Prediction of Carry- over Coefficient for Fluid Flow through Teeth on Rotor Labyrinth Seals Using Computational Fluid Dynamics

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Abstract
The analysis presented in this work deals with the computational fluid dynamics (CFD) based modeling for the carry-over coefficient of flow through teeth on rotor straight through labyrinth seal. A two dimensional model for the flow through the seal was built in FLUENT 6.3.26. A grid independence study was carried out showing the influence of grid refinement on the fluid mass flow rate. The effect of turbulence modeling, seal geometry and shaft rotational speed have been taken into consideration. Three CFD models to predict the carry-over coefficient considering the effect of the parameters mentioned before have been presented. The prediction models have been validated against experimental works conducted by Scharrer (1988) and Eser (2004). The results obtained in the present work for the effect of different parameters, namely, (Number of teeth, Reynolds number, rotational speed and seal geometry) for two labyrinth seals with (0.127mm) and (0.33mm) tooth clearance have been presented. A prediction equation for the carry-over coefficient of the labyrinth seal has been proposed through this work. Verification for some of the results obtained through this work with that published by other workers has been carried out. The results obtained through the present work found to be in a good agreement with the results of other workers.
Key words: Labyrinth seal, Carry-over coefficient, CFD.

1-Introduction

The main function of the labyrinth seals is to reduce the leakage flow between components of different pressure and prevent the rotor from contacting the stator at a very high speed, because any contact between the rotor and stator causes seal damage, deterioration of seal performance and engine failure. Labyrinth seals have proved to be quite effective and long-lived as evidenced by their widespread use, chiefly on rotating machinery such as gas compressors and turbines. The straight type labyrinth seal is the most used in turbomachinery because of its greater kinetic energy carry over in comparison with other types of seals. Most previous works show that the modeling of seals was mainly based on the bulk flow model (Child, D.W and Scharrer, J. (1986), Esser, D., Kazakia, J.Y., (1995), Yilmaz, D. and Esser, D., (2004), Gamal, A.M., (2007)).The main drawback of this model, due to the simplifying assumptions, is that sometimes it fails to predict the flow of labyrinth seal. Due to the bulk flow model limitation, its claimed by Pugachev, (2012) that the computational fluid dynamics (CFD) method for solving Navier- Stokes equations were applied to obtain more satisfactory predictions for various seal types at different boundary and operating conditions. The previous studies on the application of (CFD) methods focus on modeling the fluid flow through labyrinth seals of small clearances with teeth on the stator, Saikishan and Morrison (2009), Anand and Woo (2011) and Sunil,(2011) (2010). Vamish (2011) studied the effect of shaft rotation on flow parameters without modeling carry-over coefficient. A model for labyrinth seal with large clearance has been developed. In the present work a (CFD) model for the carry-over coefficient of teeth on rotor straight labyrinth seal has been developed in order to propose a numerical correlation for the carry over coefficient of the seal. Effects of flow parameters and seal geometry with and without rotational speed effect, for compressible and incompressible fluid flow on carry-over coefficient of labyrinth seal have been investigated.

2. Mathematical Modeling

2-1 Labyrinth Seal Geometry and Coordinates.

The analysis presented here is applicable to straight labyrinth seal with teeth on the rotor. Figure (1) shows the geometry and coordinates system for a single cavity with two teeth on rotor labyrinth seal and axisymmetric flow (i.e., two dimensional flow simulation in radial and axial direction).

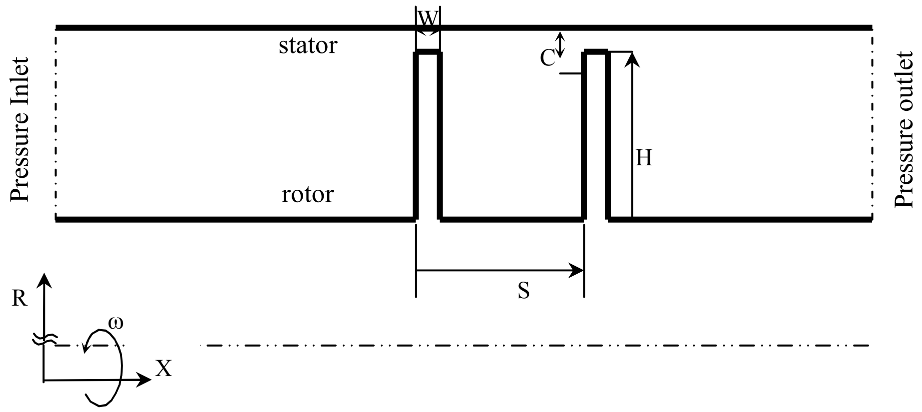


Fig.(1) Seal geometry and coordinates system.

W is the tooth width (mm). C is the clearance (mm). H is the tooth height (mm), S is the tooth pitch (mm)

2-2 Governing Equations.

The main governing equations used to model the problem of the present work are:

Continuity Equation:

The continuity equation for incompressible fluid flow can be written as**(Chain Nan .Y (1988))**:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial r} + \frac{V}{r} = 0 \qquad (1)$$

For compressible fluid flow the effect of compressibility must be considered; hence, the continuity equation can be rewritten as **(Chain Nan .Y (1988))**:

$$\frac{\partial(\rho U)}{\partial x} + \frac{\partial \rho V}{\partial r} + \frac{\rho V}{r} = 0 \qquad (2)$$

Momentum equations

The momentum equation for 2-D axisymmetric geometry in axial direction can be written as:**(Chain Nan .Y (1988))**

$$\frac{\partial}{\partial x}(\rho UU) + \frac{1}{r}\frac{\partial}{\partial r}(\rho VrU) - \frac{\partial}{\partial x}\mu\frac{\partial U}{\partial x} - \frac{1}{r}\frac{\partial}{\partial r}r\mu\frac{\partial U}{\partial r} = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}\mu\frac{\partial U}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}r\mu\frac{\partial V}{\partial X} - \frac{1}{r}\frac{\partial}{\partial x}(r\mu\frac{2}{3}(\nabla.\vec{v})) \qquad (3)$$

While the momentum equation in radial direction can be written as**:-**

$$\frac{\partial}{\partial x}(\rho UV) + \frac{\partial}{r\partial r}(\rho VrV) - \frac{\partial}{\partial x}\mu\frac{\partial V}{\partial x} - \frac{\partial}{r\partial r}r\mu\frac{\partial V}{\partial r} = -\frac{\partial P}{\partial r} + \frac{\partial}{\partial x}\mu\frac{\partial U}{\partial r} + \frac{\partial}{r\partial r}r\mu\frac{\partial V}{\partial r} + \frac{\rho W^2}{2} - \frac{2\mu V}{r^2} - \frac{1}{r}\frac{\partial}{\partial r}(r\mu\frac{2}{3}(\nabla.\vec{v})) \qquad (4)$$

