

Mathematical Theory and Modeling ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online) Vol.3, No.14, 2013



Modeling Internally Generated Revenue (IGR) of Local Governments in Nigeria

Unyime Patrick Udoudo^{1*} Emmanuel John Ekpenyong²

*E-mail of the Corresponding author: udoudogeno@gmail.com

Abstract

This paper considers the Time Series Modeling of Internally Generated Revenue of Ikot Ekpene Local Government Area of Akwa Ibom State in Nigeria. Data for this work are monthly internally generated revenue collected from the Budget, Planning, Research and Statistics (BPRS) department of the Local Government from 1998 to 2012. Box and Jenkins method of time series analysis is used to analyse the series. From the analysis, ARIMA (2, 1, 2) is identified as appropriate model for the series and Theil Coefficient indicates that the model is adequate for a good forecast.

Keywords: Time series model, internally generated revenue, Theil coefficient, forecast, Local Government Revenue.

1. **Introduction**

Revenue generation in Nigeria Local Government is principally derived from Tax. Meanwhile tax is a compulsory levy imposed by Government on individuals and companies for the various legitimate function of the State (Olaoye, 2008). Tax is a necessary ingredient for civilization. The history of man has shown that man has to pay tax in one form or the other; that is either in cash or in kind, initially to his chieftains and later in a form of organized government, (Ojo, 2003).

Historically, the development of direct taxation in local government in Nigeria can be traced to the period before the British pre-colonial period. Under this period, the community taxes were levied on communities, (Rabiu, 2004). Recently, the revenue that accrues to local governments is derived from two broad sources – the External and Internal sources. External sources include Statutory allocation from federation account in accordance with the constitution of the Federal Republic of Nigeria. It also includes grants from state and federal governments and other financial institutions. While Internal Sources include rates from local shops and markets, fines, bicycle licence, canoe and wheelbarrow fees, motor garage fees, marriage registration, taxes, etc. Internally generated revenue (IGR) is the revenue that the local government generates within the area of its jurisdiction. The capacity of local government to generate revenue internally is one very crucial consideration for the creation of a local council.

As presently contained in the 1999 constitution, local governments receive 20 percent of the federation account. In addition, proceeds from the Value Added Tax (VAT) are also allocated to them. The 1976 local government reforms state that internally revenue sources of local governments include:

- (a) Rates, which include property rates, education rates and street lighting.
- (b) Taxes such as community, flat rates and poll tax.
- (c) Fines and fees, which include court fines and fees, motor park fees, forest fees, public advertisement fees, market fees, regulated premises fees, registration of births and deaths and licensing fees and
- (d) Miscellaneous sources such as rent on council estates, royalties, interest on investment and proceeds from commercial activities.

According to Magaji (1994), the above stated are sources through which local governments generate their revenues through their own efforts.

^{1*} Department of Statistics, Akwa Ibom State Polytechnic, Ikot Osurua, Akwa Ibom State, Nigeria

² Department of Statistics, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria.



The modern States are confronted with more political, economic and social problems than ever before. First is the issue of demographic explosion, particularly in the developing countries. For instance, the population of Nigeria, which stood at about 56 million in 1963 rose to about 80 million in 1991 and the current estimation has projected the population in the country to be about 160 million people (Ebert, 1997).

According to Adedokun (2011) stressed that Local government growth has been found not to be too effective in Nigeria and ineffectiveness has been traced to many factors, which include low funding from the central government. He further said that these problems led the local governments into a vicious circle of poverty, because inadequate functions and powers lead to inadequate funding, which results in the employment of low skilled and poorly paid staff.

Moreover, due to maladministration and employment of low skilled staff in the local government, internal revenue generation has suffered a serious set-back. Revenue in the local governments is seriously undergenerated. This is as a result of the inability of the local government to have knowledge or understanding of the behaviour and characteristics of internally generated revenue through adequate modeling. The researchers therefore seek to proffer solution to revenue generation through modeling of the revenue series of Ikot Ekpene Local Government Area so as to be able to forecast and plan for the future, thereby improving the internally generated revenue of the local government. This is in line with the work of Adesoji and Chike (2013).

Busari, et al (2013) have tried to proffer a regression model for internally generated revenue of twenty local government Areas of Lagos State in Nigeria. Howard (1990) used moving average model and exponential smoothing to forecast revenue of eight cities in Florida, USA. Fullerton (1989) also used a Composite approach to model state government revenue in Idaho. We therefore seek to consider modeling internally generated revenue of Ikot Ekpene Local government using classical Box and Jenkins approach.

