Measures of Different Reliability Parameters for a Complex Redundant System Under Head-of-Line Repair

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Abstract
The authors have considered a complex system consisting of two subsystems designated as 'A' and 'B' connected in series. Subsystem 'A' consists of N non-identical units in series, while the subsystem 'B' consists of three identical components in parallel redundancy.
Keywords: Availability/Reliability Analysis, Repairable Parallel System, Laplace transform, cost profit function, Head-of-line Repair,

1. Introduction

In this paper the authors have considered a complex system consisting of two subsystems designated as '*A*' and '*B*' connected in series. Subsystem '*A*' consists of *N* non-identical units in series, while the subsystem '*B*' consists of three identical components in parallel redundancy. In this model it is considered that the system goes to complete breakdown state if any unit of subsystem '*A*' fails or more than 1 unit of subsystem '*B*' is in the failed condition. Also, the system works with reduced efficiency if one unit of subsystem '*B*' failed. The system as a whole can also fail from normal efficiency state if there is any failure due to environmental reasons. *Supplementary variable technique and Laplace transforms have been utilized to obtain various state probabilities and the cost incurred for the system is obtained*. Failure and repair times of the units follow exponential and general time distributions respectively. Head-of-line policy is being adopted for the repair purpose. Some particular cases have also been taken to highlight the practical importance of the model. This research is a step towards explaining the reliability application on a repairable system with three types of failure under 'head-of-line' repair policy and Gumbel-Hougaard family copula.

So in earlier research [19, 20, 21, 22], different techniques have been applied to evaluate the reliability of distribution system, including distributed generation such as an analytical technique using the load duration curve, distributed processing technique, Characteristic function based approach for computing the probability distributers of reliability indices, probabilistic method for assessing the reliability and quantity of power supply to a customer, composite load point model, practical reliability assessment algorithm, validation method and impact of substation on distribution reliability respectively.

2. Assumption

(*i*). Initially, all units are good.
(*ii*). A failed unit is repaired at a single service channel.
(*iii*). The parallel subsystem is composed of three identical units, while series subsystem is composed of N non-identical units.
(*iv*). Failures are statistically independent.
(*v*). Environmental failure rates are constant.
(*vi*). After repair, units work like new.
(*vii*). Repairs follow general time distribution.
(*viii*).First come first served (Head-of-line) repair policy is being adopted.

3. Notations

$f' / f_i / f_E$	:Constant failure rates of any unit of subsystem B / i^{th} unit of subsystem A/environmental failure.
$r_1(x) / r_2(\mathrm{y}) / r_3(z) / r_4(\alpha)$	:Repair rates with general time distribution from states S_4 to S_0 S_1 to S_0 or S_3 to S_4, S_2 to S_0, S_5 to S_0
$P_N^3(t)$	:Probability that at time 't' the system is operating in the state of normal efficiency.
$P_N^2(\mathrm{y}, t)\,\Delta$	:The probability that at time 't', the system is in degraded state due to the failure of one unit of subsystem B. The elapsed repair time lies in the interval $(\mathrm{y}, \mathrm{y}+\Delta)$
$P_N^F(z, t)\,\Delta$	: The probability that at time 't', the system is in failed state due to the failure of more than one unit of subsystem B, the elapsed repair time lies in the interval $(z, z+\Delta)$
$P_i^3(x, t)\,\Delta$	:The probability that at time 't', the system is in failed state due to the failure of i^{th} unit of subsystem A. The elapsed repair time lies in the interval $(x, x+\Delta)$
$P_i^2(\mathrm{y}, t)\,\Delta$	:The probability that at time 't' the repair time lies in the interval $(\mathrm{y}, \mathrm{y}+\Delta)$
$P_E(\alpha, t)\,\Delta$	:The probability that at time 't', the system is in failed state, due to the environmental failure, the elapsed repair time lies in the interval $(\alpha, \alpha+\Delta)$

Figure 1 represents the state transition diagram of the system.