The governing equations of fluid flow for a steadily rotating frame can be written as follows:

Conservation of mass:

$$\nabla.\rho\overrightarrow{v_r} = 0 \qquad (5)$$

Conservation of momentum:

$$\nabla.(\rho\overrightarrow{v_r}\,\vec{v}) + \rho(\vec{\omega} * \vec{v}) = -\nabla P + \nabla\overline{T_{eff}} + \vec{F} \qquad (6)$$

The fluid velocities can be transformed from the stationary frame (stator) to the rotating frame (rotor) using the following relations:

$$\overrightarrow{v_r} = \vec{v} - \overrightarrow{u_r} \qquad (7)$$

$$\overrightarrow{u_r} = \vec{\omega} * \vec{r} \qquad (8)$$

Where:

$\vec{\omega}$ = angular velocity relative to a stationary reference frame (rpm).

$\overrightarrow{v_r}$ = the velocity viewed from the rotating frame (relative velocity).

$\vec{v}$ = the velocity viewed from the stationary frame (absolute velocity) .

$\overrightarrow{u_r}$ = velocity due to the moving frame (m/sec).

$\vec{r}$ = position vector from the origin of rotating frame (rotor) to the axis of rotation.

Turbulence model

The standard k- ε turbulence model is used. The turbulence kinetic energy, *k*, and the specific dissipation energy, ε, are obtained from the following transport equations [**Vamish (2011)**]:

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_i}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + G_k + G_b - \rho\epsilon - Y_M + S_K \qquad (9)$$

$$\frac{\partial}{\partial t}(\rho\varepsilon) + \frac{\partial}{\partial x_i}(\rho\varepsilon u_i) = \frac{\partial}{\partial x_i}\left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + C_{1\varepsilon}\frac{\varepsilon}{k}(G_k + C_{3\varepsilon}G_b) - C_{2\varepsilon}\frac{\varepsilon^2}{K} + S_\varepsilon \qquad (10)$$

Where

G_k is the production of κ and is modeled as $G_k = -\rho\overline{\grave{u}_i\grave{u}_j}\,\frac{\partial u_j}{\partial x_i}$

G_b represents generation of κ due to buoyancy.

Y_M represents compressibility effects on turbulence and is modeled as $Y_M = 2\rho\epsilon\frac{k}{KRT}$.

S_K and S_ε are user-defined source terms.

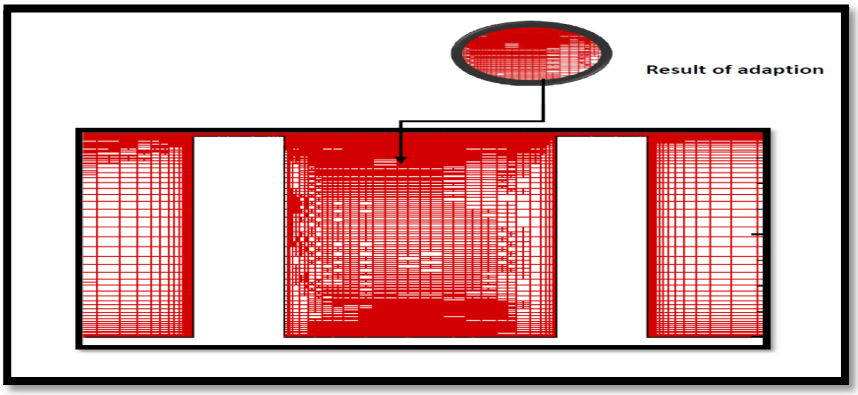
σ_k and σ_ε are the turbulent Prandtl numbers for k and ε, respectively, and have default values of 1.0 and 1.3 respectively.

$C_{1\varepsilon}$ and $C_{2\varepsilon}$ are constants with default values of 1.44 and 1.92.

2.3 Solution procedure

Grid generation

Initially the seal geometry is meshed by using **GAMBIT 2.3.16**. A long entrance and exit regions before and after the seal are considered to insure an equilibrium condition before the flow enters the cavity. The mesh has been adapted with maximum pressure gradient set to **0.1**, near the rotor surface and the stator wall to insure that the $\mathbf{Y^+}$ values are near or less than **5** to resolve the laminar sub-layer as shown in **Fig.(2)**.



Fig(2). Computational mesh.

A grid independence was obtained by observing the mass flow rate predicted for a given pressure ratio with various number of grids. It can be shown from fig.(3) that the change in mass flow rate was less than **0.5%** as the number of grids increase more than **20000**

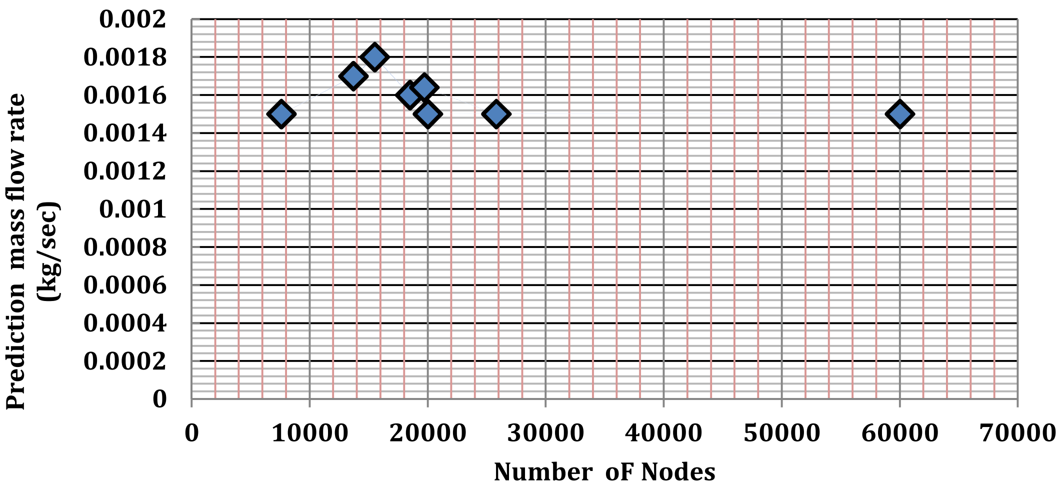


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Figure (3). Accuracy of mass flow rate prediction against number of nodes.

