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Heat And Mass Transfer Of Magnetohydrodynamic (Mhd) And Dissipative Fluid Flow Past A Moving Vertical Porous Plate With Variable Suction

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ABSTRACT

An investigation was carried out on the radiation effect on unsteady heat and mass transfer of MHD and dissipative fluid flow past a moving vertical porous plate with variable suction in the presence of heat generation and chemical reaction. The dimensionless governing equations for this model were solved analytically using perturbation method. The effects of various parameters on the velocity, temperature and concentration fields as well as the Coefficient of skin-friction, Nusselt number and Sherwood number were presented graphically and in tabulated forms.

Keywords: Chemical reaction, Unsteady, Porous medium, MHD, Radiation, Mass transfer and Heat source

Subject Classification: 76w05.

1 INTRODUCTION

The effect of thermal radiation is significant in some industrial application such as glass production and furnace design and in space technology application such as cosmical flight, aerodynamics rocket, propulsion system, plasma physics which operate at high temperature. Consequently, Chamkha (2003) studied the MHD flow of a numerical of uniformly stretched vertical permeable surface in the presence of heat generation/ absorption and a chemical reaction. Chamkha, Takhar and soundalgekar (2001) studied the radiation effect on the free convection flow past a semi-infinite vertical plate with mass transfer. F.M. Hady et al. (2006) researched on the problem of free convection flow along a vertical wavy surface embedded in electrically conducting fluid saturated porous media in the presence of internal heat generation or absorption effect. Gnaneshwara and Bhaskar (2009) investigated the radiation and mass transfer effects on an unsteady MHD free convection flow past a heated vertical porous plate with viscous dissipation. Kim and Fedorov (2004) studied transient mixed radiative convection flow of a micro polar fluid past a moving semi-infinite vertical porous plate while K.Vajravelu and Hadjinicolaou(1993) studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation.

The study of heat generation or absorption effects in moving fluids is important in view of several physical problems such as fluids undergoing exothermic or endothermic or transfer chemical reactions. M.A. Hossain et.al.(2004) investigated the problem of natural convection flown along a vertical wavy surface with uniform surface temperature in the presence of heat generation/ absorption. In this direction M.A. Alam et al.(2006) studied the problem of free convection heat and mass transfer flow past an inclined semi-infinite heated surface of an electrically conducting and steady viscous incompressible fluid in the presence of a magnetic field and heat generation. Md Abdus and Mohammed M.R.(2006) considered the thermal radiation interaction with unsteady MHD flow past a vertical porous plate immersed in a porous medium. The importance of radiation in the fluid led Muthucumaraswamy and Chandrakala (2006) to study radiative heat and mass transfer effect on moving isothermal vertical plate in the presence of chemical reaction. Muthucumaraswamy and Senthih (2004) considered a Heat and Mass transfer effect on moving vertical plate in the presence of thermal radiation.

In many chemical engineering processes, the chemical reaction do occur between a mass and fluid in which plate is moving. These processes take place in numerous industrial applications such as polymer production, manufacturing of ceramics or glassware and food processing. In the light of the fact that, the combination of heat and mass transfer problems with chemical reaction are of importance in many processes, and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries. For example, in the power industry, among the methods of generating electricity is one in which electrical energy is extracted directly from the moving conducting fluid. Naving Kumar and Sandeep Gupta (2008) investigated the effect of variable permeability on unsteady two-dimensional free convective flow through a porous bounded by a vertical porous surface. P.R. Sharma, Navin and Pooja (2011) have studied the Influence of chemical reaction on unsteady MHD free convective flow and mass transfer through viscous incompressible fluid past a heated vertical plate immersed in porous medium in the presence of heat source. R. Muthucumaraswamy and Ganesan(2001) studied the effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. R.A Mohammed (2009) studied double-diffusive convection-radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and soret effects. Soundalgakar.V.M.(1972) have Studied the Viscous dissipative effects on unsteady free convective flow past a vertical porous plate with constant suction. Soundalgakar V.M et al (1979), considered the effect of mass transfer and free convection effect on MHD stokes problem for a vertical plate.

Base on these investigations, work has been reported in the field. In particular, the study of heat and mass transfer, heat radiation is of considerable importance in chemical and hydrometallurgical industries. Mass transfer process is evaporation of water from a pound to the atmosphere the diffusion of chemical impurities in lakes, rivers and ocean from natural or artificial sources. Magneto hydrodynamic mixed convection heat transfer flow in porous plate and non-porous media is of considerable interest in the technical field due to its frequent occurrence in industrial technology and geothermal application, high temperature plasma application to nuclear fusion energy conversion, liquid metal fluid and MHD power generation systems combined heat mass transfer in natural convective flows on moving vertical porous plate. Soundalgakar.V.M and Tarkhar (1993) analyzed the radiation effect on free convection flow past a semi-infinite vertical plate. V.Srinvasa Rao and L.An and Babu(2010), studied the finite element analysis of radiation and mass transfer flow past semi-infinite moving vertical plate with viscous dissipation. Jimoh .A. (2012), Heat and mass transfer of magneto hydrodynamic (MHD) and dissipative fluid flow pass a moving vertical porous plate with variable suction.

Despite all these studies, the unsteady MHD for a heat generating fluid with thermal radiation and chemical reaction has little attention. Hence, the main objective of the present investigation is to study the effect of a second-order homogeneous chemical reaction, thermal radiation, heat source and dissipative on the unsteady MHD fluid flow past a vertical porous plate with variable suction. It is assumed that the plate is embedded in a uniform porous medium and moves with a constant velocity in the flow direction in the presence of a transverse magnetic field with oscillating free stream.

2 MATHEMATICAL ANALYSIS

Consider unsteady two-dimensional hydro magnetic laminar, incompressible, viscous, electrically conducting and heat source past a semi-infinite vertical moving heated porous plate embedded in a porous medium and subjected to a uniform transverse magnetic field in the presence of thermal diffusion, chemical reaction and thermal radiation effects. According to the coordinate system, the x-axis is taken along the plate in upward direction and y-axis is normal to the plate. The fluid is assumed to be a gray, absorbing-emitting but non-scattering medium. The radiative heat flux in the x-direction is considered negligible in comparison with that in the y-direction [3]. It is assumed that there is no applied voltage of which implies the absence of an electric field. The transversely applied magnetic field and magnetic Reynolds number are very small and hence the induced magnetic field is negligible. Viscous and Darcy resistance terms are taken into account the constant permeability porous medium. The MHD term is derived from an order-of-magnitude analysis of the full Navier-Stokes equation. It is assumed here that the whole size of the porous plate is significantly larger than a characteristic microscopic length scale of the porous medium. The chemical reactions are taking place in the flow and all thermo physical properties are assumed to be constant of the linear momentum equation which is an approximation. The fluid properties are assumed to be constants except that the influence of density variation with temperature and concentration has been consider in the body-force. Since the plate is semi-infinite in length, therefore all physical quantities are functions of y and t only. Hence, by the usual boundary layer approximations, the governing equations for unsteady flow of a viscous incompressible fluid through a porous medium are:

Continuity equation

$$\frac{\partial v^*}{\partial y^*} = 0 \qquad (1)$$

Linear momentum equation

$$\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{\partial U^*}{\partial t^*} + v\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta(T^* - T^*_\infty) + g\beta^*(C^* - C^*_\infty)\frac{\sigma\beta_0^2}{\rho}(U^* - u^*) + \frac{v}{K^*}(U^* - u^*) \qquad (2)$$

Energy Equation

$$\frac{\partial T^*}{\partial t^*} + v^*\frac{\partial T^*}{\partial y^*} = \alpha\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{V}{C_P}\left(\frac{\partial u^*}{\partial y^*}\right)^2 - \frac{1}{\rho C_P}\frac{\partial q_r}{\partial y^*} + \frac{Q_0}{\rho C_P}(T^* - T_\infty^*) \quad (3)$$

Diffusion Equation

$$\frac{\partial \emptyset^*}{\partial t^*} + v^*\frac{\partial \emptyset^*}{\partial y^*} = D\frac{\partial^2 \emptyset^*}{\partial y^{*2}} - k_r^{*2}(C^* - C_\infty^*) \quad (4)$$

The boundary conditions for the velocity, temperature and concentration fields are

