

Analysis of Single Server Fixed Batch Service Queueing System under Multiple Vacation with Catastrophe

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Abstract

Consider a single server fixed batch service queueing system under multiple vacation with a possibility of catastrophe in which the arrival rate λ follows a Poisson process and the service time follows an exponential distribution with parameter μ. Further we assume that the catastrophe occur at the rate of ν which follows a Poisson process and the length of time the server in vacation follows an exponential distribution with parameter α. Assume that the system initially contains k customers when the server enters in to the system and starts the service immediately in a batch of size k. After completion of a service, if he finds less than k customers in the queue, then the server goes for a multiple vacation of length α. If there are more than k customers in the queue then the first k customers will be selected from the queue and service will be given as a batch. We are analyzing the possibility of catastrophe that is whenever a catastrophe occurs in the system, all the customers who are in the system will be completely destroyed and system becomes an empty and server goes for a multiple vacation. This model is completely solved by constructing the generating function and we have derived the closed form solutions for probability of number of customers in the queue during the server busy and in vacation. Further we are providing the analytical solution for mean number of customers and variance of the system. Numerical studies have been done for analysis of mean and variance of number of customers in the system for various values of λ, μ, ν and k and also various particular cases of this model have been discussed.

Keywords: Single server queue , Fixed batch service , Catastrophe, Multiple vacation, Steady state distribution

1. Introduction

Batch service queues have numerous applications to traffic, transportation, production, and manufacturing systems. Bailey (1954) obtained the transform solution to the fixed-size batch service queue with Poisson arrival. Miller (1959) studied the batch arrival and batch service queues and Jaiswal (1964) have studied the batch service queues in which service size is random. Neuts (1967) proposed the "general bulk service rule" in which service initiates only when a certain number of customers in the queue are available. Neuts general bulk service rule was extended by Borthakur and Medhi (1973). Studies on waiting time in a batch service queue were also rendered by Downton (1955), Cohen (1980), Medhi (1975) and Powell (1987). Fakinos (1991) derived the relation between limiting queue size distributions at arrival and departure epochs. Briere and Chaudhry (1989), Grassmann and Chaudhry (1982), and Kambo and Chaudhry (1985) used numerical approaches to obtain the performance measures. Chaudhry and Templeton (1983) gave more extensive study on batch arrival/service queues.

The notion of catastrophes occurring at random, leading to annihilation of all the customers there and the momentary inactivation of the service facility until a new arrival of a customer is not common in many practical problems. The catastrophes may come either from outside the system or from another service station. In computer systems, if a job is infected, this job transmits a virus when it is transferred to other processors. Infected files in floppy diskettes, for instance, may also arrive at the processors according to some random process. The infected jobs may be modeled by the catastrophe. Hence, computer networks with a virus may be modeled by queueing networks with catastrophes.

Vacation queues have been extensively studied by many researchers. Comprehensive surveys can be found in Doshi (1986) and Takagi (1991). Most of the studies on vacation queues have been concerned with single-unit service systems such as M/G/1 or M^X /G/1 queues. The well-known result concerning vacation queues is the "decomposition property" by Fuhrmann and Cooper (1985) which states that the Probability Generating Function (PGF) of the queue length of a vacation system can be factorized into the queue length of ordinary queue without vacation and "something else", the "something else" depends on the system characteristics. Lee et al. (1994) have analyzed the operating characteristics of batch arrival queues with N-policy and vacations, and obtained the queue length and waiting time distributions.

For batch service queues with vacations, there have been a few related works. Dhas (1989) considered Markovian batch service systems and obtained the queue length distributions by matrix-geometric method. Lee et al. [1994] obtained various performance measures for $M/G^B/1$ queue with single vacation. Dshalalow and

Yellen (1996) have considered a non-exhaustive batch service system with multiple vacations in which the server starts a multiple vacation whenever the queue drops below a level r and resumes service at the end of a vacation segment when the queue accumulates to at least r. They called such a system (r, R)-quorum system. They have applied the theory of the first excess level (Dshalalow (1996)). Lee et al. [1994] showed that for some batch service queues, mean queue length may even decrease in systems with server vacations. This has an implication that for some batch service queues, customers do not have to complain about unavailability of the server. Instead, they would rather force the server to take a vacation.

In this paper we are analyzing a special batch service queue called the fixed size batch service queue with catastrophes under multiple vacations. The model is described in Section 2. In Section 3, we have derived the system steady state equations and using these equations, the probability generating functions for number of customers in the queue when the server is busy or in vacation are derived and also obtained steady state probability distributions. Section 4 deals with stability condition of the system. In section 5, we discuss the particular case. Closed form solutions of system performance measures are obtained in 6. A numerical study is carried out in Section 7 to test the effectiveness of the system. We are providing the analytical solution for mean number of customers and variance of the system. Numerical studies have been done for analysis of mean and variance for various values of λ, μ, α ν and k and also various particular cases of this model have been discussed.