Equations from 1to 8 are solved by **TDM** with residual for each property equal to (10^{-4}) using different under relaxation factor from 0.1 to 0.8.

Computational Method

The above model is numerically solved in order to calculate the effect of flow and geometric parameters on the carry over coefficient. The solution is based on the finite volume method by using the commercial (CFD) code **FLUENT 6.3.26** .The continuity and momentum equations have been discretized using standard κ-ε turbulence model which has been proved to be accurate for modeling the flow through seals with and without rotational effect(**Morrison and Al-Ghasem, 2007**) .

Carry over Coefficient

The carry over coefficient can be defined as the portion of the undissipated kinetic energy of the fluid exiting at the seal clearance that is converted to the next cavity **(Woo, 2011)**. The carry over coefficient, **γ**, and the percentage of the kinetic energy carried over into the next cavity, **χ**, are defined by **(Hodkinson, 1939)** as follows:

$$\gamma^2 = \frac{1}{1-\chi} \tag{11}$$

Where:

$$\chi = \frac{c}{c+s\tan\beta} \tag{12}$$

Where **β** is the divergence angle defined by noting the position on the downstream tooth where the radial velocity is zero **Saikishan** and **Morrison (2009)**, **Jeng, (2011)**, and **Sunil,(2011).** The exact position needs to be accurately determined for accurate determination of **(β).** Hence **Tecplot 360** software has been used for this purpose after suitable magnification and limiting the range of measurement. The probe tool was used to detect and extract regions of least radial velocity which correspond to stagnation points in the cavity. Referring to **Fig.(4)** the angle **(β)** can be calculated as follows:

$$\beta = \tan^{-1}\left(\frac{I_2}{I_1}\right) \tag{13}$$

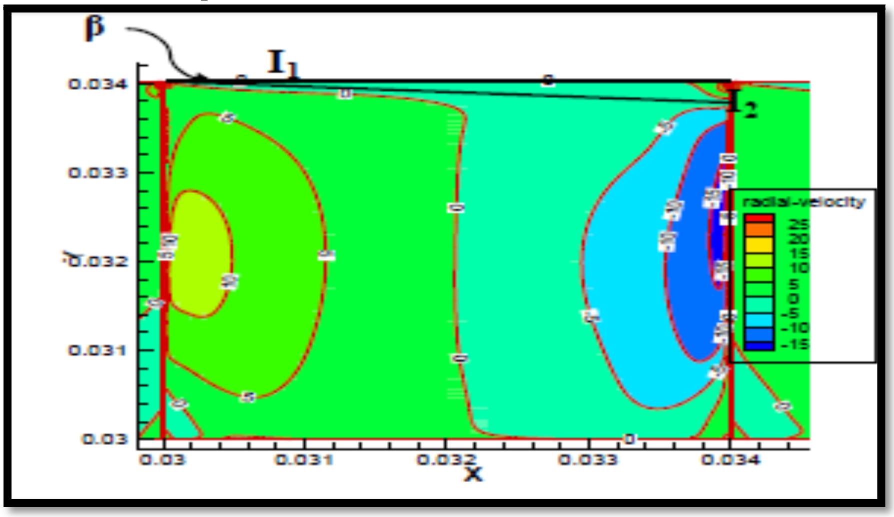


Fig.(4):Shows The Contour Plots of The Radial Velocity and Measurement of β.

The above definition of **γ** assumes one major recirculation zone in the cavity. This assumption is valid until rotor shaft speeds impart significant tangential velocities which produce a body force and a secondary recirculation zone which lead to an invalid definition of **γ.** The carry over coefficient cannot be less than one according to this definition. Since it is difficult to define **γ** when there is no dividing streamline that separates the flow in the cavity from the flow above tooth, the carry over coefficient **(γ)** is assumed to be unity when stream line is carried out into the cavity due to centrifugal effects.

3. Results and Discussions

In the present work a number of simulations, as presented in table (1), are performed with fixed seal geometry and varying flow conditions. The carry over coefficient is calculated for each case. The effects of different flow parameters (Reynolds number, pressure ratio,…etc) on kinetic energy carry over are studied. Also, the effect of shaft rotation on the carry over coefficient is studied by respective simulations. The initial cases considered are for a single cavity (two teeth) seal. While a seal with multiple cavities will be considered later. The seal geometry has been fixed so as to consider only the effect of flow parameters. The initial analysis utilizes water to eliminate compressibility effects and considers a stationary shaft. Results obtained are compared with that obtained for labyrinth seal with air flows operating under the same conditions. The Reynolds number based on the seal clearance can be evaluated as in **(Dereli and Eser (2004))**.

$$Re = \frac{2\,\dot{m}}{\pi\, D_m \mu} \qquad (14)$$

Where:
D_m = (D+2B+C) .
D = Shaft diameter (m).
B = Tooth height (mm).
C = Clearance between stator and rotor (mm).

$$\mu = \text{Dynamic viscosity of fluid} \begin{Bmatrix} = 0.001003\ N.\frac{s}{m^2} & for\ water \\ = 1.7894\times10^{-5}\ N.\frac{s}{m^2} & for\ air \end{Bmatrix}$$

The effect of different flow parameters affecting the carry-over coefficient of straight through labyrinth seals working with incompressible fluid flow have been discussed as follows:

3.1. Effect of Reynolds number

A prediction model for carry over coefficient of incompressible fluid, needs a study to the relation between the carry over coefficient and Reynolds number at different exit pressures. Fig (5) shows such study for seal geometry case (1) in table (1).

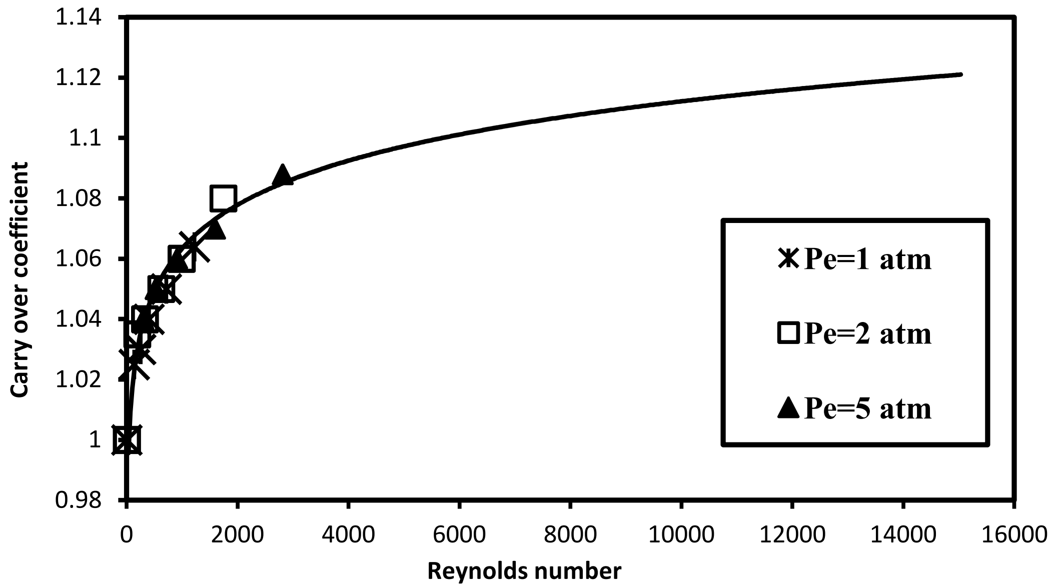


Figure (5). Carry over coefficient vs. Reynolds Number for water at different exit pressures for case (1) in Table (1).