2. Method of Analysis

The main methods of analysis for this work are to identify the most appropriate stationary process to be fitted to the data and to use a statistical package to fit the model.

2.1: Autoregressive Models [AR(p)]

The general model of an AR(p) is given by;

$$X_{t} = \phi_{1}X_{t-1} + \phi_{2}X_{t-2} + \dots + \phi_{p}X_{t-p} + e_{t}$$

$$= \sum_{i=1}^{p} \phi_{i} X_{t-i} + e_{t}$$
(1)

where X_t = monthly internally generated revenue

 ϕ_p are the autoregressive model parameters

 X_{t-i} are prior observations and ℓ_t is a purely random process

If
$$p = 2, X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + e_t$$
 (2)

(2) is called Autoregressive model of order two [AR(2)]

If
$$p = 1, X_t = \phi_1 X_{t-1} + e_t$$
 (3)

(3) is termed autoregressive model of order one [AR(1)]



An assessment of the autocorrelation and partial autocorrelation functions would help in determining whether an AR(p) model is to be fitted to the data.

2.2. Moving Average Model [MA(q)]

The general model of a Moving Average with order q is given by:

$$X_{t} = e_{t} - \theta_{1}e_{t-1} - \theta_{2}e_{t-2} - \dots - \theta_{a}e_{t-a}$$
 (4)

$$= e_1 - \sum_{i=1}^q \theta_i e_{t-i} \tag{5}$$

where X_t is the series

 θ_i are Moving Average parameters

 e_{t-i} prior random shocks.

If
$$q = 2$$
, $X_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$ (6)

Equation (6) is called a Moving Average model of order two [MA(2)]

If
$$q = 1$$
, $X_t = e_t - \theta_1 e_{t-1}$ (7)

Equation (7) is termed a Moving Average model of order one [MA(1)]

2.3: Autoregressive Integrated Moving Average Model (ARIMA)

The general model for ARIMA (p, d, q) is given by:

$$B(1-B)^{d} X_{t} = b(B) e_{t}$$
 (8)

where B is the shift operator and d is the dth difference.

2.4: Model Identification

Models are identified through the study of the behaviour of autocorrelation and partial autocorrelation functions. The table below gives the summary of the identification

Table 1: Identification of models

	AR PROCESS	MA PROCESS	ARMA PROCESS
Autocorrelation	Infinite (damped exponential and/or damped sine waves). Tails off	Finite. Cuts off	Infinite (damped exponential and/or damped sine wave after first q-p lags). Tails off
Partial autocorrelation function	Finite. Cuts off	Infinite (damped exponential and/or damped sine wave). Tails off	Infinite (damped exponential and/or damped sine waves after first p-q lags). Tails off.



2.5: The Backward Shift Operator

The backward shift operator, B is defined by $BX_t = X_{t-1}$. Hence

$$B^m X_t = X_{t-m} (9)$$

For instance an AR(p) model can be expressed as:

$$(1 - \phi_1 B - \phi_2 B^2 + \dots + \phi_p B^p) X_t = e_t$$
(10)

2.6: Forward Shift Operator, F

The operator is performed by the forward shift operator $F = B^{-1}$ and is given by $Fx_t = X_{t+1}$. Hence

$$F^m X_t = X_{t+m} ag{11}$$

2.7: Backward Difference Operator, ∇

Another important operator that is relevant to this work is the backward difference operator, ∇ , which can be written in terms of B, since $\nabla X_t = X_t - X_{t-1} = (1-B)X_t$

Hence

$$\nabla^m X_t = (1 - B)^m X_t \tag{12}$$

2.8: Non-Stationary Series and Models

If a time series contains trend, the values of r_k will not come down to zero except for very large values of the lag. This is because an observation on one side of the overall mean tends to be followed by a large number of further observations on the same side of the mean because of the trend.

Little can be inferred from a correlogram of a non-stationary time series as trend dominates all other features. In fact the sample autocorrelation function, $\{r_k\}$, should only be calculated for stationary time series and so any trend should be removed before calculating $\{r_k\}$.

Many experimental time series behave as though they had no fixed mean. Even so, they exhibit homogeneity in the sense that apart from local level, or perhaps local level and trend, one part of the series behaves much like any other part. Models which describe such homogeneous non-stationarity behaviour can be obtained by supposing some suitable difference of the process to be stationary.