$u^* = 0$, $T^* = T_w^* + \epsilon(T_w^* - T_\infty^*)e^{n^*t^*}, C^* = C_w^* + \epsilon(T_w^* - T_\infty^*)e^{n^*t^*} \quad at\ y = 0$

$u^* \to 1, T^* \to T_\infty^*, C^* \to C_\infty^*$, as $y^* \to \infty$ (5)

Where x and y are dimensional coordinates, u* and v* are dimensionless velocities, t* is dimensionless time, T* is the dimensional temperature, C* is dimensional concentration, g- the acceleration due to gravity, β - the volumetric coefficient of thermal expansion, β * is the volumetric coefficient of thermal expansion with concentration, ρ - the density of the fluid, C_p is the specific heat at constant pressure, D is the species diffusion coefficient, K* is the permeability of the porous medium, q_r is the radiation heat flux, Q_0 is the heat generation/absorption constant, k_r^2 is the chemical reaction parameter, B_0- magnetic induction, ν- the kinematic viscosity, $\alpha$is the thermal diffusivity, U_o is the scale of free stream velocity, T_w^* and C_w^* are wall dimensional temperature and concentration respectively, T_∞^* the free stream temperature far away from the plate, C_∞^*- the free stream concentration in fluid far away from the plate, n* -the constant. The radiative heat flux term by using the Rosseland approximation is given by

$$q_w^* = -\frac{4\sigma^*}{3k_r^*}\left(\frac{\partial T^{*4}}{\partial y^*}\right)_{y=0} \quad (6)$$

$where\ \sigma^*\ is\ the\ Stefan - Boltzmann\ constant\ and\ k_r^*\ the\ mean\ absorption\ coefficent.$

It should be noted that by using the Rosseland approximation the present analysis is limited to optically thick fluids. If temperature differences within the flow are sufficiently small, then Equation (6) can be linearized by expanding T* into the Taylor series about T_∞^*, which after neglecting higher order terms takes the form

$$T^{*4} \cong 4T_\infty^{*3}T^* - 3T^{*4} \quad (7)$$

$$q_w^* = -\left(\frac{16\sigma_\infty^*}{3K_r^*}\right) \quad (8)$$

Substituting equation (8), into equation (3), gives

$$\frac{\partial T^*}{\partial t^*} + v^*\frac{\partial T^*}{\partial y^*} = \alpha\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{1}{\rho C_P}\frac{16\sigma_s}{3k_e}T_\infty^{*3}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{v}{C_P}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + Q_0(T^* - T) \quad (9)$$

3 METHOD OF SOLUTION

Introducing the following non-dimensional quantities into the equations (2), (4) and (9)

$$u = \frac{u^*}{U_0},\ y = \frac{V_0 y^*}{v} t = \frac{V_0^2 t^*}{v},\ n = \frac{vn^*}{V_0}, \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, \emptyset = \frac{C - C_\infty}{C_w - C_\infty}, Pr = \frac{v\rho C_p}{k} = \frac{v}{\alpha}, S_c = \frac{v}{D}, Gr = g\beta v \frac{(T_w - T_\infty)}{U_0 V_0^2}, Gm = g\beta^* v \frac{(C_w - C_\infty)}{U_0 V_0^2}, Ec = \frac{V_0^2}{C_P (T_w - T_\infty)}, k_r^2 = \frac{k^{*2r v}}{V_0^2}, R = \frac{4\sigma_s T_\infty'^3}{k_e k},\ M = \frac{\sigma B_0^2 u v}{\rho V_0^2},\ \eta = \frac{vQ_0}{V_0^2} \quad (10)$$

Equation (1) is identically satisfied, we obtain the following set of differential equations:

$$\frac{\partial u}{\partial t} - (1 - \epsilon A e^{nt}) \frac{\partial \theta}{\partial y} = \frac{\partial U}{\partial t} + \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\emptyset - \left(M + \frac{1}{k}\right)(u - U) \quad (11)$$

$$\frac{\partial \theta}{\partial t} - (1 + \epsilon A e^{nt}) \frac{\partial \theta}{\partial y} = \frac{1}{Pr}\left(1 + \frac{4R}{3}\right)\frac{\partial^2 \theta}{\partial y^2} + Ec\left(\frac{\partial \theta}{\partial y}\right)^2 + \eta\theta \quad (12)$$

$$\frac{\square \emptyset}{\partial t} - (1 + \epsilon A e^{nt}) \frac{\partial \emptyset}{\partial y} = \frac{1}{S_c}\frac{\partial^2 \emptyset}{\partial y^2} - k_r^2 \emptyset \quad (13)$$

Where u and v are dimensionless velocities, t is dimensionless time, T_w^* and C_w^* are wall dimensional temperature and concentration respectively, T_∞^* is the free stream temperature far away from the plate, C_∞^* is the free stream concentration in fluid far away from the plate, n*is a constant, θ is dimensionless temperature function, Φ is dimensionless concentration function, U_0 is the scale of free stream velocity, R_e is the Reynolds number, R is the radiation parameter, Pr is Prandlt number, U is velocity, Sc is Schmidt number, n is the frequency, M is the Hartmann number, K is the permeability parameter, Gr is thermal Grashof number and Gm is species Grashof number, η the heat source parameter, k_r^2 is the chemical reaction parameter and Ec is Eckert number, A is a real positive constant of suction velocity parameter $<\epsilon$, and ϵA<1 are small less than unity, i.e ϵA<<1, V_0 is a scale of suction velocity normal to the plate

The boundary conditions (5) are given by the following dimensionless form.

$$y = 0:\quad u = 0:,\quad \theta = (1 + \epsilon A e^{nt}), \Phi = (1 + \epsilon A e^{nt});$$

$$y \to \infty:\ u \to 0, \theta \to 0,\ \ \emptyset \to 0 \quad (14)$$

In order to reduce the above system of partial differential equations to a system of ordinary differential equations in dimensionless form, the velocities, momentum, temperature, free stream velocity and mass are perturbed [12] as:

$$u(y,t) = u_0(y) + \epsilon e^{nt} u_1(y) + O(\epsilon^2) + \cdots \quad (15)$$

$$\theta(y,t) = \theta_0(y) + \epsilon e^{nt} \theta_1(y) + O(\epsilon^2) + \cdots \quad (16)$$

$$\emptyset(y,t) = \emptyset_0(y) + \epsilon e^{nt} \emptyset_1(y) + O(\epsilon^2) + \cdots \quad (17)$$

The free stream velocity is expressed as

$$U(t) = 1 + \epsilon e^{nt} \tag{18}$$

Substituting equations (15)-(18) into equations (11)-(13) and neglecting the coefficient of like powers of ϵ we get the following set of differential equations.

$$u_0''(y) + u_0'(y) - \left(M + \frac{1}{k}\right)u_0(y) = -\left(M + \frac{1}{k}\right) - Gr\theta_0 - Gm\emptyset_0(y) \tag{19}$$

$$u_1''(y) + u_1'(y) - \left(M + \frac{1}{k} + n\right)u_1(y) = \left(M + \frac{1}{k} + n\right) - Au_0'(y) - Gr\theta_1(y) - Gm\emptyset_1(y) \tag{20}$$

$$(3+4R)\theta_0''(y) + 3Pr\theta_0'(y) - 3Pr\eta\theta_0(y) = -3PrEc(u_0')^2(y) \tag{21}$$

$$(3+4R)\theta_1''(y) + 3Pr\theta_1'(y) - 3Pr\theta_1\eta(y) - 3Prn\theta_1(y) = -3PrA\theta_0'(y) - 6PrEc\big(u_0'(y)\big)u_1'(y) \tag{22}$$

$$\phi_0''(y) + Sc\phi_0'(y) - Sck_r^2\phi_0(y) = 0 \tag{23}$$

$$\phi_1''(y) + Sc\phi_1'(y) - Sc(n + k_r^2)\phi_1(y) = -ASc\phi_0' \tag{24}$$

and the corresponding boundary conditions reduced to

$$y = 0: \; u_0 = 0, \quad u_1 = 0, \quad \theta_0 = 1, \quad \theta_1 = 0, \quad \phi_0 = 1, \quad \phi_1 = 0,$$

$$as\; y \to \infty: \; u_0 \to 1, \quad u_1 \to 1, \quad \theta_0 \to 0, \quad \theta_1 \to 0, \quad \phi_0 \to 0, \quad \phi_1 \to 0 \tag{25}$$