The effect of the Reynolds number on the carry over coefficient is effectively modeled using a power law curve fit of the following form

$$\gamma = C_1[\,Re + C_1^{\frac{-1}{c_2}}\,]^{C_2} \qquad (15)$$

Where C_1 and C_2 are possibly functions of seal geometry and other flow parameters yet to be determined. The presence of $C_1^{\frac{-1}{c_2}}$ in the model ensures that γ becomes 1 when the Reynolds number is 0. This is essential as the minimum value of γ is 1. For this baseline geometry (case 1 in table (1)), C_1=0.9322 and C_2=0.019.

3.2. Effect of seal geometry

Effect of the following main geometrical parameters on carry over coefficients has been studied as follows:

3.2.1. Effect of clearance

Seal clearance represents one of the major geometric parameter affecting the carry over coefficient with the change of Reynolds number. Hence the constants C_1 and C_2 depend on the clearance as shown in Figure (6).

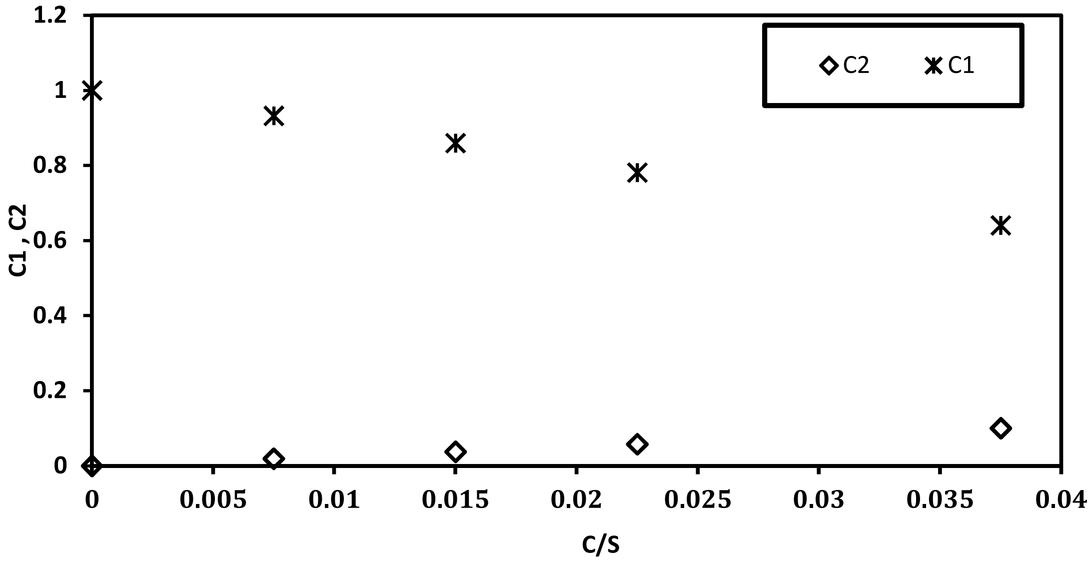


Figure (6). Variation of C1 and C2 with c/s ratio.

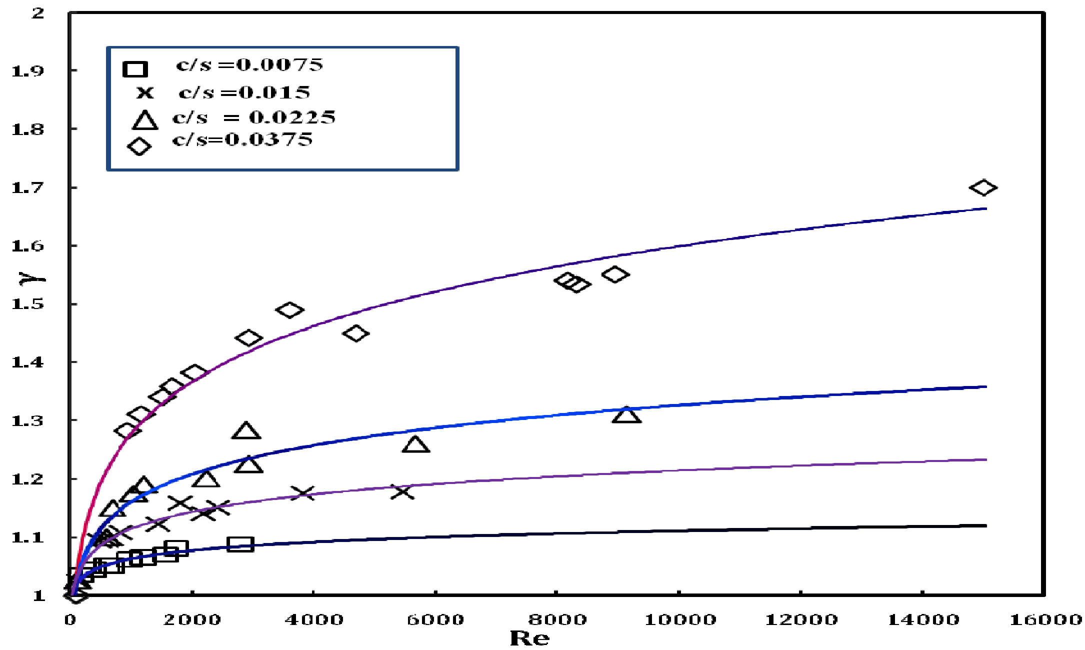


Figure (7). Reynolds number VS. carry over coefficient for cases (1- 4) of Table (1)

It can also be seen from figure (7) that the carry over coefficient increases as the seal clearance increases for the same Reynolds number. This is can be attributed to the higher inertial force of the jet. The lower viscous force at the tip of the tooth causes smaller divergence angle of the jet. A large portion of the jet is travelling the cavity and passing above the downstream tooth. A relatively smaller portion of the kinetic energy is dissipated in the cavity producing a higher carry- over coefficient. In order to access the effect of c/s ratio, seals with four different clearances are simulated for a range of Reynolds numbers with a fixed pitch corresponding to the cases (1 to 4) presented in Table (1). The power law given by equation (15) is applied to each data series and constants C_1 and C_2 are found for the best fit.

