Mathematical Theory and Modeling www.iiste.org ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online) Vol.3, No.6, 2013-Selected from International Conference on Recent Trends in Applied Sciences with Engineering Applications

Some Fixed Point Theorem for Expansive Type Mapping in Dislocated Metric Space

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Abstract

The purpose of this paper is to present some fixed point theorem in dislocated quasi metric space for expansive type mappings.

Mathematics Subject Classification: 54H25

Keywords: Dislocated Quasi Metric space, fixed point.

Introduction and Preliminaries:

It is well known that Banach Contraction mappings principle is one of the pivotal results of analysis. Generalizations of this principle have been obtained in several directions .Dass and Gupta [1] generalized Banach's Contraction principle in metric space. Also Rhoades [2] established a partial ordering for various definitions of contractive mappings. In 2005, Zeyada Salunke [4] proved some results on fixed point in dislocated quasimetric spaces. In 2005, Zeyada et al.[3] established a fixed point theorem in dislocated quasimetric spaces. In 2008, Aage and Salunke [4] proved some results on fixed point in dislocated quasimetric spaces. Recently, Isufati [5], proved fixed point theorem for contractive type condition with rational expression in dislocated quasimetric spaces. The following definitions will be needed in the sequel.

Definition 1.1(See [3]). Let *X* be a nonempty set, and let $d: X \times X \rightarrow [0, \infty)$ be a function, called a distance function. One needs the following conditions:

 $(M1) d(x, x) = 0$,

 $(M2) d(x, y) = d(y, x) = 0$, then $x = y$

 $(M3) d(x, y) = d(y, x),$

 $(M4) d(x, y) \leq d(x, z) + d(z, y),$

 $(M4)$ ['] $d(x, y) \le \max\{d(x, z), d(z, y)\}\,$, for all $x, y, z \in X$.

If d satisfies conditions (M1)-(M4), then it is called a metric on X . If d satisfies conditions (M1), (M2), and (M4), it is called a quasimetric on X . If it satisfies conditions (M2)-(M4) ((M2) and (M4)), it is called a dislocated metric (or simply d-metric) (a dislocated quasimetric (or simply dq-metric)) on *X* , respectively. If a metric d satisfies the strong triangle inequality (M) , then it is called an ultrametric.

Definition 1.2 (See [3]). A sequence $\{x_n\}_{n\in\mathbb{N}}$ in dq-metric space (dislocated quasimetric space) (X,d) is called a Cauchy sequence if, for given $\varepsilon > 0$, there exists $n_0 \in N$ such that $d(x_m, x_n) < \varepsilon$ or $d(x_n, x_m) < \varepsilon$, that is, $\min\{d(x_m, x_n), d(x_n, x_m)\} < \varepsilon$ for all $m, n \ge n_0$.

Definition 1.3 (See [3]). A sequence $\{x_n\}_{n\in\mathbb{N}}$ in dq-metric space [d-metric space] is said to be d-converge to $x \in X$ provided that

$$
\lim_{n \to \infty} d(x_n, x) = \lim_{n \to \infty} d(x, x_n) = 0
$$
\n(1.1)

In this case, *x* is called a dq-limit [d-limit] of $\{x_n\}$ and we write $x_n \to x$.

Definition 1.4 (See [3]). A dq-metric space (X,d) is called complete if every Cauchy sequence in it is a dqconvergent.

Main Results

In this paper, we prove some fixed point theorem for continuous mapping satisfying expansion condition in complete dq-metric space.

Theorem 2.1: Let (X,d) be a complete dislocated metric space and T a continuous mappings satisfying the following condition:

Mathematical Theory and Modeling www.iiste.org

ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)

Vol.3, No.6, 2013-Selected from International Conference on Recent Trends in Applied Sciences with Engineering Applications

$$
d(Tx,Ty) + \alpha \left[\frac{d(x,Ty) + d(y,Tx)}{1 + d(x,Ty)d(y,Tx)} \right] \ge \beta \frac{d(x,Tx)[1 + d(y,Ty)]}{1 + d(x,y)} + \gamma d(x,y) \tag{2.1}
$$

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For all $x, y \in X$, $x \neq y$, where $\alpha, \beta, \gamma \geq 0$ are real constants and $\beta + \gamma > 1 + 2\alpha$, $\gamma > 1 + \alpha$. Then *T* has a fixed point in *X* .

