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Bianchi Type-I Wet Dark Universe in Bimetric Relativity

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Abstract
In this paper , we have investigated the role of Wet Dark Fluid in Bianchi Type-I cosmological model within the framework of bimetric theory of relativity proposed by Rosen N. Here we have used a new equation of state for the matter dark energy component of the universe known as Wet Dark Fluid given by $p_{WDF}= \Upsilon(\rho_{WDF}- \rho_*)$ which can describe a liquid for example water. It is further observed the non existence of Wet Dark Fluid in bimetric relativity.
Keywords: Bianchi Type-I Universe , Wet Dark Fluid , Bimetric Relativity
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1. INTRODUCTION:-

Einstein's theory of general relativity is one of the most beautiful structures of theoretical physics which describes the theory of gravitation in terms of geometry. In the last decades, several theories of gravitation have been proposed as alternatives to Einstein's theory of general relativity.The most popular among them is Rosen's (1973) bimetric theory of relativity.
Rosen(1) proposed the bimetric theory of relativity to remove some of the unsatisfactory features of the general theory of relativity. In the bimetric theory , there exist two metric tensors at each point of space-time , g_{ij} which describes the gravitation and the background metric γ_{ij} which enters into the field equation and interacts withg_{ij} , *but does not interact directly with the matter.*
One can regard γ_{ij} *as describing the geometry that exists* no matter were present. Accordingly , at each space-time point ,one has two line elements:

$$ds^2 = g_{ij}\, dx^i\ dx^j$$

and $$d\sigma^2 = \gamma_{ij}\, dx^i\ dx^j$$

where ds is the interval between two neighbouring events as measured by means of a clock and a measuring rod. The interval $d\sigma$ is an abstract or geometrical quantity not directly measurable .One can regard it as describing the geometry that would exist if no matter were present.
When we study the Bianchi type models , we observed that the models contain isotropic special cases and they permit arbitrarily small anisotropic levels at some instant of cosmic times. Hence these models are to be known as suitable models of our universe. Therefore study of Bianchi type models creates much more interest.
One of the mysteries of cosmology is the nature of dark energy component of the universe. There is , definitely no lack of cosmological constant, phantom energy , quintessence k-essence etc. To explain the acceleration of the universe, modified Friedmann equation such as cardassion expansion and also what might be derived from brane cosmology have been used. In this work , Wet Dark Fluid (WDF) as a model for dark energy is used in Bimetric Relativity. This model is generalized Chaplygin gas(GCG) where an equation of state is offered with properties relevant for the dark energy problem.
Riess et al (2) , Perimuttar et al (3) and Sahani(4) have studied the nature of the dark component of the universe as one of the deepest mysteries of cosmology. We are motivated to use the wet dark fluid (WDF) as a model for dark energy which stems from an equation of state proposed by Tait(5) and Hayward(6) to treat water and aqueous solutions.
The equation of state for Wet Dark Fluid is $p_{WDF}= \Upsilon(\rho_{WDF}- \rho_*)$ (1.1)
Where the parameters Υ and ρ_* *taken to be positive and we restrict ourselves to*

$$0 \le \gamma \le 1.$$

We have energy conservation equation as:

$$\rho_{WDF} + 3H\left(p_{WDF} + \rho_{WDF}\right) = 0 \qquad (1.2)$$

Using equation of state and $3H = \frac{\dot{V}}{V}$ in the above equation , we get

$$\rho_{WDF} = \frac{\gamma}{1+\gamma} \rho_* + \frac{c}{v\,(1+\gamma)} \quad (1.3)$$

where c is the constant of integration and v is the volume expansion.

Wet Dark Fluid (WDF) has two components : one behaves as cosmological constant and other as standard fluid with equation of state p= $\gamma\rho$ Type equation here.

If we take c>0 then this fluid will not violet the strong energy condition $p + \rho \geq 0$;

$$(p_{WDF} + \rho_{WDF}) = (1+\gamma)\,\rho_{WDF} - \gamma\,\rho_*$$

$$(p_{WDF} + \rho_{WDF}) = (1+\gamma)\,\frac{c}{v\,(1+\gamma)} \quad (1.4)$$

Holman and Naidu (7) used the wet dark fluid as dark energy in the homogeneous isotropic FRW case . Singh and Chaubey (8) studied Bianchi type-I universe with wet dark fluid. And also Deo S. D. (9) studied Bianchi type-I universe with Wet Dark Energy in Bimetric Relativity. This work concludes that the Bianchi Type-I cosmological model in Bimetric Relativity does not accommodate Wet Dark Energy .

2.METRIC AND SOLUTIONS OF FIELD EQUATIONS:-

We consider Bianchi type-I Metric in the form:

$$ds^2 = A^2(dx^2 - dt^2) + B^2 dy^2 + C^2 dz^2 \quad (2.1)$$

where A, B, C are the functions of t only.

The background flat metric corresponding to equation (2.1) is

$$d\sigma^2 = -dt^2 + dx^2 + dy^2 + dz^2 \quad (2.2)$$

The field equations in bimetric theory of gravitation proposed by Rosen (1973) are:

$$N_i^j - \frac{1}{2}\delta_i^j = -8\pi k T_i^j \quad (2.3)$$

$$\text{where } N_i^j = \frac{1}{2}\gamma^{\alpha\beta}\,(\,g^{hj}.\,g_{hi}\,|_\alpha)\,|\beta$$

and $k=(\frac{g}{\gamma})^{\frac{1}{2}}$

together with g=det(g_{ij}) and $\gamma = \det(\gamma_{ij})$

Here the vertical bar ($|$) denotes the covariant differentiation with respect to γ_{ij}

and T_i^j is the energy momentum tensor of the matter field.