It is well known that the carry over coefficient approaches 1 as Re approaches 0 according to equation (12). In order to satisfy this, C_1 must be equal to (1) while C_2 should be equal to zero at c/s = 0 (as zero clearance implies no flow and zero Re). Considering the above observation, C_1 and C_2 in equation (15) are expressed in the form:

$$C_1 = 1 - R_1\left[\frac{C}{S}\right] \quad (16)$$

and

$$C_2=R_2\left[\frac{C}{S}\right] \tag{17}$$

The constants R_1 and R_2 are determined as 9.67 and 2.6277 for the RMS of relative error which equals 1.08%. Substituting equations (16) and (17) in equation (15), leads to the following model for the carry over coefficient:

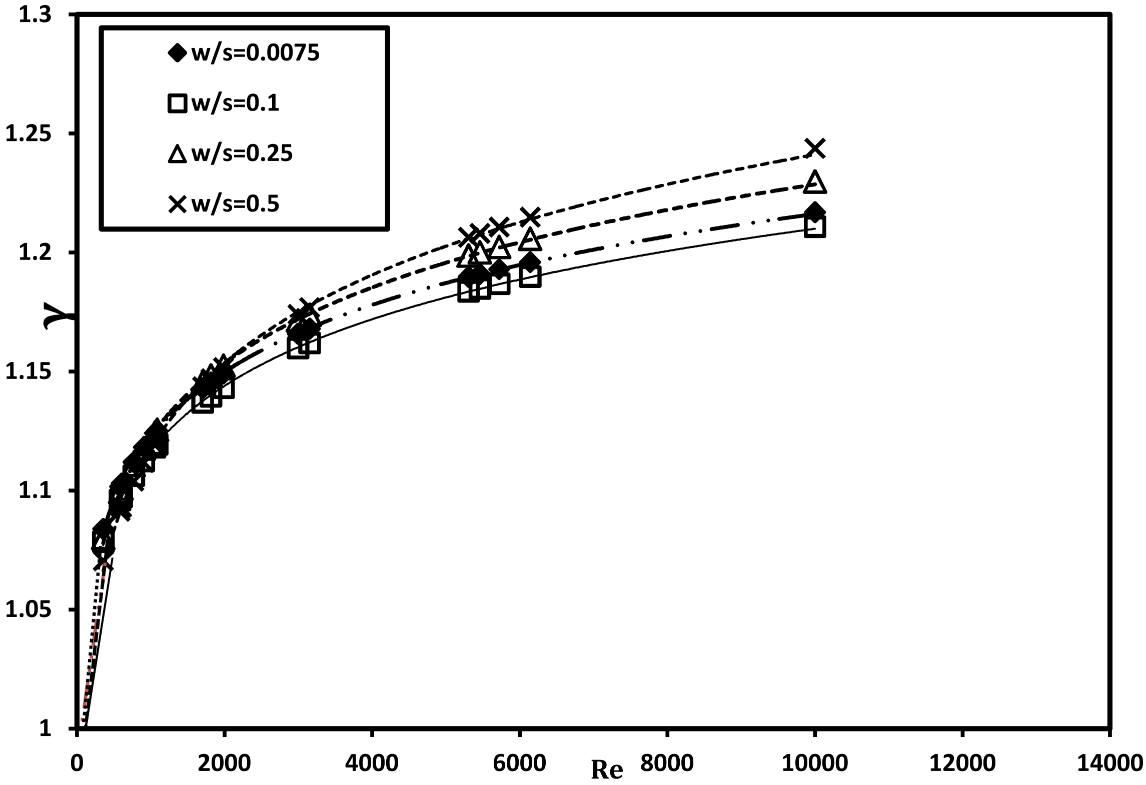
$$\gamma=\left(1-9.436565\left(\frac{C}{S}\right)\right)\left(Re+\left(1-9.436565(c/s)\right)^{-1/2.6277(C/S)}\right)^{2.6277(C/S)} \tag{18}$$

3.2.2.Effect of Tooth Width

The effect of (w/s) on the carry over coefficient for the cases (2, 5, 6 and 7) of table (1) can be shown in figure (8). It can be deduced from this figure that the tooth width has a little effect on the carry over coefficient, spatially for Reynolds number higher than 2000. It has been found that the relationship between the carry over coefficient and the Reynolds number can be represented by power law relation as follows

$\gamma = c_1(Re+R_0)^{C2}$.

The relation between constants C_1 and C_2 and tooth width (w/s) for a certain (c/s=0.015) has been studied as shown in figure (9).



Figure(8). Relationship between carry over coefficient and Reynolds number for different tooth widths (for c/s = 0.015).

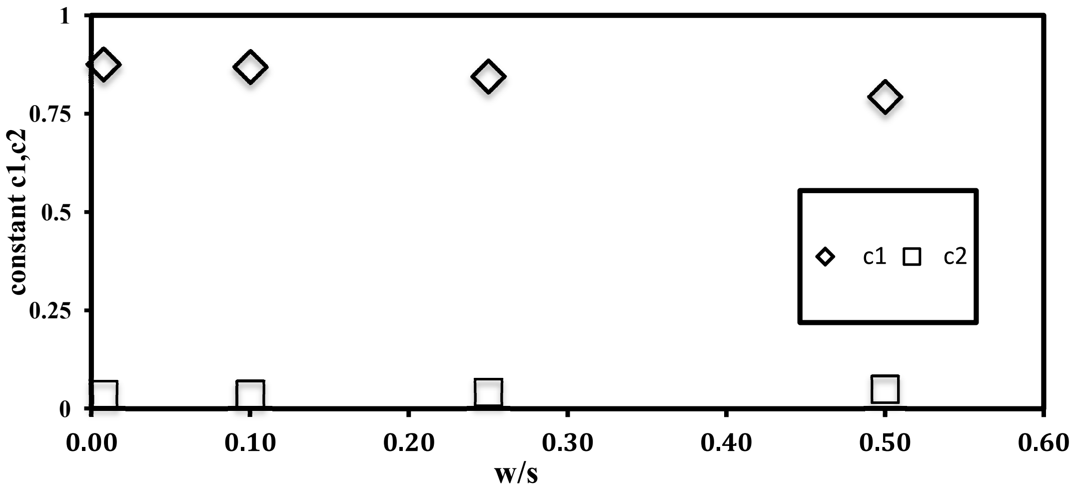


Figure (9) .Variation of c_1, c_2 with tooth width (for c/s = 0.015).

