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# A Common Fixed Point Theorem for Two Random Operators using Random Mann Iteration Scheme

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**Abstract:** In this paper, we proved that if a random Mann iteration scheme is defined by two random operators is convergent under some contractive inequality the limit point is a common fixed point of each of two random operators in Banach space.
**Keywords:** Mann iteration, fixed point, measurable mappings, Banach space.
**AMS Subject Classification:** 47H10, 47H40.

## 1.Introduction and Preliminaries:

Kasahara [8] had shown that if an iterated sequence defined by using a continuous linear mapping is convergent under certain assumption, then the limit point is a common fixed point of each of two non-linear mappings. Ganguly [6] arrived at same conclusion by taking the same contractive condition and using the sequence of Mann iteration [9].

It this note, it is proved that if a random Mann iteration scheme is defined by two operators is convergent under some contractive inequality the limit point is a common fixed point of each of two random operators in a Banach space.

The study of random fixed point has been an active area of contemporary research in mathematics. Random iteration scheme has been elaborately discussed by Choudhury ([1], [2], [3], [4]). Looking to the immense applications of iterative algorithms in signal processing and image reconstruction, it is essential to venture upon random iteration.

We first review the following concepts, which are essential for our study.

Throughout this paper ( $\Omega, \sum$ ) denotes a measurable space and $X$ stands for a separable Banach space. $C$ is a nonempty subset of $X$.

A mapping $f:\Omega \to C$ is said to be measurable if $f^{-1}(B \cap C) \in \sum$ for every Borel subset $B$ of $X$.

A mapping $F:\Omega \times C \to C$, is said to be a random operator, if $F(.,x):\Omega \to C$ is measurable for every $x \in C$.

A measurable mapping $g:\Omega \to C$ is said to be a random fixed point of the random operator $F:\Omega \times C \to C$, if $F(t,g(t)) = g(t)$ for all $t \in \Omega$.

A random operator $F:\Omega \times C \to C$ is said to be continuous if, for fixed $t \in \Omega$, $F(t,.):C \to C$ is continuous.

**Definition 1 (Random Mann Iteration scheme):** Let $S,T:\Omega \times C \to C$ be two random operators on a nonempty convex subset $C$ of a separable Banach space $X$. Then the sequence $\{x_n\}$ of random Mann iterates associates with $S$ or $T$ is defined as follows:

(1) Let $x_0:\Omega \to C$ be any given measurable mapping.

(2) $x_{n+1}(t) = (1-c_n)x_n + c_n S(t,x_n(t))$ for $n > 0$, $t \in \Omega$, or

(3) $x_{n+1}(t) = (1-c_n)x_n + c_n T(t,x_n(t))$ for $n > 0$, $t \in \Omega$,

where $c_n$ satisfies:

(4) $c_0 = 1$ for $n = 0$

(5) $0 < c_n \le 1$ for $n > 0$

(6) $\lim_{n\to\infty} c_n = h > 0$

**2.Main Result:**

**Theorem 1:** Let $S, T : \Omega \times C \to C$, where $C$ is a nonempty closed convex subset of a separable Banach space $X$, be two continuous random operators which satisfy the following inequality: for all $x, y \in C$ and $t \in \Omega$,

$$(7)\quad \|S(t,x) - T(t,y)\| \le \alpha \max\{\|x(t) - y(t)\|, \|x(t) - T(t, y(t))\|, \|y(t) - S(t, x(t))\|\}$$
$$+ \beta \max\{\|x(t) - S(t, x(t))\|, \|y(t) - T(t, y(t))\|\}$$
$$+ \gamma \max\{\|x(t) - S(t, x(t))\| + \|x(t) - y(t)\|,$$
$$\|y(t) - T(t, y(t))\| + \|x(t) - y(t)\|\}$$

where $\alpha, \beta, \gamma \ge 0$ and $\alpha + \beta + \gamma < 1$ .

If the sequence $\{x_n(t)\}$ of random Mann iterates associated with $S$ or $T$ satisfying (1)-(6) converges, then it converges to a common random fixed point of both $S$ and $T$.

**Proof:** We may assume that the sequence $\{x_n(t)\}$ defined by (2) is pointwise convergent, that is, for all $t \in \Omega$,

$$(8)\qquad x(t) = \lim_{n\to\infty} x_n(t)$$

Since $X$ is a separable Banach space, for any continuous random operator $A : \Omega \times C \to C$, and any measurable mapping $f : \Omega \to C$, the mapping $x(t) = A(t, f(t))$ is measurable mapping [7].

Since $x(t)$ is measurable and $C$ is convex, it follows that $\{x_n(t)\}$ constructed in the random iteration from (2)-(6) is a sequence of measurable mappings. Hence being limit of measurable mapping sequence is also measurable. Now for $t \in \Omega$, from (2), (6) and (7) we obtain

$$\|x(t) - T(t, x(t))\| \le \|x(t) - x_{n+1}(t)\| + \|x_{n+1}(t) - T(t, x(t))\|$$
$$\le \|x(t) - x_{n+1}(t)\| + \|(1 - c_n)x_n(t) + c_n S(t, x_n(t)) - T(t, x(t))\|$$
$$\le \|x(t) - x_{n+1}(t)\| + (1 - c_n)\|x_n(t) - T(t, x(t))\|$$
$$+ c_n \|S(t, x_n(t)) - T(t, x(t))\|$$