Proof: Choose $x_0 \in X$ be arbitrary, to define the iterative sequence $\{x_n\}_{n\in N}$ as follows and $Tx_n = x_{n-1}$ for $n = 1, 2, 3, \dots$ Then, using (2.1) we obtain

$$
d(Tx_{n+1}, Tx_{n+2}) + \alpha \left[\frac{d(x_{n+1}, Tx_{n+2}) + d(x_{n+2}, Tx_{n+1})}{1 + d(x_{n+1}, Tx_{n+2}) d(x_{n+2}, Tx_{n+1})} \right] \geq \beta \frac{d(x_{n+1}, Tx_{n+1}) [1 + d(x_{n+2}, Tx_{n+2})]}{1 + d(x_{n+1}, x_{n+2})}
$$

+ $\gamma d(x_{n+1}, x_{n+2})$

$$
\Rightarrow d(x_n, x_{n+1}) + \alpha \left[\frac{d(x_{n+1}, x_{n+1}) + d(x_{n+2}, x_n)}{1 + d(x_{n+1}, x_{n+1}) + d(x_{n+2}, x_n)} \right] \geq \beta \frac{d(x_{n+1}, x_n)[1 + d(x_{n+2}, x_{n+1})]}{1 + d(x_{n+1}, x_{n+2})} + \gamma d(x_{n+1}, x_{n+2})
$$

\n
$$
\Rightarrow d(x_n, x_{n+1}) + \alpha d(x_{n+2}, x_n) \geq \beta d(x_n, x_{n+1}) + \gamma d(x_{n+1}, x_{n+2})
$$

\n
$$
\Rightarrow d(x_n, x_{n+1}) + \alpha d(x_n, x_{n+1}) + \alpha d(x_{n+1}, x_{n+2}) \geq \beta d(x_n, x_{n+1}) + \gamma d(x_{n+1}, x_{n+2})
$$

\n
$$
\Rightarrow (1 + \alpha - \beta) d(x_n, x_{n+1}) \geq (\gamma - \alpha) d(x_{n+1}, x_{n+2})
$$

\nThe last inequality gives

The last inequality gives

$$
d(x_{n+1}, x_{n+2}) \le \left(\frac{1+\alpha-\beta}{\gamma-\alpha}\right) d(x_n, x_{n+1})
$$

\$\le kd(x_n, x_{n+1})\$ (2.2)

Where $k = \frac{(1 + \alpha)^2}{2} < 1$ $(\gamma - \alpha)$ $\frac{(1+\alpha-\beta)}{2}$ − $=\frac{(1+\alpha-1)}{2}$ $\gamma - \alpha$ $k = \frac{(1+\alpha-\beta)}{n} < 1$. Hence by induction, we obtain

$$
d(x_{n+1}, x_{n+2}) \leq k^{n+1} d(x_0, x_1)
$$

Note that, for $m, n \in N$ such that $m > n$ we have

$$
d(x_m, x_n) \le d(x_m, x_{m-1}) + d(x_{m-1}, x_{m-2}) + \dots + d(x_{n+1}, x_n)
$$

\n
$$
\le [k^{m-1} + k^{m-2} + \dots + k^{n}] d(x_0, x_1)
$$

\n
$$
\le k^n (1 + k + k^2 + \dots + k^{m-n-1}) d(x_0, x_1)
$$

\n
$$
\le k^n \sum_{r=0}^{\infty} k^r d(x_0, x_1)
$$

\n
$$
= \frac{k^n}{1 - k} d(x_0, x_1)
$$
 (2.3)