The effect of tooth width can be added to the previous model of carry over coefficient presented in equation (18) as follows:

$$\gamma = (1-9.436565\left[\frac{c}{s}\right]-R_3f_1)(Re+R_O)^{(2.677(C/S)+R_4f_2)}$$

Where:

$$Ro=\left(1-9.436565\left[\frac{c}{s}\right]-R_3f_1\right)^{-(2.677(C/S)+R_4f_2)} \qquad (19)$$

Both f_1 and f_2 represent the functions of w/s. R_3 and R_4 are constants for a given c/s, which are functions of c/s. The variation of C_1 and C_2 with tooth width for different values of c/s can be shown in figure (11) for the cases (1, 4,8 and 9) explained in Table (1).

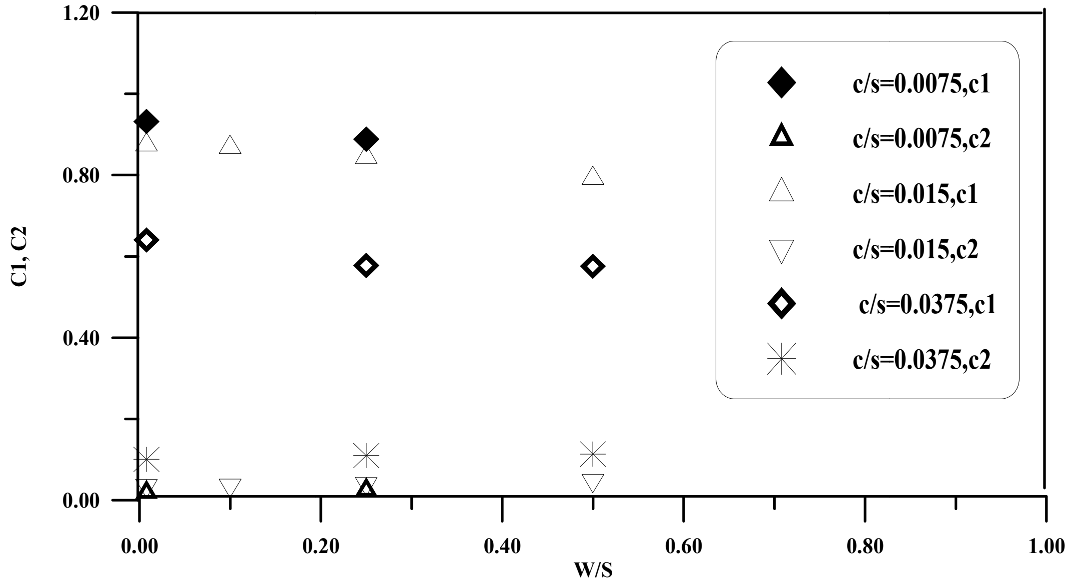


Figure (10) . Variation of $c_1$and c_2 with tooth width for different values of c/s.

The effect of tooth width on the carry over coefficient can be included in equation (19) as follows:

$$\gamma = (1-9.436565\left[\frac{c}{s}\right]-A(c/s)^a(w/s)^b)(Re+R_O)^{\left(2.677\left(\frac{c}{s}\right)+B\left(\frac{C}{s}\right)^c\left(\frac{w}{s}\right)^d\right)} \qquad (20)$$

The a, b, c and d have been calculated using SPSS software. Hence equation (20) can be re written as:

$$\gamma = (1-9.436565\left[\frac{c}{s}\right]-0.401307(c/s)^{0.2958546}(w/s)^{0.91784897})\ *$$
$$(Re+R_O)^{\left(2.6277\left(\frac{c}{s}\right)+0.2954893\left(\frac{C}{s}\right)^{0.6863094}\left(\frac{w}{s}\right)^{0.964921808}\right)} \qquad (19)$$

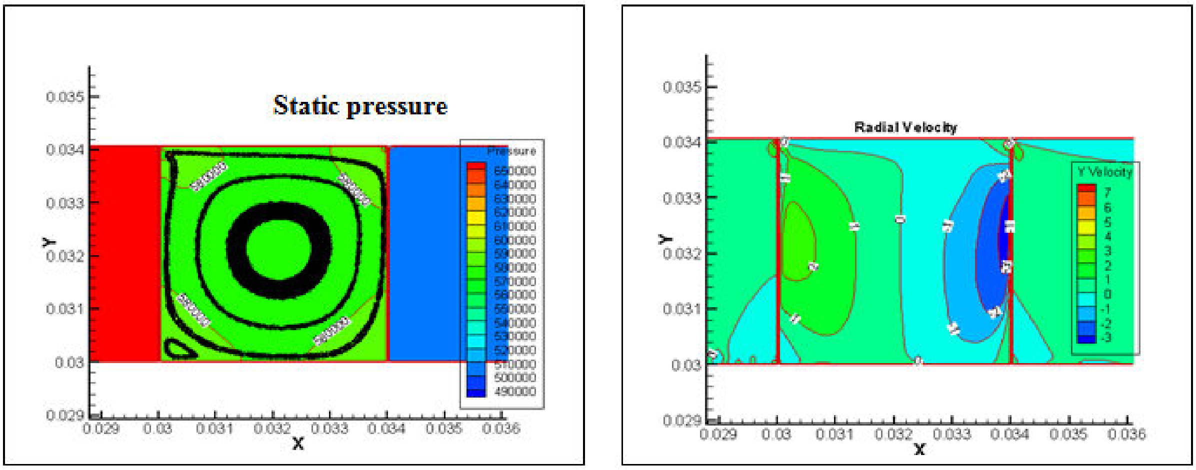
$$R_O = (1-9.436565(c/s)-0.401307(c/s)0.2958546(w/s)0.91784897)^{cc}$$

$$cc = \frac{-1}{2.6277\frac{C}{S} + 0.2954893(\frac{C}{S})^{0.6863094}(\frac{W}{S})^{0.964921808}}$$