2. DESCRIBE OF THE MODEL

Consider a single server fixed batch service queueing system under multiple vacation with a possibility of catastrophe in which the arrival rate λ follows a Poisson process and the service time follows an exponential distribution with parameter μ. Further we assume that the catastrophe occur at the rate of ν which follows a Poisson process and the length of time the server in vacation follows an exponential distribution with parameter α. Assume that the system initially contains k customers when the server enters in to the system and starts the service immediately in a batch of size k. After completion of a service, if he finds less than k customers in the queue, then the server goes for a multiple vacation of length α. If there are more than k customers in the queue then the first k customers will be selected from the queue and service will be given as a batch. We are analyzing the possibility of catastrophe that is whenever a catastrophe occurs in the system, all the customers who are in the system will be completely destroyed and system becomes an empty and server goes for a multiple vacation. If there are less than k customers in the queue upon his return from the vacation, he immediately leaves for another vacation and so on until he finally finds k or more customers in the queue.

Let < N(t),C(t) > be a random process where N(t) be the random variable which represents the number of customers in queue at time t and C(t) be the random variable which represents the server status (busy/vacation) at time t.

We define

- $P_{n,1}(t)$ - Probability that the server is in busy if there are n customers in the queue at time t.
- $P_{n,2}(t)$ - Probability that the server is in vacation if there are n customers in the queue at time t

The Chapman- Kolmogorov equations are

$$P_{0,1}^{'}(t) = -(\lambda+\nu+\mu)P_{0,1}(t) + \mu P_{k,1}(t) + \alpha P_{k,2}(t) \quad (1)$$

$$P_{n,1}^{'}(t) = -(\lambda+\nu+\mu)P_{n,1}(t) + \lambda P_{n-1,1}(t) + \mu P_{n+k,1}(t) + \alpha P_{n+k,2}(t) \text{ for } n = 1,2,3,.. \quad (2)$$

$$P_{0,2}^{'}(t) = -(\lambda+\nu)P_{0,2}(t) + \mu P_{0,1}(t) + \nu \quad (3)$$

$$P_{n,2}^{'}(t) = -(\lambda+\nu)P_{n,2}(t) + \lambda P_{n-1,2}(t) + \mu P_{n,1}(t) \text{ for } n = 1,2,3,...,k-1 \quad (4)$$

$$P_{n,2}^{'}(t) = -(\lambda+\nu+\alpha)P_{n,2}(t) + \lambda P_{n-1,2}(t) \text{ for } n \geq k \quad (5)$$

3. EVALUATION OF STEADY STATE PROBABILITIES

In this section, we are finding the closed form solutions for number of customers in the queue when the server is busy or in vacation by using Generating function.

When steady state prevails, the equations (1) to (5) becomes

$$(\lambda+\nu+\mu)P_{0,1} = \mu P_{k,1} + \alpha P_{k,2} \quad (6)$$

$$(\lambda+\nu+\mu)P_{n,1} = \lambda P_{n-1,1} + \mu P_{n+k,1} + \alpha P_{n+k,2} \text{ for } n = 1,2,3,... \quad (7)$$

$$(\lambda+\nu)P_{0,2} = \mu P_{0,1} + \nu \quad (8)$$

$$(\lambda+\nu)P_{n,2} = \lambda P_{n-1,2} + \mu P_{n,1} \text{ for } n = 1,2,... k-1 \quad (9)$$

$$(\lambda + \nu + \alpha) P_{n,2} = \lambda P_{n-1,2} \quad \text{for } n \geq k \tag{10}$$

Generating functions for the number of customers in the queue when the server is busy or in vacation are defined as

$$G(z) = \sum_{n=0}^{\infty} P_{n,1} z^n \quad \text{and} \quad H(z) = \sum_{n=0}^{\infty} P_{n,2} z^n$$

Adding equations (6) and (7) after multiplying by 1, z^n (n = 0 to ∞) respectively we get

$$G(z)[\lambda z^{k+1} - (\lambda+\mu+\nu) z^k + \mu] + \alpha H(z) = \mu \sum_{n=0}^{k-1} P_{n,1} z^n + \alpha \sum_{n=0}^{k-1} P_{n,2} z^n \tag{11}$$

Adding equations (8), (9) and (10) after multiplying by 1, z^n and z^n (n = 1,2,3, . . .) respectively, we get

$$H(z)[\alpha + \nu + \lambda(1-z)] - \nu = \mu \sum_{n=0}^{k-1} P_{n,1} z^n + \alpha \sum_{n=0}^{k-1} P_{n,2} z^n \tag{12}$$

$$H(z) = \frac{\mu \sum_{n=0}^{k-1} P_{n,1} z^n + \alpha \sum_{n=0}^{k-1} P_{n,2} z^n + \nu}{\alpha + \nu + \lambda(1-z)} \tag{13}$$

Equation (13) represents the probability generating function for the number of customers in the queue when the server is in vacation.

