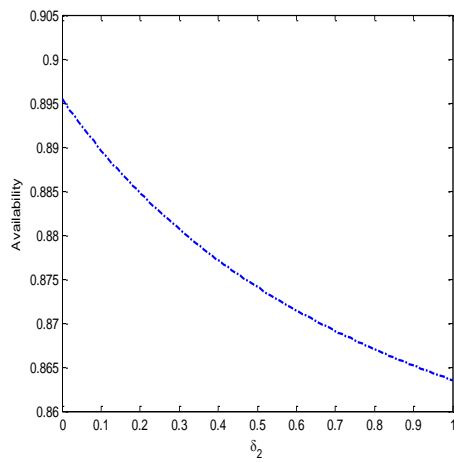


Fig. 14 Effect of δ_2 on availability

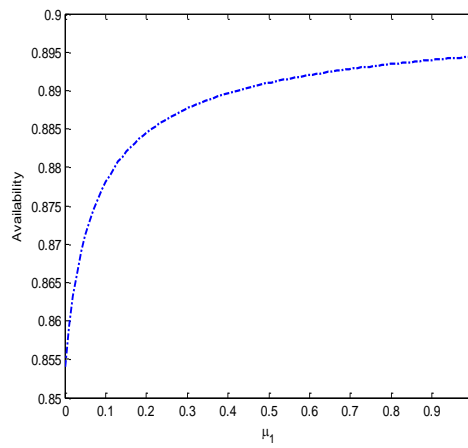


Fig. 15 Effect of μ_1 on availability

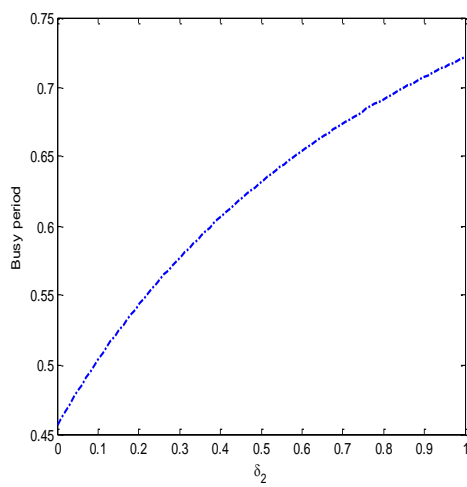


Fig. 16 Effect of δ_2 on Busy period

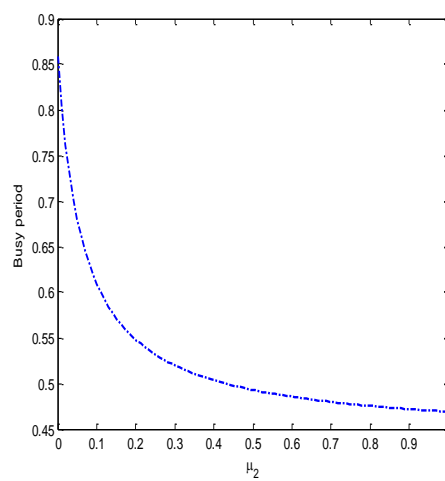


Fig. 17 Effect of μ_2 on Busy period

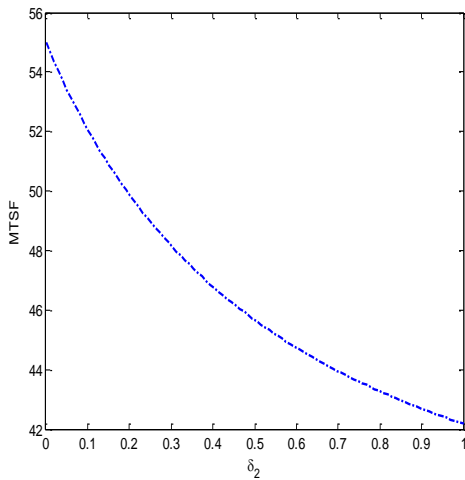


Fig. 18 Effect of δ_2 on MTSF

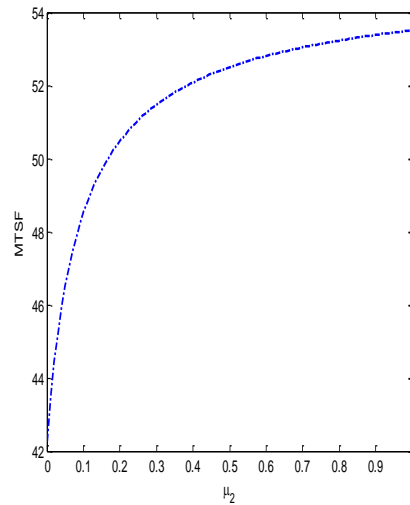


Fig. 19 Effect of μ_2 on MTSF

The impact of δ_1 on MTSF, steady-state availability and busy period can be observed in Figures 2,9 and 13. From Figures 2, and 9, it is evident that the MTSF and steady-state availability decreases as δ_1 increases while in Fig. 13, busy period increases with increase in δ_1 . Similar results can be observed in Figures 14,16 and 18 on steady-state availability, busy period and MTSF with respect to δ_2 . From Figures 14 and 18 steady-state availability and MTSF decreases as δ_2 increases and the busy period increases with increase in δ_2 from Fig. 16. Results of MTSF, steady-state availability and busy period with respect to β_1 are given in Figures 4,8 and 12. It is evident from Figures 4,8 and 12 that as β_1 increases, the MTSF, steady-state availability and busy period decreases while from Figures 5,6 and 10, the MTSF, steady-state availability and busy period increases with increase in α_1 . Furthermore, the impact of μ_1 on MTSF, steady-state availability and busy period can be observed in Figures 3,7 and 11. From Figures 3 and 7, it is evident that MTSF and steady-state availability increases with increase in μ_1 while busy period decreases as μ_1 increase from Fig. 11. Similarly, the effect of μ_2 on steady-state availability, busy period and MTSF can be seen in Figures 15,17 and 19. In Figures 15 and 19, the steady-state availability and MTSF increase as μ_2 increases and the busy period decreases with increase in μ_2 from Fig. 17.

5. Conclusion

In this paper, we constructed a two-stage deteriorating three layers reinforce concrete structure to study the effectiveness of the structure. Explicit expressions MTSF, steady-state availability and busy period were derived. We performed numerical investigation to see the effect of slow deterioration rate, fast deterioration rate, minor minimal maintenance, major minimal maintenance, failure and repair rates on MTSF, steady-state availability and busy period of repairman. It is evident from the results that repair rate, minor minimal maintenance rate and major minimal maintenance rates prolong the life span of the reinforce concrete structure while slow deterioration, fast deterioration and failure rate decrease the life span of the structure.

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