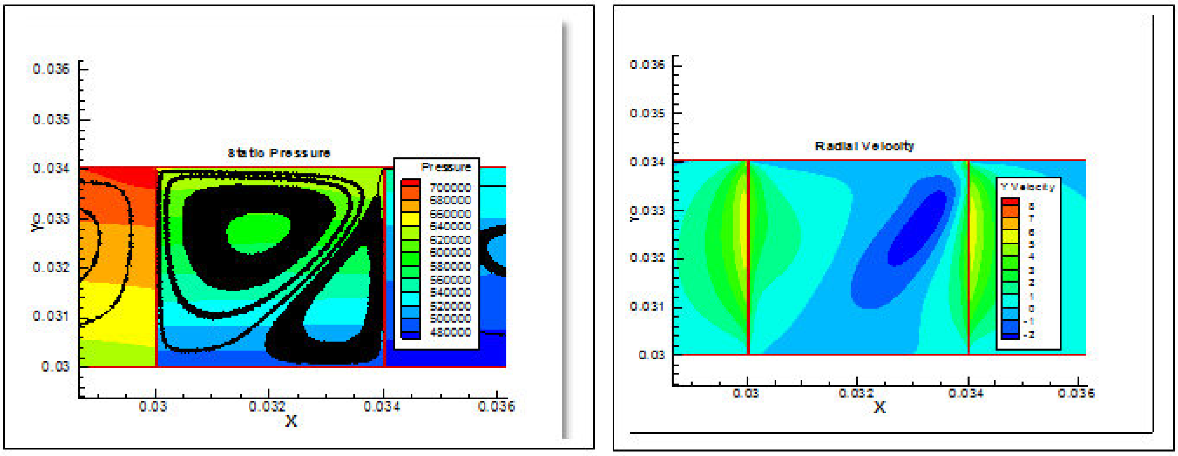
The above modification equation has been proved to be a better equation for the model used since the relative error reduced from 1.07% to 0.9%.

3.2.3. Effect of Shaft Rotational Speed

A shaft rotational speed may changes the flow pattern within the seal which influences the carry over coefficient. The effect of rotational speed on the carry-over coefficient of the labyrinth seal has been studied by considering different shaft speeds at a given flow condition and seal geometry for the cases 1, 2 and 4 of table (1). Reynolds number is assumed to be 1000 as shown in figure (12). It can be seen from this figure that the carry over coefficient is independent of shaft speed. The carry over coefficient seems to be constant for the range of shaft speeds (0 -10000) rpm as shown in figure (11-a). For the range of shaft speeds (11000-20000) rpm the fluid is subjected to higher radial pressure gradient within the cavity leading to detachment of the streamlines that exit at the preceding tooth and being drawn into forming a secondary vortex. Hence for cases like that in figure (11-b) the carry over coefficient cannot be defined by Hokinson's equation and the carry over coefficient is taken as the least possible value (γ=1 in this case).



a-Contour of Static pressure and Radial Velocity at(zero rpm).



b-Contour of Static pressure and Radial Velocity at(15000 rpm).

Figure (11). Influence of Rotation on Stream lines in Cavity at Various Shaft Speeds.

Table (1). Seal geometries used for simulation

Case No.	No. of Teeth	Clearance (mm)	Pitch (mm)	Tooth Width (mm)	Tooth Height (mm)	Shaft Diameter (mm)
1	2	0.03	4	0.03	4	60
2	2	0.06	4	0.03	4	60
3	2	0.09	4	0.03	4	60
4	2	0.15	4	0.03	4	60
5	2	0.06	4	0.4	4	60
6	2	0.06	4	1	4	60
7	2	0.06	4	2	4	60
8	2	0.03	4	1	4	60
9	2	0.15	4	2	4	60

3.3. Validation

The CFD models developed in the present work are validated against previous experimental and theoretical works conducted by other workers. Figure (12) shows a comparison between the results obtained by the leakage mass flow rate of air through the straight - through labyrinth seal model with that obtained experimentally by **Scharrer (1988) , Derli and Eser (2004) .** The seal geometry and the operating conditions used in validation process are summarized in table (2). The root mean square error between the results obtained with that for **Scharrer** and **Eser** was to be 5% and 11%, respectively.

Table (2) . Seal Geometries and Operating Conditions

Seal Geometry	Operating Conditions
Number of teeth = 5,10,15,16	Pin = 7.00E+5 & 3.08 E+5 N/m2
Radius of Shaft = 0.0756 m	Pout = 1.01 E+5 N/m2
Height of Teeth =0.03175 m	Temperature =300 0 K
Clearance = 0.000127 & 0.00033 m	Gas Constant 287.06 Nm/kg .0 k
Width of teeth = 2 E-4	Rotational velocity 16000 & 20000 RPM
Pitch = 0.002175 m & 0.003175 m	Fluid Air

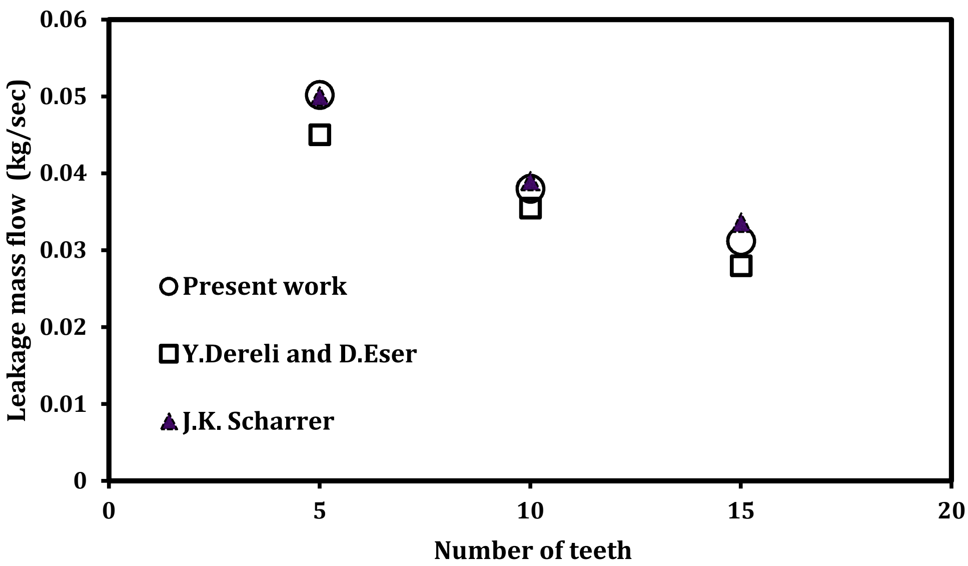


Figure (12). Comparison of leakage flow rate at Pin=7.00E+5 N/m^2.