Consider the ARMA model given below:

$$\phi(B) X_t = \theta(B) e_t \tag{13}$$

With

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p$$

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_a B^q$$

To ensure stationarity, the roots of $\Phi(B) = 0$ must lie outside the unit circle, and exhibit explosive non-stationarity if the roots lie inside the unit circle. The only other case open to us is that for which the roots of $\Phi(B) = 0$ lie on the unit circle. It turns out that the resulting models are of great value in representing homogeneous non-stationary time series.



Consider the model

$$\varphi(B)X_t = \theta(B) e_t$$

where $\varphi(B)$ is a nonstationary autoregressive operator, such that d of the roots of $\varphi(B) = 0$ are unity and the remainder lie outside the unit circle. Then we can express the model (3.14) in the form

$$\varphi(B) X_{t} = \phi(B) (1 - B)^{d} X_{t} = \theta(B)e_{t}$$
(14)

where $\varphi(B)$ is a stationary autoregressive operator.

Equivalently, the process (3.16) is defined by the two equations

$$\phi(B) W_t = \theta(B)e_t$$
 and

$$W_t = (I - B)^d X_t = \nabla^d X_t \tag{15}$$

Thus, we see that the model corresponds to assuming that the dth difference of the series can be represented by a stationary, invertible ARMA process.

2.9: Differencing

A special type of filtering which is particularly useful for removing a trend is simply to difference a given time series until it becomes stationary. This method is particularly stressed by Box and Jenkins (1976).

For non-seasonal data, first order differencing is usually sufficient to obtain apparent stationarity so that the new series $\{Y_1, Y_2, ..., Y_{n-1}\}$ is formed from the original series $\{X_1, X_2, ..., X_n\}$ by

$$Y_t = \nabla X_t = X_t - X_{t-1} \tag{16}$$

For instance if X_t has a linear trend, then ∇X_t does not.

Occasionally, second – order differencing is required using the operator ∇^2 , where

 $\nabla^2 X_t = (I - B)^2 X_t = X_t - 2X_{t-1} + X_{t-2}$ which removes quadratic trends. Generally, the operator

$$\nabla^{\mathbf{p}} = (1 - B)^{\mathbf{p}} \tag{17}$$

removes a polynomial trend of order p.

2.10: Test for Adequacy of the Model

Theil coefficient would be used to test for the adequacy of the estimated model. A statistical package – MINITAB would be used to carry out the analysis. Theil coefficient U is given by:

$$U = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_{t} - \hat{X}_{t})^{2}}}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}X_{t}^{2} + \sqrt{\frac{1}{n}\sum_{i=1}^{n}\hat{X}_{t}^{2}}}}$$
(18)

If U is closer to zero, then the estimates are adequate, otherwise they are not adequate.



3. **Data Analysis**

A clear insight on the plot of the original internally generated revenue and its autocorrelation function show that the series is not stationary, because the autocorrelation function has large values. Hence, there is need for differencing.

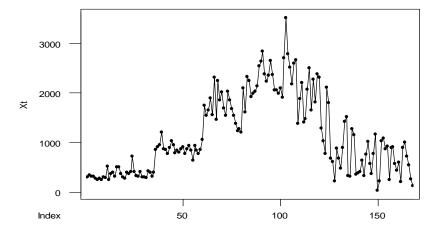
1.0 0.8 0.6 Autocorrelation 0.4 0.2 0.0 -0.2 -0.4 -0.6 -0.8 -1.0 2 12 22 32 42 LBQ Lag 1683.92 0.86 11.11 125.76 0.60 2.12 1146.08 0.30 0.90 1623.46 37 -0.07 -0.19 13 25 0.78 6.44 230.56 14 0.57 1.99 1207.15 26 0.27 0.80 1637.84 38 -0.07 -0.22 1685.15 1687.65 0.77 5.21 333.90 15 0.56 1.88 1264.82 27 0.25 0.75 1650.91 39 -0.11 -0.31 0.78 4.57 439.64 16 0.53 1.75 1317.23 28 0.22 0.64 1660.51 40 -0.14 -0.40 1691.72 540.86 1669.84 1696.32 0.76 3.99 17 0.52 1.70 1368.73 29 0.21 0.63 41 -0.14 -0.420.70 3.35 626.30 18 0.48 1.54 1412.64 30 0.19 0.55 1677.06 -0.16 -0.47 1702.09 710.54 0.47 1680.10 0.69 3.11 19 1.47 1454.21 31 0.12 0.35 0.68 2.92 793.87 20 0.45 1.41 1493.84 32 0.07 0.21 1681.22 1681.77 0.66 2.68 871.39 21 0.43 1.32 1529.22 33 0.05 0.15 10 0.63 22 0.40 0.05 0.15 1682.31 2.45 942.10 1.21 1560.04 34 11 0.61 2.30 1009.46 23 0.35 -0.01 -0.03 1682.33 1.06 1584.42 35