Rosen's field equation in bimetric theory for the metric (2.1) with the help of equations (2.2)-(2.3) , can be written in the form:

$$\left(\frac{\dot{B}}{B}\right)^{\cdot} + \left(\frac{\dot{C}}{C}\right)^{\cdot} = -16\pi k T_1^1 \quad (2.4)$$

$$2\left(\frac{\dot{A}}{A}\right)^{\cdot} - \left(\frac{\dot{B}}{B}\right)^{\cdot} + \left(\frac{\dot{C}}{C}\right)^{\cdot} = -16\pi k T_2^2 \quad (2.5)$$

$$2\left(\frac{\dot{A}}{A}\right)^{\cdot} + \left(\frac{\dot{B}}{B}\right)^{\cdot} - \left(\frac{\dot{C}}{C}\right)^{\cdot} = -16\pi k T_3^3 \quad (2.6)$$

$$\left(\frac{\dot{B}}{B}\right)^{\cdot} + \left(\frac{\dot{C}}{C}\right)^{\cdot} = -16\pi k T_4^4 \quad (2.7)$$

Here the overhead dot(.) denotes differentiation with respect to t.

The energy momentum tensor by Singh and Chaubey(2008) for the Wet Dark Fluid source is given by,

$$T_i^j = (p_{WDF} + \rho_{WDF})u_i u^j - p_{WDF}\delta_i^j \quad (2.8)$$

Here p_{WDF} *is the isotopic pressure and* ρ_{WDF} is the matter density and u^i is the flow vector of the fluid. The flow of the matter is taken orthogonal to the hyper surfaces of the homogenity so that

$$g_{ij}\,.\,u_i u^j = 1 \quad (2.9)$$

In commoving system of coordinates , from equation (2.8) &(2.9) *we have ,*

$$T_1^1 = T_2^2 = T_3^3 = -p_{WDF} \;\&\; T_4^4 = \rho_{WDF} \quad (2.10)$$

Now using equation (2.10), equations (2.4)-(2.7) becomes,

$$\left(\frac{\dot{B}}{B}\right)^{\cdot} + \left(\frac{\dot{C}}{C}\right)^{\cdot} = 16\pi k p_{WDF} \quad (2.11)$$

$$-\left(\frac{\dot{B}}{B}\right)^{\cdot} + \left(\frac{\dot{C}}{C}\right)^{\cdot} = 16\pi k p_{WDF} \quad (2.12)$$

$$2\left(\frac{\dot{A}}{A}\right)^{\cdot} + \left(\frac{\dot{B}}{B}\right)^{\cdot} - \left(\frac{\dot{C}}{C}\right)^{\cdot} = 16\pi k p_{WDF} \quad (2.13)$$

$$\left(\frac{\dot{B}}{B}\right)^{\cdot} + \left(\frac{\dot{C}}{C}\right)^{\cdot} = -16\pi k \rho_{WDF} \quad (2.14)$$

Using equations (2.11) & (2.14) ,we get

$$p_{WDF} + \rho_{WDF} = 0 \quad (2.15)$$

For the reality conditions , the relations ,

$$p_{WDF} > 0 \text{ and}$$

$$\rho_{WDF} > 0 \text{ must hold.}$$

The above equation (2.15) *satisfies only when ,*

$$p_{WDF} = 0 \text{ and } \rho_{WDF} = 0 \quad (2.16)$$

which means that the physical parameters viz. Wet Dark Fluid pressure (p_{WDF}) and wet dark fluid energy density(ρ_{WDF}), both are identically zero. Thus Bianchi type-I universe with wet dark fluid in bimetric theory of relativity does not survive and hence only vacuum model is obtained.

For vacuum case, $p_{WDF} = 0 = \rho_{WDF}$, the field equations (2.11)-(2.14),gives the solution of the form,

$$A=B=C=e^{nt} \quad (2.17)$$

Where n is the constant of integration.

Thus in view of equation (2.17), the metric (2.1) takes the form:

$$ds^2 = e^{2nt}(dx^2 + dy^2 + dz^2 - dt^2) \quad (2.18)$$

This can be transformed through a proper choice of coordinates to

$$ds^2 = e^{2T}(dX^2 + dY^2 + dz^2 - dT^2) \quad (2.19)$$

3.CONCLUSION:-

Here we have constructed Bianchi Type-I cosmological model in Rosen's bimetric theory of relativity with a new equation of state for the dark energy component of the universe known as Wet Dark Fluid. Further, we conclude the nil contribution of Wet Dark Fluid in Bianchi Type-I cosmological model in Rosen's theory and hence only vacuum model is obtained.

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REFERENCES:-

(1) Rosen , N: Gen. Rel.Grav.,4,435(1973).
(2)Riess , A.G. et al. 1009(1998).
(3)Perimutter , S.et al : Astrophys.J.,157,565(1998).
(4)Sahni,V.:arXiv:astro-ph/0403324(2004).
(5)Tait,P.G.: The Voyage of HMS Challenger(H.M.S.O. , London,1998).
(6)Hayward,A.T.J.,Brit.J.Appl.Phys.18,965,(1967).
(7)Holman,R. and Naidu,S., arXiv: Astro-phy/0408102(preprint)(2005).
(8)Singh,T. and Chaubey,R.: Pramana Journal of Physics,71,No.3(2008).
(9)Deo,S.D.: OIIRJ,II(1), 104-107(2012).