From equations (11) and (13), we get

$$G(z) = \frac{\left(\mu \sum_{n=0}^{k-1} P_{n,1} z^n + \alpha \sum_{n=0}^{k-1} P_{n,2} z^n + \nu \right) (\nu + \lambda(1-z)) - \nu}{(\alpha + \nu + \lambda(1-z)) \left(\lambda z^{k+1} - (\lambda+\mu+\nu) z^k + \mu \right)} \tag{14}$$

The generating function G(z) has the property that it must converge inside the unit circle $|z| < 1$. We notice that the expression in the denominator of G(z) , $\lambda z^{k+1} - (\lambda+\mu+\nu) z^k + \mu$ has k+1 zeros. By Rouche's theorem, we notice that k zeros of this expression lies inside the circle $|z| < 1$ and must coincide with k zeros of numerator of G(z) and one zero lies outside the circle $|z| < 1$. Let z_0 be a zero which lies outside the circle $|z| < 1$.

As G(z) converges, k zeros of numerator and denominator will be cancelled, we get

$$G(z) = \frac{A}{(\alpha + \nu + \lambda - \lambda z) \lambda (z - z_0)} \tag{15}$$

When z = 0,

$$G(0) = \frac{A}{(\alpha + \nu + \lambda) \lambda (-z_0)}$$

$$A = P_{0,1} (\alpha + \nu + \lambda) \lambda (-z_0)$$

Then

$$G(z) = \frac{(\alpha + \nu + \lambda) z_0 P_{0,1}}{(\alpha + \nu + \lambda - \lambda z)(z_0 - z)} \tag{16}$$

By applying partial fractions, we get

$$G(z) = \frac{(\alpha + \nu + \lambda) z_0 P_{0,1}}{(\alpha + \nu + \lambda - \lambda z_0)} \left[\sum_{n=1}^{\infty} \frac{z^n}{z_0^{n+1}} - \sum_{n=1}^{\infty} \left(\frac{\lambda}{(\alpha + \nu + \lambda)} \right)^{n+1} z^n \right] \tag{17}$$

Comparing the coefficient of z^n on both sides of the equation (17), we get

$$P_{n,1} = P_{0,1} s^n \left(1 + \left(\frac{r}{s}\right) + \ldots + \left(\frac{r}{s}\right)^n\right) \quad \text{for n = 1,2,3,...} \quad \text{Where } r = \frac{1}{z_0} \text{ and } s = \frac{\lambda}{\lambda+\nu+\alpha} \quad (18)$$

Using equation (18) in (8) and (9) , apply recursive for n = 1,2, ...,k-1 and we get

$$P_{n,2} = \frac{\mu}{\lambda+\nu} P_{0,1} \left[s^n \left(1 + \left(\frac{r}{s}\right) + \ldots \left(\frac{r}{s}\right)^n\right) + \frac{\lambda}{\lambda+\nu} s^{n-1} \left(1 + \left(\frac{r}{s}\right) + \ldots \left(\frac{r}{s}\right)^{n-1}\right) + \ldots + \left(\frac{\lambda}{\lambda+\nu}\right)^n \right] + \left(\frac{\lambda}{\lambda+\nu}\right)^n \frac{\nu}{\lambda+\nu}$$

for n = 0,1,2,...,k-1 (19)

$$P_{n,2} = \left(\frac{\lambda}{\lambda+\alpha+\nu}\right)^{n-k+1} P_{k-1,2} \quad \text{for } n \geq k \quad (20)$$

The normalized condition is

$$P_{0,1} + \sum_{n=1}^{\infty} P_{n,1} + \sum_{n=0}^{k-1} P_{n,2} + \sum_{n=k}^{\infty} P_{n,2} = 1 \quad (21)$$