Autocorrelation Function for Xt

Figure 1: Autocorrelation Function of Actual Revenue series

1605.54

0.33



1683.00

Figure 2: Plot of Original Revenue series

1080.45

0.62

2.28

We difference the set of data once and observe the behaviours of the autocorrelation and partial autocorrelation functions to determine the model to be fitted to the data.



Autocorrelation Function for Xt-1

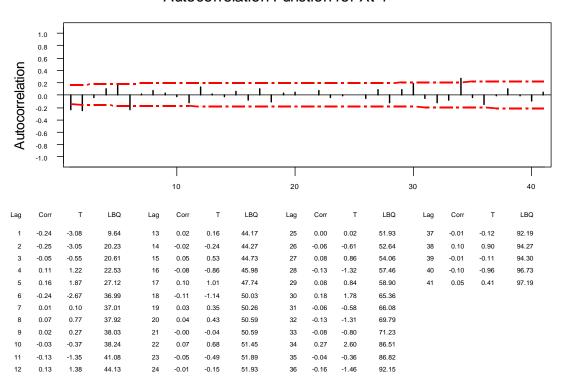


Figure3: Autocorrelation function of the Differenced Series

Partial Autocorrelation Function for Xt-1 1.0 0.8 0.6 0.4 0.2 0.0 -0.2 -0.4 -0.6 -0.8 Partial Autocorrelation 10 20 30 40 PAC PAC PAC PAC Lag Lag Lag Lag 13 25 0.03 37 -0.24 -3.08 -0.02 -0.22 0.33 -0.04 -0.51 -0.32 -4.19 0.14 -0.99 0.04 0.46 14 0.01 -0.08 15 -1.35 -0.09 -1.13 16 -0.02 -0.26 28 -0.15 -1.93 -0.02 -0.30 0.12 1.58 17 0.07 0.91 29 0.01 0.06 0.06 0.79 -2.00 18 -0.97 30 2.87 -0.15 -0.08 0.22 -0.02 19 0.01 31 0.16 2.07 -0.26 0.08 -0.01 -0.18 21 0.04 0.58 -0.06 -0.78 10 -0.02 -0.30 22 0.09 1.21 34 0.09 1.20

Figure 4: Partial autocorrelation Function of the differenced series

0.10

1.24

23

11

-0.10

0.01

-1.33

From the figures 1 and 2, ARIMA (2, 1, 2) is suggested to be fitted to the data. The model is given as:

35

-0.03

-0.40



$$(1 - \phi_1 B - \phi_2 B^2) \nabla X_t = (1 - \theta_1 B - \theta_2 B^2) \nabla e_t$$

$$\Rightarrow (1 - 0.304801B + 0.33799B^2) \nabla X_t = (1 - 0.712383B + 0.176146B^2) \nabla e_t$$
(19)

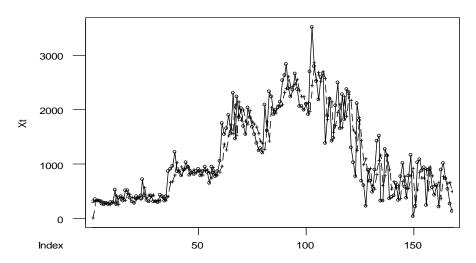
$$\Rightarrow X_{t} - 1.304801X_{t-1} + 0.642791X_{t-2} - 0.33799X_{t-3} = e_{t} - 1.712383e_{t-1} + 0.888529e_{t-2} - 0.176146e_{t-3}$$
(20)

