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Fixed Point Theorem in Fuzzy Metric Space with E.A Property

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Abstract
In this paper, we prove common fixed point theorems in Fuzzy Metric spaces for weakly compatible mappings along with property (E.A.) .Property (E.A.) buys containment of ranges without any continuity requirement besides minimizing the commutatively conditions of the maps to commutatively at their point of coincidence. Moreover, property (E.A.) allows replacing the completeness requirement of the space with a more natural condition of closeness of the range.
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1. INTRODUCTION

It proved a turning point in the development of Mathematics when the notion of Fuzzy set was introduced by Zadeh [23] which laid the foundation of Fuzzy Mathematics. Fuzzy set theory has applications in applied sciences such as neural network theory, stability theory, Mathematical Programming, Modeling theory, Engineering Sciences, Medical Sciences (medical genetics, nervous system), image processing, control theory, communication etc.

Kramosil and Michalek [9] introduced the notion of a Fuzzy Metric space by generalizing the concept of the probabilistic metric space to the fuzzy situation. George and Veeramani[5] modified the concept of fuzzy metric spaces introduced by Kramosil and Michalek[9]. There are many view points of the notion of the metric space in fuzzy topology for instance one can refer to Kaleva and Seikkala [8], Kramosil and Michalek [9], George and Veeramani [5], the E.A property is introduced by Aamri, M. and Moutawakil [1] Regan and Abbas [2] obtained some necessary and sufficient conditions for the existence of common fixed point in fuzzy metric spaces .Popa ([14]- [15]) introduced the idea of implicit function to prove a common fixed point theorem in metric spaces . Singh and Jain[7] further extended the result of Popa ([14]- [15]) in Fuzzy Metric spaces. For the reader convenience, we recall some terminology from the theory of Fuzzy Metric spaces.

2. PRELIMINARIES

Definition 2.1. ([23])

Let X be any non empty set. A Fuzzy set M in X is a function with domain X and values in [0, 1].

Definition 2.2. ([17])

A mapping $*: [0, 1]\times[0, 1] \rightarrow [0, 1]$ is called a continuous t-norm if ([0, 1],*) is an abelian topological monoid with unit 1such that, $a *b \le c* d$, for $a \le c$, $b \le d$.

Example :2.1

1) **$a*b = ab$**
2) **$a*b= \min\{a,b\}$**

Definition 2.3. ([9])

The 3 − tuple (X,M,*) is called a Fuzzy metric space in the sense of Kramosil and Michalek if X is an arbitrary set,* is a continuous t − norm and M is a Fuzzy set in $X^2\times[0,\infty)$ satisfying the following conditions:

(a) $M(x, y, t) > 0$,
(b) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,
(c) $M(x, y, t) = M(y, x, t)$,
(d) $M(x, y, t) *M(y, z, s) \le M(x, z, t + s)$,
(e) $M(x, y, .) : [0,\infty) \rightarrow [0, 1]$ is a continuous function,
for all $x, y, z \in X$ and $t, s > 0$.

M(x, y, t) can be thought as degree of nearness between x and y with respect to t.

It is known that M(x,y, .) is non decreasing for all $x, y \in X$([5]).

Definition 2.4.

A sequence $\{x_n\}$ in X converges to x if and only if for each $t >0$ there exists $n_0 \in N$,such that, $M(x_n, x, t) = 1$, for all $n \ge n_0$.

Definition 2.5

The sequence $\{x_n\}$ $n \in N$ is called Cauchy sequence if $\lim_{n\to\infty} M(x_n, x_{n+p}, t) = 1$, for all $t > 0$ and $p \in N$.

Definition 2.6
A Fuzzy Metric space X is called complete if every Cauchy sequence is convergent in X.

Definition 2.7. [20]
Two Self mappings A and S of a Fuzzy Metric space (X, M, *) are said to be weakly compatible if they commute at their coincidence points, i.e., $Ax = Sx$ implies
$ASx = SAx$.

Definition 2.8 [1]
Two Self mappings A and S on a fuzzy metric space (X,M,*) are said to satisfy the property (E.A) if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.

MAIN RESULT

THEOREM 3.1 :
Let A,B ,S & T be self map on a fuzzy metric space (X,M,*) where * is a continuous norm such that $a*b=\min\{a,b\}$ satisfying the following condition

(i) $A(x) \cup B(x) \subset S(x) \cap T(x)$
(ii) $\{A,T\}$ & $\{B,S\}$ are satisfy E.A Property &weakly compatible
(iii) There exists $k \in (0,1)$ such that
$M(Ax,By,t) \geq M(Tx,Sy,kt)*M(Tx,Ax,kt)*M(By,Ax,kt)*\max\{M(Ax,Sy,kt) *M(By,Sy,kt)\}$ -------- (3.1.1) Equation
(iv) T(x) is closed subspace of X
Then A ,B, S, T have unique common fixed point.