In order to obtain the solutions of above coupled differential equations from (19) to (24), we expand u_0, u_1, θ_0, θ_1, $\emptyset_0$ and $\emptyset_1$ in powers of Ecket number Ec. assuming that it is very small

$$u_0(y) = u_{00}(y) + Ecu_{01}(y) + O(\epsilon^2).$$

$$u_1(y) = u_{10}(y) + Ecu_{11}(y) + O(\epsilon^2).$$

$$\theta_0(y) = \theta_{00}(y) + Ec\theta_{01}(y) + O(\epsilon^2),$$

$$\theta_1(y) = \theta_{10}(y) + Ec\theta_{11}(y) + O(\epsilon^2).$$

$$\emptyset_0(y) = \emptyset_{00}(y) + Ec\emptyset_{01}(y) + O(\epsilon^2),$$

$$\emptyset_1(y) = \emptyset_{10}(y) + Ec\emptyset_{11}(y) + O(\epsilon^2), \tag{26}$$

Substituting (26) into equations (19) to (24), equating the coefficients of like powers of Ec and neglecting the higher order terms of Ec, we get

$$u_{00}''(y) + u_{00}'(y) - \left(M + \frac{1}{k}\right)u_{00}(y) = -\left(M + \frac{1}{k}\right) - Gr\theta_{00}(y) - Gm\emptyset_{00}(y) \tag{27}$$

$$u_{01}''(y) + u_{01}'(y) - \left(M + \frac{1}{k}\right)u_{01}(y) = -Gr\theta_{01}(y) - Gm\emptyset_{01}(y) \tag{28}$$

$$u''_{10}(y) + u'_{10}(y) - \left(M + \frac{1}{k} + n\right) u_{10}(y) = \left(M + \frac{1}{k} + n\right) - Au'_{00}(y) - Gr\theta_{10}(y) - Gm\emptyset_{10}(y) \quad (29)$$

$$u''_{11}(y) + u'_{11}(y) - \left(M + \frac{1}{k} + n\right) u_{11}(y) = -Au'_{01}(y) - Gr\theta_{11}(y) - Gm\emptyset_{11}(y) \quad (30)$$

$$(3 + 4R)\theta''_{00}(y) + 3Pr\theta'_{00}(y) - 3Pr\eta\theta_{00}(y) = 0 \quad (31)$$

$$(3 + 4R)\theta''_{01}(y) + 3Pr\theta'_{01}(y) - 3Pr\eta\theta_{01}(y) = -3PrPr(u'_{00})^2 \quad (32)$$

$$(3 + 4R)\theta''_{10}(y) + 3Pr\theta'_{10}(y) - 3Pr\square\theta_{10}(y) - 3Prn\theta_{10}(y) = -3PrPrA\theta'_{0}(y) \quad (33)$$

$$(3 + 4R)\theta''_{11}(y) + 3Pr\theta'_{11}(y) - 3Pr\theta_{11}(\eta + n)(y) = -3PrA\theta'_{01}(y) - 6Pr(u'_{00}(y)u'_{10})(y) \quad (34)$$

$$\emptyset''_{00}(y) + Sc\emptyset'_{00}(y) - Sck_r^2\emptyset_{00}(y) = 0 \quad (35)$$

$$\emptyset''_{01}(y) + Sc\emptyset'_{01}(y) + Sck_r^2\emptyset_{01}(y) = 0 \quad (36)$$

$$\emptyset''_{10}(y) + Sc\emptyset'_{10}(y) - Sc(n + k_r^2)\emptyset_{10}(y) = -ASc\emptyset'_{00}(y) \quad (37)$$

$$\emptyset''_{11}(y) + Sc\emptyset'_{11}(y) - Sc(n + k_r^2)\emptyset_{11}(y) = -ASc\emptyset'_{01}(y) \quad (38)$$

with the corresponding boundary conditions:

$$y = 0:\ u_{00}(y) = 0, u_{01}(y) = 0, u_{10}(y) = 0, u_{11}(y) = 0, \theta_{00}(y) = 1, \theta_{01}(y) = 0, \theta_{10}(y) = 0, \theta_{11}(y) = 0, \phi_{00}(y) = 1, \phi_{01}(y) = 0, \phi_{10}(y) = 0\ \ \phi_{11}(y) = 0, \quad (39)$$

$$as\ y \to \infty:\ u_{00}(y) \to 1, u_{01}(y) \to 0, u_{10}(y) \to 0, u_{11}(y) \to 0,\ \ \theta_{00}(y) \to 1, \theta_{01}(y) \to 0, \theta_{10}(y) \to 0, \theta_{11}(y) \to 0,\ \ \phi_{00}(y) \to 0, \phi_{01}(y) \to 0, \phi_{10}(y) \to 0, \square_{11}(y) \to 0 \quad (40)$$

The solutions of equations (27)-(38) subject to the boundary conditions (39) and (40) are respectively

$$U_{00} = e^{-b_8 y}(-1 - L_1 - L_2) + 1 - L_1 e^{-b_2 y} + L_2 e^{-b_4 y} \quad (41)$$

$$U_{01} = e^{-b_{10} y}(-L_6 - L_7 - L_8 - L_9) + L_6 e^{-2b_{12} y} + L_7 e^{-2b_8 y} + L_8 e^{-2b_2 y} + L_9 e^{-2b_4 y} \quad (42)$$

$$U_{10} = e^{-b_{18} y}(-1 - L_{12} - L_{13} - L_{14} - L_{15} - L_{16}) + 1 + L_{12} e^{-b_8 y} + L_{13} e^{-b_2 y} + L_{14} e^{-b_4 y} + L_{15} e^{-b_{14} \square} + + L_{16} e^{-b_{16} y} \quad (43)$$

$$U_{11} = e^{-b_{24}y}(-1 - L_{33} - L_{34} - L_{35} - L_{36} - L_{37} - L_{38} - L_{39} - L_{40} - L_{41} - L_{42} - L_{43} - L_{44} - L_{45} - L_{46} - L_{47} - L_{48} - L_{49} - L_{50}) + 1 + L_{33}e^{-b_{10}y} + L_{34}e^{-b_{12}y} + L_{35}e^{-2b_8y} + L_{36}e^{-2b_2y} + L_{37}e^{-2b_4y} + L_{38}e^{-b_{20}y} + L_{39}e^{-(b_8y+b_{18}y)} + L_{40}e^{-(b_2y+b_8y)} + L_{41}e^{-(b_4y+b_8y)} + L_{42}e^{-(b_8y+b_{14}y)} + L_{43}e^{-(b_8y+b_{16}y)} + L_{44}e^{-(b_2y+b_{18}y)} + L_{45}e^{-(b_2y+b_4y)} + L_{46}e^{-(b_2y+b_{14}y)} + L_{47}e^{-(b_2y+b_{16}y)} + L_{48}e^{-(b_4y+b_{18}y)} + L_{49}e^{-(b_4y+b_{14}y)} + L_{50}e^{-(b_4y+b_{16}y)} \quad (44)$$

$$\theta_{00} = e^{-b_2y} \quad (45)$$

$$\theta_{01} = e^{-b_{12}y}(-L_3 - L_4 - L_5) + L_3e^{-2b_8y} + L_4e^{-2b_2y} + L_5e^{-2b_4y} \quad (46)$$

$$\theta_{10} = L_{10}\left(-e^{-b_{14}y} + e^{-b_2y}\right) \quad (47)$$

$$\theta_{11} = e^{-b_{20}y}(-L_{17} - L_{18} - L_{19} - L_{20} - L_{21} - L_{22} - L_{23} - L_{24} - L_{25} - L_{26} - L_{27} - L_{28} - L_{29} - L_{30} - L_{31} - L_{32}) + L_{17}e^{-(2b_8y)} + L_{18}e^{-(2b_2y)} + L_{19}e^{-(2b_4y)} + L_{20}e^{-(b_{12}y)} + L_{21}e^{-(b_8y+b_{18}y)} + L_{22}e^{-(b_2y+b_8y)} + L_{23}e^{-(b_4y+b_8y)} + L_{24}e^{-(b_8y+b_{14}y)} + L_{25}e^{-(b_8y+b_{16}y)} + L_{26}e^{-(b_2y+b_{18}y)} + L_{27}e^{-(b_2y+b_4y)} + L_{28}e^{-(b_2y+b_{14}y)} + L_{29}e^{-(b_2y+b_{16}y)} + L_{30}e^{-(b_4y+b_{18}y)} + L_{31}e^{-(b_4y+b_{14}y)} + L_{32}e^{-(b_4y+b_{16}y)} \quad (48)$$