Theil coefficient, U is therefore used to determine the adequacy of the model. Therefore,

$$\frac{1}{n}\sum_{i=1}^{n}(X_{t}-\hat{X_{t}})^{2} = 128952 \qquad \frac{1}{n}\sum_{i=1}^{n}X_{t}^{2} = 1925158 \qquad \frac{1}{n}\sum_{i=1}^{n}\hat{X_{t}}^{2} = 1853198$$

$$U = \frac{\sqrt{128952}}{\sqrt{1287450 + \sqrt{1257450}}} = 0.13$$

Since U is farther away from 1, but closer to zero, then the model is adequate. Therefore, the plot of the original Revenue and Estimated Revenue series is given as:



ACTUALS IN CIRCLE ESTIMATES IN PLUS

Figure 5: Plot of the Actual and Estimated Revenue series

4. Conclusion

We have been able to fit an appropriate model to the internally generated revenue of Ikot Ekpene Local Government Area of Akwa Ibom State. This model is Autoregressive Integrated Moving Average model [ARIMA(2, 1, 2)]. This model can then be used to forecast future values of the revenue. Moreover, knowing implementation of such revenue. In summary, the model fits well to the data.

References

Adedokun, A. A (2011): The Development of Local Government in Nigeria since pre-colonial Era to 1999 constitution. *Polycom* 2(2).

Adedeji, A.(2006): Nigeria Federal Finance. African Publishing Company, New York.



Adesoji, A.A and Chike, F.O. (2013): The Effect of Internal Revenue Generation on infrastructural Development. A study of Lagos State Internal Revenue Service. *Journal of Education and Social Research*, 8(2), 419-436.

Box,G.E.P and Jenkin,G.M.(1997):Time Series Analysis: Forecasting and Control. San Francisco:Holden-Day Publishers.

Busari, G.A, Akinbo R.O; and Adeboye, N.O. (2013): Impact of Internally Generated Revenue (IGR) on the growth of Local Government in Nigeria. *Indian Journal of Research, PARIPEX*; 2(11), 267-268.

Fullerton, T. M. (1989): A Composite Approach to Forecasting State Government Revenues: Case of the Idako Sales Tax. *International Journal of Forecasting*, *5*, *373-380*.

Howard, A. F. (1990): Municipal Revenue Forecasting with Time Series Models: A Florida Case Study. *The American Review of Public Administration*, 20(1), 45-59.

Magaji, Y. (1994): Revenue Generation and Collection in markets: An unpublished Project submitted to the Department of Local Government Studies, Ahmadu Bello University, Zaria.

Nigeria Local Government Bye laws: Local Government Bye laws published in 1991.

Ola, C.S (1988): Boosting Revenue Generation by State Governments in Nigeria. Strategies for enhanced Tax Revenue in Nigeria, 8(4).

Olaoye, C.O, (2008): Concepts of Taxation Nigeria. Clement Publishing, Ijagbo/Ofa, Kwara State.

Olusola, O.O. (2011): Boosting 'Internally Generated Revenue of Local Governments in Ogun State, Nigeria: European Journal of Humanities and Social Sciences, 8(1), 339-348.

Ojo, S. (2003): Principles of Nigerian Tax. Sagriba Publishing, Illorin.

Rabiu, S.A, (2004): Personal Income Tax in Nigeria: Procedures and Problem. 1st Edition. Adebanyo Publishers Limited. Lagos.

This academic article was published by The International Institute for Science, Technology and Education (IISTE). The IISTE is a pioneer in the Open Access Publishing service based in the U.S. and Europe. The aim of the institute is Accelerating Global Knowledge Sharing.

More information about the publisher can be found in the IISTE's homepage: http://www.iiste.org

CALL FOR JOURNAL PAPERS

The IISTE is currently hosting more than 30 peer-reviewed academic journals and collaborating with academic institutions around the world. There's no deadline for submission. Prospective authors of IISTE journals can find the submission instruction on the following page: http://www.iiste.org/journals/ The IISTE editorial team promises to the review and publish all the qualified submissions in a fast manner. All the journals articles are available online to the readers all over the world without financial, legal, or technical barriers other than those inseparable from gaining access to the internet itself. Printed version of the journals is also available upon request of readers and authors.

MORE RESOURCES

Book publication information: http://www.iiste.org/book/

Recent conferences: http://www.iiste.org/conference/

IISTE Knowledge Sharing Partners

EBSCO, Index Copernicus, Ulrich's Periodicals Directory, JournalTOCS, PKP Open Archives Harvester, Bielefeld Academic Search Engine, Elektronische Zeitschriftenbibliothek EZB, Open J-Gate, OCLC WorldCat, Universe Digtial Library, NewJour, Google Scholar

