Substitute (18),(19) and (20) in (21), we get,

$$P_{0,1} = \frac{Nr}{Dr} \quad (22)$$

$$Nr = 1 - \sum_{n=0}^{k-1} \left(\frac{\lambda}{\lambda+\nu}\right)^n \frac{\nu}{\lambda+\nu} - \frac{\nu}{\alpha+\nu}\left(\frac{\lambda}{\lambda+\nu}\right)^k$$

$$Dr = \frac{1}{(1-s)(1-r)} + \frac{\mu}{\lambda+\nu}\left[\sum_{n=0}^{k-1}\left[s^n \left(1 + \frac{r}{s} + \ldots + \left(\frac{r}{s}\right)^n\right) + \frac{\lambda}{\lambda+\nu} s^{n-1} \left(1 + \frac{r}{s} + \ldots \left(\frac{r}{s}\right)^{n-1}\right) + \ldots + \left(\frac{\lambda}{\lambda+\nu}\right)^n \right]\right]$$

$$+ \frac{\mu}{\lambda+\nu}\left[s^{k-1} \left(1 + \frac{r}{s} + \ldots + \left(\frac{r}{s}\right)^{k-1}\right) + \frac{\lambda}{\lambda+\nu} s^{k-2} \left(1 + \frac{r}{s} + \ldots \left(\frac{r}{s}\right)^{k-2}\right) + \ldots + \left(\frac{\lambda}{\lambda+\nu}\right)^{k-1} \right] \frac{\lambda}{\alpha+\nu}$$

Equations (18),(19),(20) and (22) represent the steady state probabilities for number of customers in the queue , when the server is in busy or in vacation.

4. STABILITY CONDITION

The necessary and sufficient condition for the system to the stable is $\frac{\lambda}{k\mu} < 1$.

5. PARTICULAR CASE

If we take $\nu = 0$, the results coincides with the results of the model single server batch service under multiple vacation.

6. SYSTEM PERFORMANCE MEASURES

In this section we will list some important performance measures along with their formulae. These measures are used to bring out the qualitative behaviour of the queueing model under study. Numerical study has been dealt in very large scale to study the following measures.

1. $$P_{0,1} = \frac{Nr}{Dr}$$

$$Nr = 1 - \sum_{n=0}^{k-1} \left(\frac{\lambda}{\lambda+\nu}\right)^n \frac{\nu}{\lambda+\nu} - \frac{\nu}{\alpha+\nu}\left(\frac{\lambda}{\lambda+\nu}\right)^k$$

$$Dr=\frac{1}{(1-s)(1-r)}+\frac{\mu}{\lambda+\nu}\left[\sum_{n=0}^{k-1}\left[s^n\left(1+\frac{r}{s}+\ldots+\left(\frac{r}{s}\right)^n\right)+\frac{\lambda}{\lambda+\nu}s^{n-1}\left(1+\frac{r}{s}+\ldots+\left(\frac{r}{s}\right)^{n-1}\right)+\ldots+\left(\frac{\lambda}{\lambda+\nu}\right)^n\right]\right]$$

$$+\frac{\mu}{\lambda+\nu}\left[s^{k-1}\left(1+\frac{r}{s}+\ldots+\left(\frac{r}{s}\right)^{k-1}\right)+\frac{\lambda}{\lambda+\nu}s^{k-2}\left(1+\frac{r}{s}+\ldots+\left(\frac{r}{s}\right)^{k-2}\right)+\ldots+\left(\frac{\lambda}{\lambda+\nu}\right)^{k-1}\right]\frac{\lambda}{\alpha+\nu}$$

2. $P_{n,1} = P_{0,1}s^n\left(1+\left(\frac{r}{s}\right)+\ldots+\left(\frac{r}{s}\right)^n\right)$ for n = 0,1,2,...

3. $P_{n,2} = \frac{\mu}{\lambda+\nu}P_{0,1}\left[s^n\left(1+\left(\frac{r}{s}\right)+\ldots+\left(\frac{r}{s}\right)^n\right)+\frac{\lambda}{\lambda+\nu}s^{n-1}\left(1+\left(\frac{r}{s}\right)+\ldots\left(\frac{r}{s}\right)^{n-1}\right)+..+.\left(\frac{\lambda}{\lambda+\nu}\right)^n\right]+\left(\frac{\lambda}{\lambda+\nu}\right)^n\frac{\nu}{\lambda+\nu}$

for n = 0,1,2,...,k-1

4. $P_{n,2}=\left(\frac{\lambda}{\lambda+\alpha+\nu}\right)^{n-k+1}P_{k-1,2}$ for $n \geq k$

5. $L_s=\sum_{n=0}^{\infty}(n+k)P_{n,1}+nP_{n,2}$

6. $L_q=\sum_{n=0}^{\infty}n(P_{n,1}+P_{n,2})$

7. $V(x)=\left(\sum_{n=0}^{\infty}(n+k)^2P_{n,1}+\sum_{n=0}^{\infty}n^2P_{n,2}\right)-(L_s)^2$