Since $0 \le k < 1$, then as $n \to \infty$, $k^{n}(1-k)^{-1} \to 0$. Hence, $d(x_m, x_n) \to 0$ as $m, n \to \infty$. This forces that $\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence in X. But X is a complete dislocated metric space; hence, $\{x_n\}_{n\in\mathbb{N}}$ is dconverges. Call the d-limit $x^* \in X$. Then, $x_n \to x^*$ as $n \to \infty$. By continuity of *T* we have,

$$
Tx^* = T\Big(d - \lim_{n \to \infty} x_n\Big) = d - \lim_{n \to \infty} Tx_n = d - \lim_{n \to \infty} x_{n-1} = x^*
$$
\n(2.4)

That is, $Tx^* = x^*$; thus, T has a fixed point in X. **Uniqueness**

Let y^* be another fixed point of *T* in *X*, then $Ty^* = y^*$ and $Tx^* = x^*$.now,

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Vol.3, No.6, 2013-Selected from International Conference on Recent Trends in Applied Sciences with Engineering Applications

$$
d(Tx^*, Ty^*) + \alpha \left[\frac{d(x^*, Ty^*) + d(y^*, Tx^*)}{1 + d(x^*, Ty^*) d(y^*, Tx^*)} \right] \geq \beta \frac{d(x^*, Tx^*) + d(y^*, Ty^*)}{1 + d(x^*, y^*)} + \gamma d(x^*, y^*) \qquad (2.5)
$$

This implies that
\n
$$
d(x^*, y^*) + \alpha \left[\frac{d(x^*, y^*) + d(y^*, x^*)}{1 + d(x^*, y^*) d(y^*, x^*)} \right] \geq \beta \frac{d(x^*, x^*) + d(y^*, y^*)}{1 + d(x^*, y^*)} + \gamma d(x^*, y^*)
$$
\n
$$
\Rightarrow d(x^*, y^*) + \frac{2\alpha d(x^*, y^*)}{1 + [d(x^*, y^*)]^2} \geq \gamma d(x^*, y^*)
$$
\n
$$
\Rightarrow d(x^*, y^*) + [d(x^*, y^*) + 2\alpha d(x^*, y^*) \geq \gamma d(x^*, y^*) + \gamma [d(x^*, y^*)]^3
$$
\n
$$
\Rightarrow (1 + 2\alpha - \gamma) d(x^*, y^*) \geq (\gamma - 1)[d(x^*, y^*)]^3
$$
\n
$$
d(x^*, y^*) \leq \left(\frac{1 + 2\alpha - \gamma}{\gamma - 1} \right)^{\frac{1}{3}} d(x^*, y^*) \qquad (2.6)
$$
\n
$$
\gamma = \gamma \left(\frac{1 + 2\alpha - \gamma}{\gamma - 1} \right) \frac{d(x^*, y^*)}{1 + \gamma - 1} \tag{2.6}
$$

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This is true only when $d(x^*, x^*) = 0$. Similarly $d(y^*, x^*) = 0$. Hence $d(x^*, y^*) = d(y^*, x^*) = 0$ and so $x^* = y^*$. Hence, *T* has a unique fixed point in *X*.

Theorem 2.2: Let (X, d) be a complete dislocated metric space and *T* a sujective mapping satisfying the condition (2.1) for all $x, y \in X, x \neq y$, where $\alpha, \beta, \gamma \geq 0$ are real constants and $\beta + \gamma > 1 + 2\alpha$, $\gamma > 1 + \alpha$. Then, *T* has a fixed point in *X*.

Proof : Choose $x_0 \in X$ to be arbitrary, and define the iterative sequence $\{x_n\}_{n\in N}$ as follows : $Tx_n = x_{n-1}$ for $n = 1, 2, 3, \dots$ Then, using (2.1), we obtain, sequence $\{x_n\}_{n \in \mathbb{N}}$ is a Cauchy sequence in *X* .But *X* is a complete dislocated metric space ; hence $\{x_n\}_{n\in\mathbb{N}}$ is a d-converges. Call the d-limit $x^* \in X$. Then, $x_n \to x^*$ as $n \to \infty$.