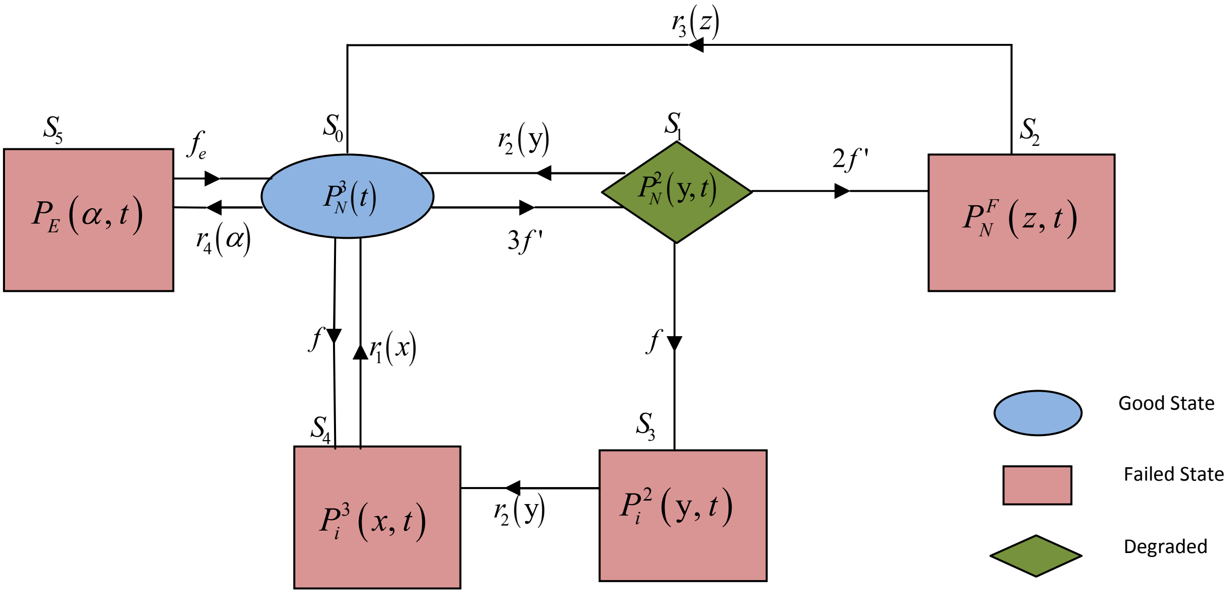

Figure 1: Transition State Diagram

4. Formulation of the Mathematical Model

Viewing the nature of the problem, we obtain the following set of difference-differential equations:

$$\left[\frac{\partial}{\partial t} + 3f' + f + f_e\right] P_N^3(t) = \int_0^\infty P_i^2(\mathrm{y}, t)\, r_2(\mathrm{y})\, d\mathrm{y} + \int_0^\infty P_N^F(z, t)\, r_3(z)\, dz$$

$$+\int_0^\infty P_i^3(x,t)\,r_1(x)\,dx+\int_0^\infty P_E(\alpha,t)\,r_4(\alpha)\,d\alpha \tag{1}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial \mathrm{y}}+2f'+f+r_2(\mathrm{y})\right]P_N^2(\mathrm{y},t)=0 \tag{2}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+r_3(z)\right]P_N^F(z,t)=0 \tag{3}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+r_1(z)\right]P_i^3(x,t)=0 \tag{4}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial \mathrm{y}}+r_2(\mathrm{y})\right]P_i^2(\mathrm{y},t)=f\,P_N^2(\mathrm{y},t) \tag{5}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial \alpha}+r_4(\alpha)\right]P_E(\alpha,t)=0 \tag{6}$$