7. NUMERICAL STUDIES

Table 1: Steady state probabilities for various batch size when λ = 2, μ =10, ν =1, α = 5

P_i	k=1	k=2	k=3
$P_{0,1}$	0.0915	0.0382	0.0202
$P_{1,1}$	0.0391	0.0155	0.0081
$P_{2,1}$	0.0127	0.0048	0.0025
$P_{3,1}$	0.0036	0.0013	0.0007
$P_{4,1}$	0.0010	0.0003	0.0001
$P_{5,1}$	0.0002	0.00009	0.00004
$P_{6,1}$	0.00007	0	0
$P_{0,2}$	0.6383	0.4606	0.4006
$P_{1,2}$	0.1595	0.3589	0.2943
$P_{2,2}$	0.0398	0.0897	0.2046
$P_{3,2}$	0.0099	0.0224	0.0511
$P_{4,2}$	0.0024	0.0056	0.0127
$P_{5,2}$	0.0006	0.0014	0.0031
$P_{6,2}$	0.0001	0.0003	0.0007
$P_{7,2}$	0.00003	0.00008	0.0001
$P_{8,2}$	0	0	0.00004

Table 2: Steady state probabilities for various α when λ = 2, μ = 10, ν = 1 and k = 3

P_i	α = 5	α =10	α =20
$P_{0,1}$	0.0202	0.0249	0.0281
$P_{1,1}$	0.0081	0.0076	0.0067
$P_{2,1}$	0.0025	0.0017	0.0012
$P_{3,1}$	0.0007	0.0003	0.0002
$P_{4,1}$	0.0001	0.00007	0.00003
$P_{5,1}$	0.00004	0	0
$P_{6,1}$	0	0	0
$P_{0,2}$	0.4006	0.4163	0.4276
$P_{1,2}$	0.2943	0.3031	0.3072
$P_{2,2}$	0.2046	0.2079	0.2090
$P_{3,2}$	0.0511	0.0319	0.0181
$P_{4,2}$	0.0127	0.0049	0.0015
$P_{5,2}$	0.0031	0.0007	0.0001
$P_{6,2}$	0.0007	0.0001	0.00001
$P_{7,2}$	0.0001	0.00001	0
$P_{8,2}$	0.00004	0	0

Table 3: Steady state probabilities for various ν when λ = 2, μ = 10, k = 2, α = 5

P_i	ν = 1	ν = 10	ν = 50
$P_{0,1}$	0.0382	0.0038	0.0001.
$P_{1,1}$	0.0155	0.0007	0.000006
$P_{2,1}$	0.0048	0.0001	0
$P_{3,1}$	0.0013	0.00001	0
$P_{4,1}$	0.0003	0	0
$P_{5,1}$	0.00009	0	0
$P_{0,2}$	0.4606	0.8365	0.9615
$P_{1,2}$	0.3589	0.1400	0.0369
$P_{2,2}$	0.0897	0.0164	0.0012
$P_{3,2}$	0.0224	0.0019	0.00004
$P_{4,2}$	0.0056	0.0002	0
$P_{5,2}$	0.0014	0.00002	0
$P_{6,2}$	0.0003	0	0
$P_{7,2}$	0.00008	0	0

Table 4: System measures for various batch size when λ = 2, μ = 10, ν =1, α = 5

	k = 1	k = 2	k = 3
L_s	0.5111	0.78829	1.03904
L_q	0.08072	0.03065	0.0158
V(x)	0.66047	0.85402	1.2129

Table 5: System measures for various values of α when λ = 2, μ = 10, ν =1, k = 3

	α =5	α=10	α =20
L_s	1.03904	0.95427	0.90334
L_q	0.0158	0.01218	0.00982
V(x)	1.2129	1.01359	0.908116

Table 6: System measures for various values of ν when λ = 2, μ = 10, α =5, k = 3

	ν = 1	ν = 10	ν = 50
L_s	0.78829	0.18955	0.0398
L_q	0.03065	0.00013	0.00006
V(x)	0.85402	0.213820	0.04141

8. Conclusion

The Numerical studies show the changes in the system due to impact of batch size, vacation rate and catastrophes. The mean number of customers in the system increases as batch size increase. The mean number of customers in the system decreases as catastrophes rate and vacation rate increases. Various special cases have been discussed, which are particular cases of this research work. This research work can be extended further by introducing various concepts like negative arrival, breakdown and repair etc.

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