$$\emptyset_{00} = e^{-b_4y} \quad (49)$$

$$\emptyset_{01} = 0 \quad (50)$$

$$\emptyset_{10} = -L_{11}\left(e^{-b_{16}y} - e^{-b_4y}\right) \quad (51)$$

$$\emptyset_{11} = 0 \quad (52)$$

$$b_1 = \frac{3Pr}{2(3+4R)}\left[-1 + \sqrt{\frac{3\mathrm{Pr} - 4\eta(3+4R)}{3Pr}}\right], \qquad b_2 = \frac{3Pr}{2(3+4R)}\left[1 + \sqrt{\frac{3\mathrm{Pr} - 4\eta(3+4R)}{3Pr}}\right]$$

$$b_3 = \frac{1}{2}\left[-S_C + \sqrt{S_c^2 - 4S_cK_r^2}\right], \qquad b_4 = \frac{1}{2}\left[S_C + \sqrt{S_c^2 - 4S_cK_r^2}\right]$$

$$b_5 = \frac{1}{2}\left[-S_C + \sqrt{S_c^2 - 4S_cK_r^2}\right], \qquad b_6 = \frac{1}{2}\left[S_C + \sqrt{S_c^2 - 4S_cK_r^2}\right],$$

$$b_7 = \frac{1}{2}\left[-1 + \sqrt{1 + 4\left(M + \frac{1}{k}\right)}\right], \qquad b_8 = \frac{1}{2}\left[1 + \sqrt{1 + 4\left(M + \frac{1}{k}\right)}\right],$$

$$b_9 = \frac{1}{2}\left[-1 + \sqrt{1 + 4\left(M + \frac{1}{k}\right)}\right], \qquad b_{10} = \frac{1}{2}\left[1 + \sqrt{1 + 4\left(M + \frac{1}{k}\right)}\right],$$

$$b_{11} = \frac{3Pr}{2(3+4R)}\left[-1 + \sqrt{\frac{3\mathrm{Pr} - 4\eta(3+4R)}{3Pr}}\right], \qquad b_{12} = \frac{3Pr}{2(3+4R)}\left[1 + \sqrt{\frac{3\mathrm{Pr} - 4\eta(3+4R)}{3Pr}}\right],$$

$$b_{13} = \frac{3Pr}{2(3+4R)}\left[-1 + \sqrt{\frac{3\mathrm{Pr} + 4(\eta + n)(3+4R)}{3Pr}}\right],$$

$$b_{14} = \frac{3Pr}{2(3+4R)}\left[1 + \sqrt{\frac{3\mathrm{Pr} + 4(\eta + n)(3+4R)}{3Pr}}\right],$$

$$b_{15} = \frac{1}{2}\left[-Sc + \sqrt{Sc^2 - 4Sc(n + K_r^2)}\right], \qquad b_{16} = \frac{1}{2}\left[Sc + \sqrt{Sc^2 - 4Sc(n + K_r^2)}\right],$$

$$b_{17} = \frac{1}{2}\left[-1 + \sqrt{1 + 4\left(M + \frac{1}{k} + n\right)}\right], \qquad b_{18} = \frac{1}{2}\left[1 + \sqrt{1 + 4\left(M + \frac{1}{k} + n\right)}\right],$$

$$b_{19} = \frac{3Pr}{2(3+4R)}\left[-1 + \sqrt{\frac{3\mathrm{Pr} + 4(\eta + n)(3+4R)}{3Pr}}\right],$$

$$b_{20} = \frac{3Pr}{2(3+4R)}\left[1 + \sqrt{\frac{3\mathrm{Pr} + 4(\eta + n)(3+4R)}{3Pr}}\right],$$

$$b_{21} = \frac{1}{2}\left[-Sc + \sqrt{Sc^2 + 4Sc(n + K_r^2)}\right], \qquad b_{22} = \frac{1}{2}\left[Sc + \sqrt{Sc^2 + 4Sc(n + K_r^2)}\right],$$

$$b_{23} = \frac{1}{2}\left[-1 + \sqrt{1 + 4\left(M + \frac{1}{k} + n\right)}\right], \qquad b_{24} = \frac{1}{2}\left[1 + \sqrt{1 + 4\left(M + \frac{1}{k} + n\right)},\right]$$

$$L_1 = \frac{-Gr}{b_2^2 - b_2 - \left(M + \frac{1}{k}\right)}, \qquad L_2 = \frac{-Gr}{b_4^2 - b_4 - \left(M + \frac{1}{k}\right)},$$

$$L_3 = \frac{-3Prb_8^2(1 + L_1^2 + L_2^2)}{(4(3+4R)b_8^2 - 6Prb_8 - 3Pr\eta)}, \qquad L_4 = \frac{-3Prb_2^2(L_1^2)}{(4(3+4R)\square_2^2 - 6Prb_2 - 3Pr\eta)},$$

$$L_5 = \frac{-3Prb_4^2(L_2^2)}{(4(3+4R)b_4^2 - 6Prb_4 - 3Pr\eta)}, \qquad L_6 = \frac{-Gr(-L_3 - L_4 - L_5)}{4b_{12}^2 - 2b_{12} - \left(M + \frac{1}{k}\right)},$$

$$L_7=\frac{-GrL_3}{4b_8^2-2b_8-\left(M+\frac{1}{k}\right)},\qquad L_8=\frac{-GrL_4}{4b_2^2-2b_2-\left(M+\frac{1}{k}\right)},$$

$$L_9=\frac{-GrL_5}{4b_4^2-2b_4-\left(M+\frac{1}{k}\right)},\qquad L_{10}=\frac{3PrAb_2}{(3+4R)b_2^2-3Prb_2-3Pr(\eta+n)},$$

$$L_{11}=\frac{AScb_4}{b_4^2-Scb_4-SC(\eta+k_r^2)},\qquad L_{12}=\frac{Ab_8(-L_1-L_2)}{b_8^2-b_8-\left(M+\frac{1}{k}+n\right)},$$

$$L_{13}=\frac{-Ab_2L_1+Grb_2L_{10}}{b_2^2-b_2-\left(M+\frac{1}{k}+n\right)},\qquad L_{14}=\frac{A(b_2L_2)}{b_4^2-b_4-\left(M+\frac{1}{k}+n\right)},$$

$$L_{15}=\frac{-GrL_{10}b_{14}+Gmb_{14}L_{11}}{b_{14}^2-b_{14}-\left(M+\frac{1}{k}+n\right)},\qquad L_{16}=\frac{-GmL_{11}b_{16}}{b_{16}^2-b_{16}-\left(M+\frac{1}{k}+n\right)},$$

$$L_{17}=\frac{-6Prb_8^2(-1-L_1+L_2)-6PrAb_8L_{13}}{4(3+4R)b_8^2-6Prb_8^2-3Pr(\eta+n)},\quad L_{18}=\frac{-6Prb_2^2L_1L_{13}-6PrAb_2L_4,}{4(3+4R)b_2^2-6Prb_2^2-3Pr(\eta+n)},$$

$$L_{19}=\frac{-6Prb_4^2L_2L_{14}-6PrAb_4L_5}{4(3+4R)b_2^2-6Prb_2^2-3Pr(\eta+n)},\quad L_{20}=\frac{-3PrA_{12}(-L_3-L_4L_5)}{4(3+4R)b_{12}^2-3Prb_{12}-3Pr(\eta+n)}$$

$$L_{21}=\frac{-6Prb_8b_{18}(-1-L_1-L_2)(-1-L_{12}-L_{13}-L_{14}-L_{15}-L_{16})}{(3+4R)(b_8+b_{18})^2-3Pr(b_8+b_{18})-3Pr(\eta+n)}$$