4.1 Boundary Conditions

$$P_N^2(0,t)=3\,f'\,P_N^3(t) \tag{7}$$

$$P_N^F(0,t)=2\,f'\,P_N^2(t) \tag{8}$$

$$P_i^3(0,t)=f\,P_N^3(t)+\int_0^\infty P_i^2(\mathrm{y},t)\,r_2(\mathrm{y})\,d\mathrm{y} \tag{9}$$

$$P_i^2(0,t)=0 \tag{10}$$

$$P_i(0,t)=f_e\,P_N^3(t) \tag{11}$$

4.2 Initial Conditions

$$P_N^3(0)=1\text{, Otherwise zero} \tag{12}$$

5. Solution of the Model

Taking Laplace transforms of equations (1) through (11) and using initial conditions one may obtain:

$$[s+3f'+f+f_e]\,\overline{P}_N^3(s)=1+\int_0^\infty \overline{P}_N^2(\mathrm{y},s)\,r_2(\mathrm{y})\,d\mathrm{y}+\int_0^\infty \overline{P}_N^F(z,s)\,r_3(z)\,dz$$

$$+\int_0^\infty \overline{P}_N^3(x,s)\,r_1(x)\,dx+\int_0^\infty \overline{P}_E(\alpha,s)\,r_4(\alpha)\,d\alpha \tag{13}$$

$$\left[\frac{\partial}{\partial \mathrm{y}}+s+2f'+r_2(\mathrm{y})\right]\overline{P}_N^2(\mathrm{y},s)=0 \tag{14}$$

$$\left[\frac{\partial}{\partial z}+s+r_3(z)\right]\overline{P}_N^F(z,s)=0 \tag{15}$$

$$\left[\frac{\partial}{\partial x}+s+r_1(z)\right]\overline{P}_i^3(x,s)=0 \tag{16}$$

$$\left[\frac{\partial}{\partial \mathrm{y}}+s+r_2(\mathrm{y})\right]\overline{P}_i^2(\mathrm{y},s)=f\,\overline{P}_N^2(\mathrm{y},s) \tag{17}$$

$$\left[\frac{\partial}{\partial \alpha}+s+r_4(\alpha)\right]\overline{P}_E(\alpha,s)=0 \tag{18}$$

$$\overline{P}_N^2(0,s)=3f'\,\overline{P}_N^3(s) \tag{19}$$

$$\overline{P}_N^F(0,s)=2f'\,\overline{P}_N^2(s) \tag{20}$$

$$\overline{P}_i^3(0,s)=f\,\overline{P}_N^3(s)+\int_0^\infty \overline{P}_i^2(\mathrm{y},s)\,r_2(\mathrm{y})\,d\mathrm{y} \tag{21}$$

$$\overline{P}_i^2(0,s)=0 \tag{22}$$

$$\overline{P}_E(0,s)=f_e\,\overline{P}_N^3(s) \tag{23}$$

After solving the above equations, we get finally

$$\overline{P}_N^3(s) = \frac{1}{A(s)} \tag{24}$$

$$\overline{P}_N^2(s) = \frac{3f'}{A(s)} D_{r_2}(s+2f'+f) \tag{25}$$

$$\overline{P}_N^F(s) = \frac{6f'^2}{A(s)} D_{r_2}(s+2f'+f) D_{r_3}(s) \tag{26}$$

$$\overline{P}_i^2(s) = \frac{3ff'}{(2f'+f)A(s)} \left[D_{r_2}(s) - D_{r_2}(s+2f'+f) \right] \tag{27}$$

$$\overline{P}_i^3(s) = \frac{f}{A(s)} \left[1 + \frac{3f'}{2f'+f} \left\{ \overline{S}_{r_2}(s) \overline{S}_{r_2}(s+2f'+f) \right\} \right] D_{\eta}(s) \tag{28}$$

$$\overline{P}_E(s) = \frac{f_e}{A(s)} D_{r_4}(s) \tag{29}$$

Where, $A(s) = s + 3f' + f + f_e - 3f' \overline{S}_{r_2}(s+2f'+f) - 6f' D_{r_2}(s+2f'+f) \overline{S}_{r_3}(s)$

$$-f \left[1 + \frac{3f'}{2f'+f} \left\{ \overline{S}_{r_2}(s) - \overline{S}_{r_2}(s+2f'+f) \right\} \right] \overline{S}_{\eta}(s) - f_e \overline{S}_{r_4}(s) \tag{30}$$