Also the results obtained for the carry-over coefficient of labyrinth seal with air as a working fluid have been compared to that obtained by **Vamshi, Y. (2011),** as shown in figure (13)**.**

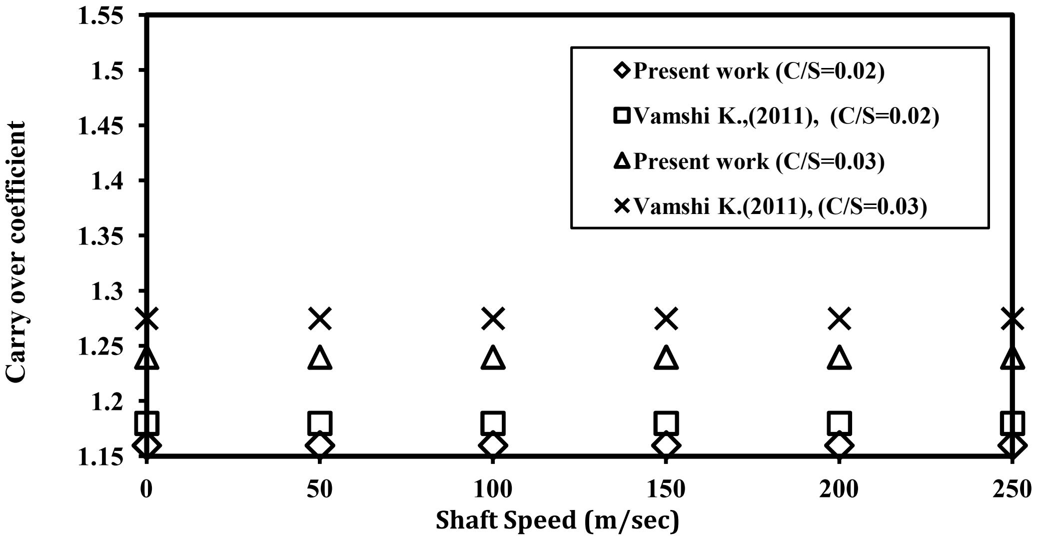


Figure (13) . Effect of shaft rotation on carry over coefficient at Re = 1000 of Air .

The carry over coefficient for the straight through labyrinth seal with a water as working fluid has been compared with that obtained by **Vamshi, Y. (2011),** as shown in figure (14).

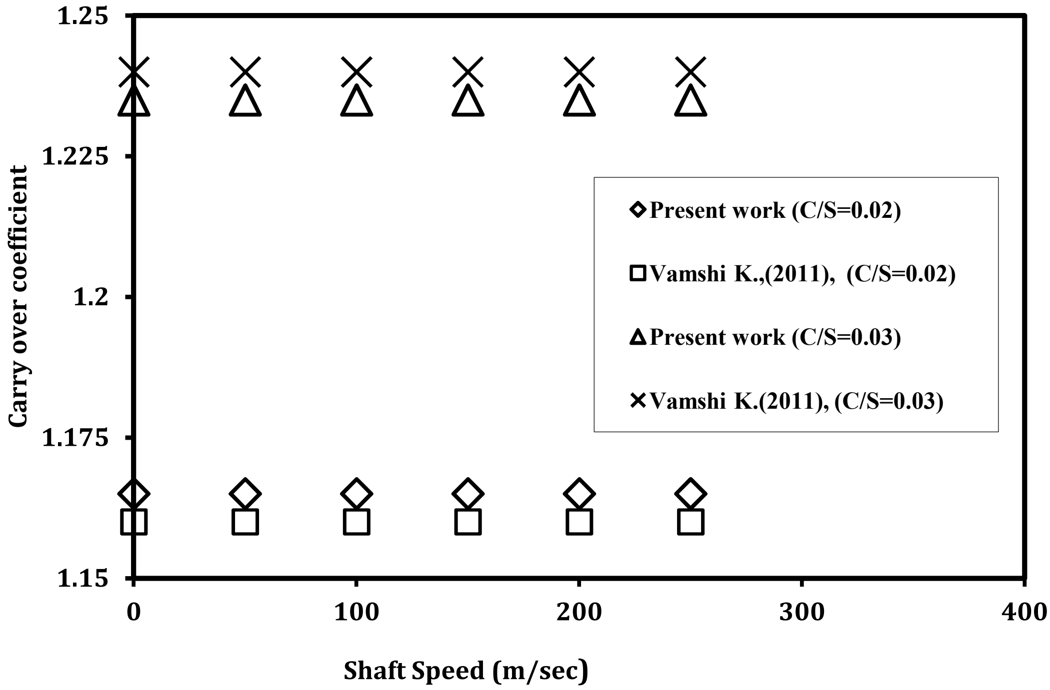


Figure (14). Effect of shaft rotation on carry over coefficient at Re = 1000 of water .

The relative error for the result in Figure (13) was found to be 2.8 %. While for figure (14) was found to be 0.4 % .

4. Conclusion

It is clear from above results that the carry over coefficient affected by the seal geometry and flow parameters such as Reynolds number. The results obtained show that the carry over coefficient increased with increasing the clearance to pitch ratio and decreased with increasing the tooth width to pitch ratio. Effect of shaft rotation on carry over coefficient has been studied and show that the carry decreased after a certain value of rotational speed (10000 rpm) due to the secondary recirculation zone which appear at higher rotational speed prevent the fluid to leakage were dissipation energy increased .

Acknowledgement
We would like to express our deep thanks to Saikishan, S., in Texas A&M for his suggestion to complete our research.

Nomenclature

Latin Symbols

Symbol	*Definition*	*Units*
A	**Clearance area**	m^2
B	**Height of tooth**	**m**
C	**Radial clearance**	**m**
$C_{\varepsilon 1}$	k-ε **turbulence model constant, 1.44**	**-**
$C_{\varepsilon 2}$	k-ε **turbulence model constant, 1.09**	**-**
C_{μ}	k-ε **turbulence model constant, 0.09**	**-**
D	**Shaft diameter**	**m**
Re	**Reynolds number based on clearance**	**-**
s	**Tooth pitch**	**m**
U	**Axial velocity**	**m/sec**
V	**Radial velocity**	**m/sec**
w	**Tooth width m**	**m**

Greek symbols

Symbol	*Definition*	*Units*
α	**Flow coefficient**	**-**
β	**Divergence angle of jet**	**radian**
x	**Percentage of kinetic energy carried over**	**-**
ε	**Dissipation of Turbulent Kinetic energy**	m^2/sec^3
γ	**Kinetic energy carry over coefficient**	**-**
k	**Turbulent Kinetic energy**	m^2/sec^3
ρ	**Fluid density at tooth inlet**	kg/m^3

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