$$L_{22}=\frac{-6Prb_2b_8(L_{13}(-1-L_1-L_2)+L_1L_2)}{(3+4R)(b_2+b_8)^2-3Pr(b_2+b_8)-3Pr(\eta+n)}$$

$$L_{23}=\frac{-6Prb_4b_8(L_{14}(-1-L_1-L_2)+L_2L_{12})}{(3+4R)(b_4+b_8)^2-3Pr(b_4+b_8)-3Pr(\eta+n)}$$

$$L_{24}=\frac{-6Prb_8b_{14}L_{15}(-1-L_1-L_2)}{(3+4R)(b_8+b_{14})^2-3Pr(b_8+b_{14})-3Pr(\eta+n)}$$

$$L_{25}=\frac{-6Prb_8b_{16}L_{16}(-1-L_1-L_2)}{(3+4R)(b_8+b_{16})^2-3Pr(b_8+b_{16})-3Pr(\eta+n)}$$

$$L_{26}=\frac{-6Prb_2b_{18}L_1(-1-L_{12}-L_{13}-L_{14}-L_{15}-L_{16})}{(3+4R)(b_2+b_{18})^2-3Pr(b_2+b_{18})-3Pr(\eta+n)}$$

$$L_{27}=\frac{-6Prb_2b_4(L_1L_{14}-L_2L_{13})}{(3+4R)(b_2+b_4)^2-3Pr(b_2+b_4)-3Pr(\eta+n)}$$

$$L_{28}=\frac{-6Prb_2b_{14}(L_1L_{15})}{(3+4R)(b_2+b_{14})^2-3Pr(b_2+b_{14})-3Pr(\eta+n)}$$

$$L_{29} = \frac{-6Prb_2b_{16}(L_1L_{16})}{(3+4R)(b_2+b_{16})^2 - 3Pr(b_2+b_{16}) - 3Pr(\eta+n)}$$

$$L_{30} = \frac{-6Prb_4b_{18}L_2(-1-L_{12}-L_{13}-L_{14}-L_{15}-L_{16})}{(3+4R)(b_4+b_{18})^2 - 3Pr(b_4+b_{18}) - 3Pr(\eta+n)}$$

$$L_{31} = \frac{-6Prb_4b_{14}L_2L_{15}}{(3+4R)(b_4+b_{14})^2 - 3Pr(b_4+b_{14}) - 3Pr(\eta+n)}$$

$$L_{32} = \frac{-6Prb_4b_{16}L_2L_{16}}{(3+4R)(b_4+b_{16})^2 - 3Pr(b_4+b_{16}) - 3Pr(\eta+n)}$$

$$L_{33} = \frac{Ab_{10}(-L_6-L_7-L_8-L_9)}{(b_{10})^2 - b_{10} - \left(M+\frac{1}{k}+n\right)}, \qquad L_{34} = \frac{2A\square_{12}L_6 - GrL_{20}}{(b_{12})^2 - b_{12} - \left(M+\frac{1}{k}+n\right)}$$

$$L_{35} = \frac{2Ab_8L_7 - GrL_{17}}{4b_8^2 - 2b_8 - \left(M+\frac{1}{k}+n\right)}, \qquad L_{36} = \frac{2Ab_2L_8 - GrL_{18}}{4b_2^2 - 2b_2 - \left(M+\frac{1}{k}+n\right)}$$

$$L_{37} = \frac{2Ab_4L_9 - GrL_{19}}{4b_4^2 - 2b_4 - \left(M+\frac{1}{k}+n\right)}$$

$$L_{38} = \frac{-Gr(-L_{17}-L_{18}-L_{19}-L_{20}-L_{21}-L_{22}-L_{23}-L_{24}-L_{25}-L_{26}-L_{27}-L_{28}-L_{29}-L_{30}-L_{31}-L_{32})}{b_{20}^2 - b_{20} - \left(M+\frac{1}{k}+n\right)}$$

$$L_{39} = \frac{-GrL_{21}}{(b_8+b_{18})^2 - (b_8+b_{18}) - \left(M+\frac{1}{k}+n\right)},$$

$$L_{40} = \frac{-GrL_{22}}{(b_2+b_8)^2 - (b_2y+b_8) - \left(M+\frac{1}{k}+n\right)}$$

$$L_{41} = \frac{-GrL_{23}}{(b_4+b_8)^2 - (b_4y+b_8) - \left(M+\frac{1}{k}+n\right)}$$

$$L_{42} = \frac{-GrL_{24}}{(b_8+b_{14})^2 - (b_8y+b_{14}) - \left(M+\frac{1}{k}+n\right)}$$

$$L_{43} = \frac{-GrL_{25}}{(b_8+b_{16})^2 - (b_8y+b_{16}) - \left(\square+\frac{1}{k}+n\right)}$$

$$L_{44} = \frac{-GrL_{26}}{(b_2+b_{18})^2 - (b_2y+b_{18}) - \left(M+\frac{1}{k}+n\right)}$$

$$L_{45} = \frac{-GrL_{27}}{(b_2+b_4)^2 - (b_2y+b_4) - \left(M+\frac{1}{k}+n\right)}$$

$$L_{46} = \frac{-GrL_{28}}{(b_2+b_{14})^2 - (b_2y+b_{14}) - \left(M+\frac{1}{k}+n\right)}$$

$$L_{47} = \frac{-GrL_{29}}{(b_2+b_{16})^2 - (b_2y+b_{16}) - \left(M+\frac{1}{k}+n\right)}$$

$$L_{48} = \frac{-GrL_{30}}{(b_4+b_{18})^2 - (b_4y+b_{18}) - \left(M+\frac{1}{k}+n\right)}$$

$$L_{49} = \frac{-GrL_{31}}{(b_4+b_{14})^2 - (b_4y+b_{14}) - \left(M+\frac{1}{k}+n\right)}$$

$$L_{50} = \frac{-GrL_{32}}{(b_4+b_{16})^2 - (b_4y+b_{16}) - \left(M+\frac{1}{k}+n\right)}$$

In view of the above solutions, the velocity, temperature and concentration distributions in the boundary layer become

$$\begin{aligned}
&U(y,t) \\
&= e^{-b_8y}(-1-L_1-L_2) + 1 + L_1e^{-b_2y} - L_2e^{-b_4y} \\
&+ Ec\left(e^{-b_{10}y}(-L_6-L_7-L_8-L_9) + L_6e^{-2b_{12}y} + L_7e^{-2b_8y} + L_8e^{-2b_2y} + L_9e^{-2b_4y} +\right) \\
&+ \epsilon e^{nt}\big[e^{-b_{18}y}(-1-L_{12}-L_{13}-L_{14}-L_{15}-L_{16}) + 1 + L_{12}e^{-b_8y} + L_{13}e^{-b_2y} + L_{14}e^{-b_4y} \\
&+ L_{15}e^{-b_{14}y} + L_{16}e^{-b_{16}y} \\
&+ Ec\big(e^{-b_{24}y}(-1-L_{33}-L_{34}-L_{35}-L_{36}-L_{37}-L_{38}-L_{39}-L_{40}-\square_{41}-L_{41}-L_{42}-L_{43} \\
&- L_{44}-L_{45}-L_{46}-L_{47}-L_{48}-L_{49}-L_{50}) + 1 + L_{33}e^{-b_{10}y} + L_{34}e^{-b_{12}y} + L_{35}e^{-2b_8y} \\
&+ L_{36}e^{-2b_2y} + L_{37}e^{-2b_4y} + L_{38}e^{-b_{20}y} + L_{39}e^{-(b_8y+b_{18}y)} + L_{40}e^{-(b_2y+b_8y)} \\
&+ L_{41}e^{-(b_4y+b_8y)} + L_{42}e^{-(b_8y+b_{14}y)} + L_{43}e^{-(b_8y+b_{16}y)} + L_{44}e^{-(b_2y+b_{18}\square)} + L_{45}e^{-(b_2y+b_4y)} \\
&+ L_{46}e^{-(b_2y+b_{14}y)} + L_{47}e^{-(b_2y+b_{16}y)} + L_{48}e^{-(b_4y+b_{18}y)} + L_{49}e^{-(b_4y+b_{14}y)} \\
&+ L_{50}e^{-(b_4y+b_{16}y)}\big)\big] \qquad (53)
\end{aligned}$$