It is interesting to note that sum of relation (24) through (29) = $\frac{1}{s}$

6. Ergodic behaviour of the system

Using Abel's Lemma $\lim_{s \to 0} s\overline{F}(s) = \lim_{t \to \infty} F(t) = F$ (say), provided the limit on the R.H.S. exists, the time independent probabilities are obtained as follows by making use above lemma in the relations (24) through (29)

$$P_N^3 = \frac{1}{A'(0)} \tag{31}$$

$$P_N^2 = \frac{3f'}{A'(0)} D_{r_2}(2f'+f) \tag{32}$$

$$P_N^F = \frac{6f'}{A'(0)} D_{r_2}(2f'+f) M_{r_3} \tag{33}$$

$$P_i^2 = \frac{3ff'}{(2f'+f)A'(0)} \left[M_{r_3} - D_{r_2}(2f'+f) \right] \tag{34}$$

$$P_i^3 = \frac{f}{A'(0)} M_{\eta} \tag{35}$$

$$P_E = \frac{f_e}{A'(0)} M_{r_4} \tag{36}$$

Where, $A'(0) = \left[\frac{d}{ds} A(s) \right]_{s=0}$ and M_k = Mean time to repair k^{th} unit

7. Evaluation of up and down state probabilities

We have,

$$\overline{P}_{up}(s) = \frac{1}{s+3f'+f+f_e} \left[1 + \frac{3f'}{s+2f'+f} \right] \tag{37}$$

On inverting

$$\overline{P}_{up}(t) = \left[1 - \frac{3f'}{f'+f_e}\right]\exp\{-(3f'+f_e+f)t\} + \frac{3f'}{f'+f_e}\exp\{-(2f'+f)t\} \tag{38}$$

$$P_{down}(t) = 1 - P_{up}(t) \tag{39}$$

8. Cost Analysis

We have,

$$G(t) = C_1 \int_0^t P_{up}(t)\,dt - C_2\,t \tag{40}$$

Where,

$G(t) =$ Expected cost for total time,

$C_1 =$ Revenue cost per unit up time and $C_2 =$ Service cost per unit time

$$G(t) = C_1\left(1 - \frac{3f'}{f'+f_e}\right)\left[\frac{1-\exp\{-(3f'+f_e+f)t\}}{3f'+f_e+f}\right] + C_1\left(\frac{3f'}{f'+f_e}\right)\left[\frac{1-\exp\{-(2f'+f)t\}}{2f'+f}\right] - C_2\,t \tag{41}$$

9. Numerical Computation

Substituting $f = 0.001,\ f' = 0.002,\ f_e = 0.003,\ C_1 = 2,\ C_2 = 1$ and all repair rates are zero.

Availability

$$P_{up}(t) = -0.2\exp(-0.010\,t) + 1.2\exp(-0.005\,t)$$

Cost function analysis

$$G(t) = -0.4\left[\frac{1-\exp(-0.010\,t)}{0.010}\right] + 2.4\left[\frac{1-\exp(-0.005\,t)}{0.005}\right] - t$$

10. Interpretation

10.1 Table 1 outlines the variation of availability of the model with time and their corresponding curve

S.No.	t	$P_{up}(t)$
1	0	1
2	1	0.996005
3	2	0.9920201
4	3	0.9880452
5	4	0.9840805
6	5	0.980126
7	6	0.9761817
8	7	0.9722477
9	8	0.9683241
10	9	0.9644107
11	10	0.9605078
12	11	0.9566154
13	12	0.9527334
14	13	0.9488619
15	14	0.9450009
16	15	0.9411506