Existence of fixed point

Since *T* is a surjective map, so there exists a point *y* in *X*, such that $x = Ty$. Consider

$$
d(x_n, x) = d(Tx_{n+1}, Ty)
$$

\n
$$
\geq -\alpha \left[\frac{d(x_{n+1}, Ty) + d(y, Tx_{n+1})}{1 + d(x_{n+1}, Ty) d(y, Tx_{n+1})} \right] + \beta \frac{d(x_{n+1}, Tx_{n+1})[1 + d(y, Ty)]}{1 + d(x_{n+1}, y)} + \gamma d(x_{n+1}, y)
$$
\n(2.7)

Taking
$$
n \to \infty
$$
, we get
\n
$$
d(x,x) \ge -\alpha \left[\frac{d(x,x) + d(y,x)}{1 + d(x,x)d(y,x)} \right] + \beta \frac{d(x,x)[1 + d(y,x)]}{1 + d(x,y)} + \gamma d(x,y)
$$
\n
$$
0 \ge -\alpha d(x,y) + \gamma d(x,y)
$$
\n
$$
\Rightarrow (\gamma - \alpha) d(x,y) \le 0
$$
\n
$$
\Rightarrow d(x,y) = 0 \text{ as } \gamma > \alpha
$$
\n(2.8)

Similarly, $d(y, x) = 0$. Hence $d(x, y) = d(y, x) = 0$ This implies $x = y$ and so $Tx = x$, that is *x* is fixed point of *T*.

Uniqueness

Let
$$
y^*
$$
 be another fixed point of T in X , then $Ty^* = y^*$ and $Tx^* = x^*$. Now,
\n
$$
d(Tx^*, Ty^*) + \alpha \left[\frac{d(x^*, Ty^*) + d(y^*, Tx^*)}{1 + d(x^*, Ty^*) d(y^*, Tx^*)} \right] \ge \beta \frac{d(x^*, Tx^*) \left[1 + d(y^*, Ty^*) \right]}{1 + d(x^*, y^*)} + \gamma d(x^*, y^*)
$$

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This implies that

$$
d(x^*, y^*) + \alpha \left[\frac{d(x^*, y^*) + d(y^*, x^*)}{1 + d(x^*, y^*) d(y^*, x^*)} \right] \geq \beta \frac{d(x^*, x^*) [1 + d(y^*, y^*)]}{1 + d(x^*, y^*)} + \gamma d(x^*, y^*)
$$

\n
$$
\Rightarrow d(x^*, y^*) + \frac{2\alpha d(x^*, y^*)}{1 + [d(x^*, y^*)]^2} \geq \gamma d(x^*, y^*)
$$

\n
$$
\Rightarrow d(x^*, y^*) + [d(x^*, y^*)]^3 + 2\alpha d(x^*, y^*) \geq \gamma d(x^*, y^*) + \gamma [d(x^*, y^*)]^2
$$

\n
$$
\Rightarrow (1 - 2\alpha - \gamma) d(x^*, y^*) \geq (\gamma - 1)[d(x^*, y^*)]^3
$$

\n
$$
\Rightarrow d(x^*, y^*) \leq \left(\frac{1 + 2\alpha - \gamma}{\gamma - 1}\right)^{\frac{1}{3}} d(x^*, y^*)
$$
\nThis is true only when $d(x^*, y^*) = 0$, similarly $d(y^*, y^*) = 0$. Hence $d(x^*, y^*) = d(y^*, y^*) = 0$ and

This is true only when $d(x^*, y^*) = 0$. Similarly, $d(y^*, x^*) = 0$. Hence $d(x^*, y^*) = d(y^*, x^*) = 0$ and so $x^* = y^*$. Hence *T* has a unique fixed point in *X*.

The proof is completed.

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