$$\begin{aligned}
\theta(y,t) = {} & e^{-b_2y} + Ec\left(e^{-b_{12}y}(-L_3-L_4-L_5) + L_3e^{-2b_8y} + L_4e^{-2b_2y} + L_5e^{-2b_4y}\right) \\
&+ \epsilon e^{nt}\big[L_{10}\left(-e^{-b_{14}y} + e^{-b_2y}\right) \\
&+ Ec\big(e^{-b_{20}y}(-L_{17}-L_{18}-L_{19}-L_{20}-L_{21}-L_{22}-L_{23}-L_{24}-L_{25}-L_{26} \\
&- L_{27}-L_{28}-L_{29}-L_{30}-L_{31}-L_{31}-L_{32}) + L_{17}e^{-(2b_8y)} + L_{18}e^{-(2b_2y)} \\
&+ L_{19}e^{-(2b_4y)} + L_{20}e^{-(b_{12}y)} + L_{21}e^{-(b_8y+b_{18}y)} + L_{22}e^{-(b_2y+b_8y)} \\
&+ L_{23}e^{-(b_4y+b_8y)} + L_{24}e^{-(b_8y+b_{14}y)} + L_{25}e^{-(b_8y+b_{16}y)} + L_{26}e^{-(b_2y+b_{18}y)} \\
&+ L_{27}e^{-(b_2y+b_4y)} + L_{28}e^{-(b_2y+b_{14}y)} + L_{29}e^{-(b_2y+b_{16}y)} \\
&+ L_{30}e^{-(b_4y+b_{18}y)} + L_{31}e^{-(b_4y+b_{14}y)} + L_{32}e^{-(b_4y+b_{16}y)}\big)\big] \qquad (54)
\end{aligned}$$

$$\emptyset(y,t) = e^{-b_4 y} + \epsilon e^{nt}\left[L_{11}\left(-e^{-b_{16} y} + e^{-b_4 y}\right)\right] \quad (55)$$

Skin-friction Coefficient is expressed as follows:

$$C_f = \left[\frac{\tau_w}{\rho U_0 V_0}\right] = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left(\frac{\partial u_0(y)}{\partial y} + \epsilon e^{nt}\frac{\partial u_1(y)}{\partial y}\right)_{y=0}$$
$$= b_8(1 + L_1 + L_2) - b_2 L_1 - b_4 L_2 + Ec(b_{10}(L_6 + L_7 + L_8 + L_9) - 2b_{12}L_6 - 2b_8 L_7 - 2b_2 L_8 - 2b_4 L_9) + \epsilon e^{nt}(b_{18}(1 + L_{12} + L_{13} + L_{14} + L_{15} + L_{16}) - b_8 L_{12} - b_2 L_{13} - L_{14} - b_{14}L_{15} - b_{16}L_{16}) + Ec(b_{24})(1 + L_{33} + L_{34} + L_{35} + L_{36} + L_{37} + L_{38} + L_{39} + L_{40} + L_{41} + L_{42} + L_{43} + L_{44} + L_{45} + L_{46} + L_{47} + L_{48} + L_{49} + L_{50}) - b_{10}L_{33} - b_{12}L_{34} - 2b_8 L_{35} - 2b_2 L_{36} - 2b_4 L_{37} - b_{20}L_{38} - (b_8 + b_{18})L_{39} - (b_2 + b_8)L_{40} - (b_4 + b_8)L_{41} - (b_8 + b_{16})L_{43} - (b_2 + b_{18})L_{44} - (b_2 + b_4)L_{45} - (b_2 + b_{14})L_{46} - (b_2 + b_{16})L_{47} - (b_4 + b_{18})L_{48} - (b_4 + b_{14})L_{49} - (b_4 + b_{16})L_{50} \quad (56)$$

The heat transfer coefficient in term of Nusselt number is as follows:

Knowing the temperature field, it is interesting to study the effect of the free convection and radiation on the rate of heat transfer q_w^*. This is given by

$$q_w^* = -K\left(\frac{\partial T^*}{\partial y^*}\right)_{y=0} - \frac{4\sigma^*}{3k_r^*}\left(\frac{\partial T^{*4}}{\partial y^*}\right)_{y=0} \quad (57)$$

Using this equation $T^{*4} \cong 4T_\infty^{*3}T^* - 3T^{*4}$ we can write equation (57) as follow

$$□_w^* = -\left(K + \frac{16\sigma^* T_\infty^{*3}}{3k_r^*}\right)\left(\frac{\partial T^*}{\partial y^*}\right)_{y=0} \quad (58)$$

which is written in non-dimensional form as:

$$q_w^* = -\left(1 + \frac{4R}{3}\right)\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \quad (59)$$

The non-dimensional Nusselt number is obtained as

$$NuRe_x^{-1} = \left(1+\frac{4R}{3}\right)\left(\frac{\partial\theta_0(y)}{\partial y} + \epsilon e^{nt}\frac{\partial\theta_1(y)}{\partial y}\right)_{y=0}$$
$$= b_2 - Ec(b_{12}(L_3+L_4+L_5) - 2b_8L_3 - 2b_2L_4 - 2b_4L_5)$$
$$-\epsilon e^{nt}[L_{10}(b_3-b_2)$$
$$+ + Ec(b_{20}(L_{17}+L_{18}+L_{19}+L_{20}+L_{21}+L_{22}+L_{23}+L_{24}+L_{25}+L_{26}$$
$$+L_{27}+L_{28}+L_{29}+L_{30}+L_{31}+L_{32}) - 2b_8L_{17} - 2b_2L_{18} - 2b_4L_{19}$$
$$-b_{12}L_{20} - b_8 + b_{18})L_{21} - (b_2+b_8)L_{22} - (b_4+b_8)L_{23} - (b_8+b_{14})L_{24}$$
$$-(b_8+b_{16})L_{25} - (b_2+b_{18})L_{26} - (b_2+b_4)L_{27} - (b_2+b_{14})L_{28}$$
$$-(b_2+b_{16})L_{29} - (b_4+b_{18})L_{30} - (b_4+b_{14})L_{31}$$
$$-(b_4+b_{16})L_{32}] \quad (60)$$

where $Re_x = V_0x/v$ is the Reynolds number.

Local Sherwood number (Sh_w) can be define as

$$Sh = \frac{Kx}{D} \quad (61)$$

with the help of these equations, one can write

$$ShRe_x^{-1} = \left(\frac{\partial\phi_0}{\partial y} + \epsilon e^{nt}\frac{\partial\phi_1}{\partial y}\right)_{y=0} = -b_4 - \epsilon e^{nt}L_{11}(b_{16}-b_4) \quad (62)$$

4. **Results and Discussions:**

The effects of thermal radiation, heat source and chemical reaction on heat and mass transfer of MHD incompressible, viscous fluid along vertical porous moving plate in a porous medium has been investigated. The numerical calculation for the distribution of the velocity, temperature and concentration across the boundary layer for various values of the parameters are obtained in this study using the following A=0.5, t=1.0, n=0.1 and ∈=0.10, while R, k_r^2, Sc, Gr, Gc, M, Pr, η, and K are varied in order to account for their effects. The boundary conditions for y are replaced with y_{max} that is when y sufficiently large and the velocity profile u approaches to the relevant free stream velocity

Fig. 1 shows the effect of radiation R on velocity. It is observed that as the value of R increases, the velocity increases with an increasing in the flow boundary layer thickness. Thus, thermal radiation enhances the flow. The effect of radiation parameter R on the temperature profiles are presented in Fig. 2 it shows that, as the value of R increases the temperature profiles increases, with an increasing in the thermal boundary layer thickness.

The influence of chemical reaction parameter k_r^2, on the velocity profiles across the boundary layer are presented in Fig. 3 it is seen that the velocity distribution across the boundary layer decreases with increasing in k_r^2. For different values of the chemical reaction parameterk_r^2, the concentration profiles are plotted in Fig. 4. It is obvious that the influence of increasing values ofk_r^2, the concentration distribution across the boundary layer decreases.

The effect of heat generation η on the velocity profiles is shown in Fig.5. From this figure it is observed that the heat is generated the buoyancy force increase which influence the flow rate to increase giving rise to the increase in the velocity profiles.