$$(9)\quad \|x(t) - T(t, x(t))\| \le \|x(t) - x_{n+1}(t)\| + (1 - h)\|x_n(t) - T(t, x(t))\|$$
$$+ c_n[\alpha \max\{\|x_n(t) - x(t)\|, \|x_n(t) - T(t, x(t))\|,$$
$$\|x(t) - S(t, x_n(t))\|\} + \beta \max\{\|x_n(t) - S(t, x_n(t))\|,$$
$$\|x(t) - T(t, x(t))\|\} + \gamma \max\{\|x_n(t) - S(t, x_n(t))\| +$$
$$\|x_n(t) - x(t)\|, \|x(t) - T(t, x(t))\| + \|x_n(t) - x(t)\|\}]$$

Now $c_n(S(t, x_n(t)) - x_n(t)) = c_n S(t, x_n(t)) - c_n x_n(t) = x_{n+1}(t) - x_n(t)$

Implies that $\|S(t, x_n(t)) - x_n(t)\| \le \frac{1}{c_n}\|x_{n+1}(t) - x_n(t)\|$

This shows that for $t \in \Omega$, $S(t, x_n(t)) - x_n(t) \to 0$ as $n \to \infty$ and so $S(t, x_n(t)) - x(t) \to 0$ as $n \to \infty$ as $S$ is continuous random operator and $x$ is a measurable mapping. Consequently from (9) on taking limit as $n \to \infty$ we obtain

$$\|x(t) - T(t, x(t))\| \le 0 + (1 - h)\|x(t) - T(t, x(t))\| + c_n[\alpha \max\{0, \|x(t) - T(t, x(t))\|, 0\}$$
$$+ \beta \max\{0, \|x(t) - T(t, x(t))\|\} + \gamma \max\{0, \|x(t) - T(t, x(t))\|\}]$$

$$\leq (1-h+h\alpha+h\beta+h\gamma)\|x(t)-T(t,x(t))\|$$

implies that $T(t,x(t))=x(t)$ for all $t\in\Omega$ $(\sin ce\, \alpha+\beta+\gamma<1)$ as $T$ is continuous random operator and $x$ is measurable.

Therefore,

$$\begin{aligned}\|S(t,x(t))-x(t)\| &= \|S(t,x(t))-T(t,x(t))\| \\ &\leq \alpha\max\{\|x(t)-x(t)\|,\|x(t)-T(t,x(t))\|,\|x(t)-S(t,x(t))\|\} \\ &\quad+\beta\max\{\|x(t)-S(t,x(t))\|,\|x(t)-T(t,x(t))\|\} \\ &\quad+\gamma\max\{\|x(t)-S(t,x(t))\|+\|x(t)-x(t)\|, \\ &\qquad\|x(t)-T(t,x(t))\|+\|x(t)-x(t)\|\}\end{aligned}$$

$$\begin{aligned}\|S(t,x(t)-x(t))\| &\leq \alpha\max\{\|x(t)-x(t)\|,\|x(t)-x(t)\|,\|x(t)-S(t,x(t))\|\} \\ &\quad+\beta\max\{\|x(t)-S(t,x(t))\|,\|x(t)-x(t)\|\} \\ &\quad+\gamma\max\{\|x(t)-S(t,x(t))\|+\|x(t)-x(t)\|, \\ &\qquad\|x(t)-x(t)\|+\|x(t)-x(t)\|\} \\ &\leq \alpha\max\{0,0,\|x(t)-S(t,x(t))\|\}+\beta\max\{\|x(t)-S(t,x(t))\|,0\} \\ &\quad+\gamma\max\{\|x(t)-S(t,x(t))\|,0\} \\ &\leq (\alpha+\beta+\gamma)\|x(t)-S(t,x(t))\|\end{aligned}$$

Since $\alpha+\beta+\gamma<1$ implies that $S(t,x(t))=x(t)$.

Uniquness:-Let $v(t)$, $v(t)\neq x(t)$ is another common fixed point of $S$ and $T$ ,then, using (7), we have

$$\begin{aligned}\|x(t)-v(t)\| &\leq \alpha\max\{\|x(t)-v(t)\|,\|x(t)-T(t,v(t))\|,\|v(t)-S(t,x(t))\|\} \\ &\quad+\beta\max\{\|x(t)-S(t,x(t))\|,\|v(t)-T(t,v(t))\|\} \\ &\quad+\gamma\max\{\|x(t)-S(t,x(t))\|+\|x(t)-v(t)\|, \\ &\qquad\|v(t)-T(t,v(t))\|+\|x(t)-v(t)\|\} \\ &\leq \alpha\max\{\|x(t)-v(t)\|,\|x(t)-v(t)\|,\|v(t)-x(t)\|\} \\ &\quad+\beta\max\{\|x(t)-x(t)\|,\|v(t)-v(t)\|\} \\ &\quad+\gamma\max\{\|x(t)-x(t)\|+\|x(t)-v(t)\|, \\ &\qquad\|v(t)-v(t)\|+\|x(t)-v(t)\|\} \\ &\leq (\alpha+\gamma)\|x(t)-v(t)\|\end{aligned}$$

$$\Rightarrow x(t)=v(t) \quad \text{as} \quad \alpha+\gamma<1$$

This complete the proof.

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