Proof :
Suppose the pair {B,S} satisfy E.A. property therefore there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Bx_n = z = \lim_{n\to\infty} Sx_n$ for some $z \in X$.
Now $B(x) \subset T(x)$ it implies that there is a sequence $\{y_n\}$ in X such that $Bx_n = Ty_n$.
Put $x = y_n$ and $y = x_n$ in Equation—(3.1.1)
$M(Ay_n,Bx_n,t) \geq M(Ty_n,Sx_n,kt)*M(Ty_n,Ay_n,kt)*M(Bx_n,Ay_n,kt)*\max\{M(Ay_n,Sx_n,kt),M(Bx_n,Sx_n,kt)\}$
$M(Ay_n,Bx_n,t) \geq M(Bx_n,Sx_n,kt)* M(Bx_n,Ay_n,kt) * M(Bx_n,Ay_n,kt)*$
$\max\{ M(Ay_n,Sx_n,kt),M(Bx_n,Sx_n,kt)\}$
Taking limit $n \to\infty$
$M(\lim_{n\to\infty}Ay_n,z,t) \geq M(z,z,kt)* M(z, \lim_{n\to\infty}Ay_n,kt) * M(z, \lim_{n\to\infty}Ay_n,kt)*$
$\max\{ M(\lim_{n\to\infty}Ay_n,z,kt), M(z,z,kt)\}$
$M(\lim_{n\to\infty}Ay_n,z,t) \geq 1* M(z, \lim_{n\to\infty}Ay_n,kt) * M(z \lim_{n\to\infty},Ay_n,kt)*$
$\max\{ M(\lim_{n\to\infty}Ay_n,z,kt),1\}$

$M(\lim_{n\to\infty}Ay_n,z,t) \geq M(\lim_{n\to\infty}Ay_n,z,kt)$
This implies a contradiction
$\lim_{n\to\infty}Ay_n = z$
Suppose T(x) is a closed subspace of X then $z=Tu$ for some $u \in X$
If $Au=z$ if not,
Put $x=u$ & $y=x_n$ in Eq –(3.1.1)
$M(Au,Bx_n,t) \geq M(Tu,Sx_n,kt)* M(Tu,Au,kt) * M(Bx_n,Au,kt)*\max\{ M(Au, Sx_n,kt), M(Bx_n, Sx_n,kt) \}$
$M(Au ,z, t) \geq M(z, z ,kt)* M(z, Au ,kt) * M(z, Au, kt)*\max\{ M(Au ,z, kt), M(z, z, kt) \}$
$M(Au ,z, t) \geq 1* M(z,Au,kt) * M(z,Au,kt)*\max\{ M(Au, z,kt),1\}$
$M(Au, z, t) \geq M(Au ,z, kt)$
This implies a contradiction
Therefore $Au=z$
Hence $Au= Tu = z$
Since $A(x) \subset S(x)$ $\exists$ $u \in X$ such that $z = Sv$
If $Bv = z$ if not,
Put $x=y_n$ & $y=v$ in Equation —3.1.1
$M(A y_n,Bv,t) \geq M(T y_n,Sv,kt)* M(Ty_n, A y_n, kt) * M(Bv,A y_n, kt)*$

$\max\{ M(A y_n ,Sv,kt), M(Bv, Sv,kt)\}$

$M(A y_n,Bv,t) \geq M(B x_n,Sv,kt)* M(B x_n, A y_n, kt) * M(Bv,A y_n, kt)*$
$\max\{ M(A y_n ,z,kt), M(Bv, z,kt),\}$

Taking limit $n \to\infty$

$M(z,Bv,t) \geq M(z,z,kt)* M(z,z, kt) * M(Bv,z, kt)*\max\{ M(z,z ,kt), M(Bv,z,kt)\}$

$M(z,Bv,t) \geq M(z,Bv,kt)$

$Sv = Bv = z$

Since $\{A,T\}$ is weakly compatible

$ATu=TAu$

$Az=Tz$

$Az = Tz = z$

Similarly $Bz=Sz=z$

$\therefore$ $z= Az = Bz = Tz = Sz$.

Thus z is a common fixed point of A,B,S&T.

Uniqueness:

Let w be another fixed point of A ,B,S, T

Put $x=z$ & $y=w$ in Equation—(3.1.1)

$M(Az,Bw,t) \geq M(Tz,Sw,kt)* M(Tz,Az,kt) * M(Bw,Az,kt)*\max\{ M(Aw, Sw,kt), M(Bw, Sw,kt)\}$

$M(z,w,t) \geq M(z,w,kt)* M(z,z,kt) * M(w,z,kt)*\max\{ M(w, w,kt), M(w,w,kt)\}$

$M(z,w,t) \geq M(z,w,kt)$

This implies a contradiction.

Therefore, $z=w$

Thus z is a unique common fixed point of A,B,S & T

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