The velocity profiles for different values of solution of Grashof number GØ are plotted in Fig. 7. In addition, the curves show that the peak value of the velocity increases rapidly near the wall of the porous plate as Grashof number increases, and then decays to the relevant free stream velocity.

Fig.8. represents the velocity profile with various Schmidt number Sc. The effect of increasing values of Sc results in a decreasing velocity distribution across the boundary layer. The concentration profiles across the boundary layer for various values of Schmidt number Sc. It is shown from fig. 9. that an increasing in Sc result in a decreasing the concentration distribution, because the smaller values of Sc are equivalent the chemical molecular diffusivity.

The velocity profiles for different values of Grashof number Gr are described in the fig.10. It is observed that an increasing in Gr leads to a rise in the values of velocity. Here the Grashof number represents the effect of the free convection currents. Physically, Gr>0 means heating of fluid of cooling of the boundary surface, Gr<0 means cooling of the fluid of heating of the boundary surface and Gr=0 corresponds to the absence of free convection current. In addition, the curves show that the peak value of velocity increases rapidly near the wall of the porous plate as Grshof number increases, and then decays to the relevant free stream velocity.

Fig.11. shows that the effect of increasing values of M parameter results in decreasing velocity distribution across the boundary layer because of the application of transfer magnetic field will result a restrictive type force(Lorenz force) similar to drag force which tends to resist the fluid and this reducing its velocity.

The velocity profiles across the boundary layer for different values of Prandlt number Pr are plotted in fig. 12. The results shows that the effect of increasing values of Pr results in a decreasing the velocity.

It is observed from fig.13. that an increase in Prandlt number results in a decreasing the thermal boundary layer thickness and more uniform temperature distribution across the boundary layer. The reason is that smaller values of Pr are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher values of Pr. Hence for smaller Pr, the rate of heat transfer is reduced.

It is observed from fig.14. that as velocity profiles for different values of the permeability K. Clearly, as K increases the peak value of velocity tends to increase. These results could be very useful in deciding the applicability of enhanced oil recovery in reservoir engineering.

Fig.15. show the effect of Eckert number on temperature, as Eckert number increases the temperature distribution across the boundary layer decreases

Table1 Effect of R on velocity n=0.1, t=1 and A=1.

∈	Pr	R	Ec	η	k_r^2	Gr	Gc	M	Sc	K	C_f
0.10	0.71	0.2	0.01	0.0	0.0	2	1	1	0.22	0.5	3.8433
0.10	0.71	0.4	0.01	0.0	0.0	2	1	1	0.22	0.5	3.9325
0.10	0.71	0.6	0.01	0.0	0.0	2	1	1	0.22	0.5	4.0031
0.10	0.71	0.8	0.01	0.0	0.0	2	1	1	0.22	0.5	4.0550
0.10	0.71	1.0	0.01	0.0	0.0	2	1	1	0.22	0.5	4.0974

Table2. Effect of Ec on velocity n=0.1, t=1 and A=1

∈	Pr	R	Ec	η	k_r^2	Gr	Gc	M	Sc	K	C_f
0.10	0.71	0.0	0.02	0.0	0.0	2	1	1	0.22	0.5	3.5704
0.10	0.71	0.0	0.04	0.0	0.0	2	1	1	0.22	0.5	3.2667
0.10	0.71	0.0	0.06	0.0	0.0	2	1	1	0.22	0.5	2.9671
0.10	0.71	0.0	0.08	0.0	0.0	2	1	1	0.22	0.5	26717
0.10	0.71	0.0	0.10	0.0	0.0	2	1	1	0.22	0.5	2.3807

Table3. Effect of k_r^2 on velocity n=0.1, t=1 and A=1.

∈	Pr	R	Ec	η	k_r^2	Gr	Gc	M	Sc	K	C_f
0.10	0.71	0.0	0.01	0.0	0.2	2	1	1	0.22	0.5	3.6838
0.10	0.71	0.0	0.01	0.0	0.5	2	1	1	0.22	0.5	3.6506
0.10	0.71	0.0	0.01	0.0	1	2	1	1	0.22	0.5	3.6215
0.10	0.71	0.0	0.01	0.0	2	2	1	1	0.22	0.5	3.5843
0.10	0.71	0.0	0.01	0.0	3	2	1	1	0.22	0.5	3.5587

Table4. Effect of Gr on velocity n=0.1, t=1 and A=1.

∈	Pr	R	Ec	η	k_r^2	Gr	Gc	M	Sc	K	C_f
0.10	0.71	0.0	0.01	0.0	0.0	4	1	1	0.22	0.5	4.4212
0.10	0.71	0.0	0.01	0.0	0.0	6	1	1	0.22	0.5	.0717
0.10	0.71	0.0	0.01	0.0	0.0	8	1	1	0.22	0.5	5.6904
0.10	0.71	0.0	0.01	0.0	0.0	10	1	1	0.22	0.5	6.2912
0.10	0.71	0.0	0.01	0.0	0.0	12	1	1	0.22	0.5	6.8909

Table5. Effect of Gc on velocity n=0.1, t=1 and A=1

.∈	Pr	R	Ec	η	k_r^2	Gr	Gc	M	Sc	K	C_f
0.10	0.71	0.0	0.01	0.0	0.0	2	2	1	0.22	0.5	4.1600
0.10	0.71	0.0	0.01	0.0	0.0	2	3	1	0.22	0.5	4.5199
0.10	0.71	0.0	0.01	0.0	0.0	2	4	1	0.22	0.5	4.8057
0.10	0.71	0.0	0.01	0.0	0.0	2	5	1	0.22	0.5	5.0115
0.10	0.71	0.0	0.01	0.0	0.0	2	6	1	0.22	0.5	5.1432

Table6. Effect of M on velocity n=0.1, t=1 and A=1.

∈	Pr	R	Ec	η	k_r^2	Gr	Gc	M	Sc	K	C_f
0.10	0.71	0.0	0.01	0.0	0.0	2	1	2	0.22	0.5	3.8420
0.10	0.71	0.0	0.01	0.0	0.0	2	1	4	0.22	0.5	4.0901
0.10	0.71	0.0	0.01	0.0	0.0	2	1	6	0.22	0.5	4.3376
0.10	0.71	0.0	0.01	0.0	0.0	2	1	8	0.22	0.5	4.5774

0.10	0.71	0.0	0.01	0.0	0.0	2	1	10	0.22	0.5	4.8078

Table7. Effect of Pr on temperature n=0.1, t=1 and A=1.

∈	Pr	R	Ec	η	k_r^2	Gr	Gc	M	Sc	K	C_f
0.10	0.71	0.0	0.01	0.0	2	2	1	1	0.6	0.5	.1748
0.10	1.0	0.0	0.01	0.0	2	2	1	1	0.6	0.5	0.6490
0.10	1.25	0.0	0.01	0.0	2	2	1	1	0.6	0.5	0.7390
0.10	1.52	0.0	0.01	0.0	2	2	1	1	0.6	0.5	0.8292

Table8. Effect of R on temperature n=0.1, t=1 and A=1.

∈	Pr	R	Ec	η	k_r^2	Gr	Gc	M	Sc	K	Nu/Re
0.10	0.71	0.2	0.01	0.0	0.0	2	1	1	0.22	0.5	16.9322
0.10	0.71	0.4	0.01	0.0	0.0	2	1	1	0.22	0.5	16.0492
0.10	0.71	0.6	0.01	0.0	0.0	2	1	1	0.22	0.5	15.5589
0.10	0.71	0.8	0.01	0.0	0.0	2	1	1	0.22	0.5	15.2596
0.10	0.71	1.0	0.01	0.0	0.0	2	1	1	0.22	0.5	15.0644

Table 9. Effect of Ec on temperature n=0.1, t=1 and A=1

∈	Pr	R	Ec	η	k_r^2	Gr	Gc	M	Sc	K	Nu/Re
0.10	0.71	0.0	0.02	0.0	0.0	2	1	1	0.22	0.5	18.7781
0.10	0.71	0.0	0.04	0.0	0.0	2	1	1	0.22	0.5	18.7330
0.10	0.71	0.0	0.06	0.0	0.0	2	1	1	0.22	0.5	18.6880
0.10	0.71	0.0	0.08	0.0	0.0	2	1	1	0.22	0.5	17.0208
0.10	0.71	0.0	0.10	0.0	0.0	2	1	1	0.22	0.5	18.5979

Table-10. Effect of η on temperature n=0.1, t=1 and A=1.