Table 1: Availability as function of time

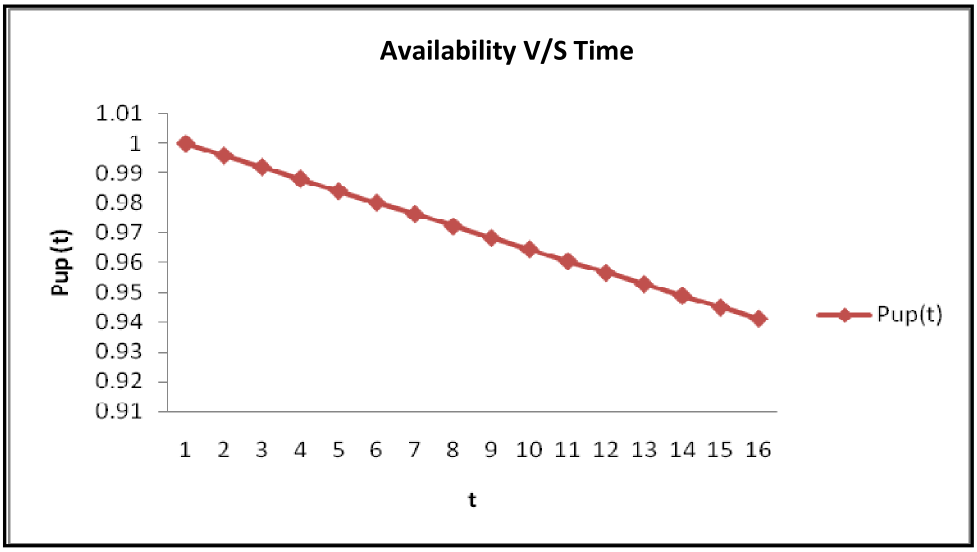

Figure 2: Availability as function of time

10.2 Table 2 exhibits expected cost function with respect to time and their corresponding curve

S.No.	*t*	*G*(*t*)
1	0	0
2	1	0.9960033
3	2	1.9840267
4	3	2.9640903
5	4	3.9362144
6	5	4.9004192
7	6	5.8567252
8	7	6.805153
9	8	7.7457231
10	9	8.6784561
11	10	9.603373
12	11	10.520494
13	12	11.429841
14	13	12.331435
15	14	13.225296
16	15	14.111446

Table 2: Cost Profit as function of time

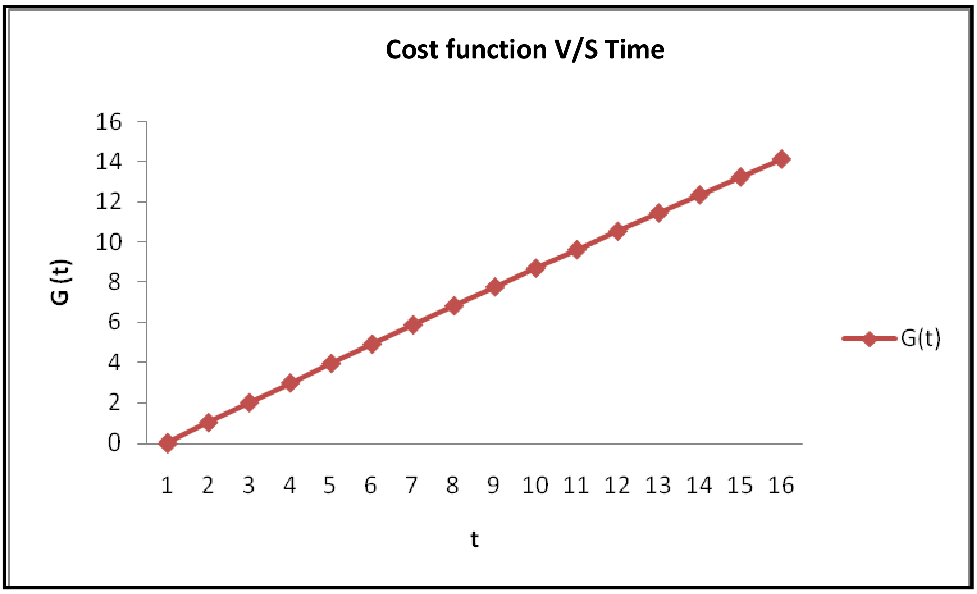

Figure 3: Cost Profit as function of

11. Conclusion

Table 1 and Figure 2 provide information how availability of the complex engineering repairable system changes with respect to the time when failure rate increases availability of the system decreases.

Table 2 and Figure 3 when revenue cost per unit time C_1 and C_2 are fixed, then one can conclude by observing this graph that as cost increases, when time increases.

Hence the present study clearly proves the importance of head-of line repair policy in comparison of [17-18] which seem to be possible in many engineering systems when it is analyzed with the help of the copula. The further research area is widely open, where one may think of the application of other members of copula family, MTTF and sensitivity analysis.

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