∈	Pr	R	Ec	η	k_r^2	Gr	Gc	M	Sc	K	Nu/Re
0.10	0.71	0.0	0.01	0.1	0.0	2	1	1	0.22	0.5	17.1935
0.10	0.71	0.0	0.01	0.1	0.0	2	1	1	0.22	0.5	18.5979
0.10	0.71	0.0	0.01	0.1	0.0	2	1	1	0.22	0.5	19.3898
0.10	0.71	0.0	0.01	0.5	0.0	2	1	1	0.22	0.5	22.9187
0.10	0.71	0.0	0.01	1.0	0.0	2	1	1	0.22	0.5	27.7513

Table 11. Effect of Sc on concentration n=0.1, t=1 and A=1.

∈	Pr	R	Ec	η	k_r^2	Gr	Gc	M	Sc	K	Nu/Re
0.10	0.71	0.0	0.01	0.0	0.0	2	1	1	0.30	0.5	-1.1102
0.10	0.71	0.0	0.01	0.0	0.0	2	1	1	0.66	0.5	-1.7746
0.10	0.71	0.0	0.01	0.0	0.0	2	1	1	0.78	0.5	-1.9678
0.10	0.71	0.0	0.01	0.0	0.0	2	1	1	1	0.5	-2.3016
0.10	0.71	0.0	0.01	0.0	0.0	2	1	1	2	0.5	-3.6433

Table-(1)- (6) The effects of the radiation parameter, chemical reaction and heat generation on the skin- friction coefficient. It is observed from this table that as radiation parameter increases, the skin-friction coefficients increases, as either the Prandlt number or chemical reaction Parameter effect increases, skin-friction coefficient decreases. Also decreases in the heat generation Parameter effect, the skin-friction coefficient increases.

Table-(7)-(10) As the radiation parameter increases the Nusselt number decreases, increases in chemical reaction, Grashof number for mass transfer, Permeability parameter, magnetic field, heat generation, Prandlt number, Eckert number, Schmidt number, Grashof number for heat transfer, the Nusselt number also increases.

Table-(11) reflects that the Sherwood number at the plate decreases with the increase of chemical reaction or Schmidt number, also an increase of Grashof number for heat transfer, Magnetic field and Permeability Parameter and Prandlt number. Sherwood remains unchanged.

5 CONCLUSIONS

Heat and mass transfer of MHD and dissipative fluid flow past a moving vertical porous plate with variable suction in the presence of chemical reaction, heat source, transfer magnetic field and oscillating free stream are carried out and the following conclusions are made.
A rise in the Grashof number causes an increase in the heavy flow of fluid velocity owing to the increase in quality of buoyancy force. The highest point of the velocity goes up quickly near the porous plate as buoyancy force for heat movement rises and rots the free stream velocity.
The size of fluid velocity reaches a very high value with the rise of buoyancy force for a large amount of movement in the drops appropriately to reach a free stream velocity.
The size of fluid velocity drops with the rise of molecular diffusivity of the magnetic field, while it rises with the increase of heat source.
A rise in the chemical reaction parameter leads to reduction of the velocity as well as the species concentration. The hydrodynamic and the concentration boundary surface get thick as the reaction parameter goes up.
The size of fluid concentration reduces with the rise of chemical reaction parameter.
A rise in the radiation heat transfer leads to a fall in the size of fluid velocity and the height of fluid temperature inside the boundary surface and also a fall in the thickness of the velocity as well as thermal boundary layer.
A rise in Prandlt number generates a fall in the thermal boundary layer and in totality less average temperature inside the boundary area being the lesser values of PR are same to the rise in the thermal conductivity of the fluid. Hency, heat is able to pass away from the heated surface quickly for higher values of PR. Due to this, for smaller Pr, the rate of heat movement are reduced.
The level of fluid temperature falls with the rise of chemical reaction parameter, viscous dissipation effect and molecular diffusivity; while it rises alongside an increase of level of magnetic field and heat source.

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APPEDICES

Fig. 1: The effect of R on velocity

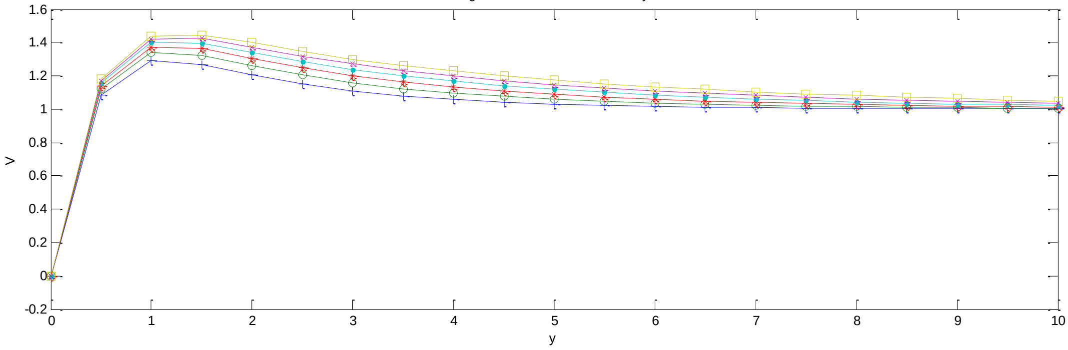

Fig.2: The effect of R on temperature

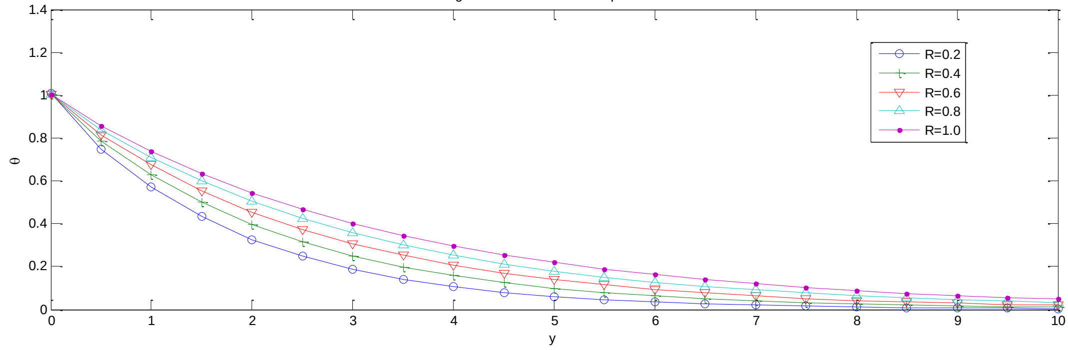

Fig.3: The effect of kr_2 on velocity

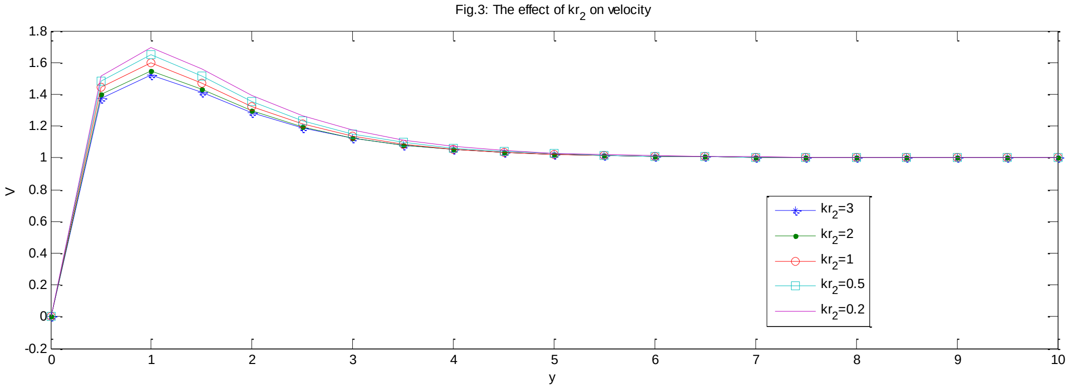

Fig.4: The effect of kr_2 on concentration

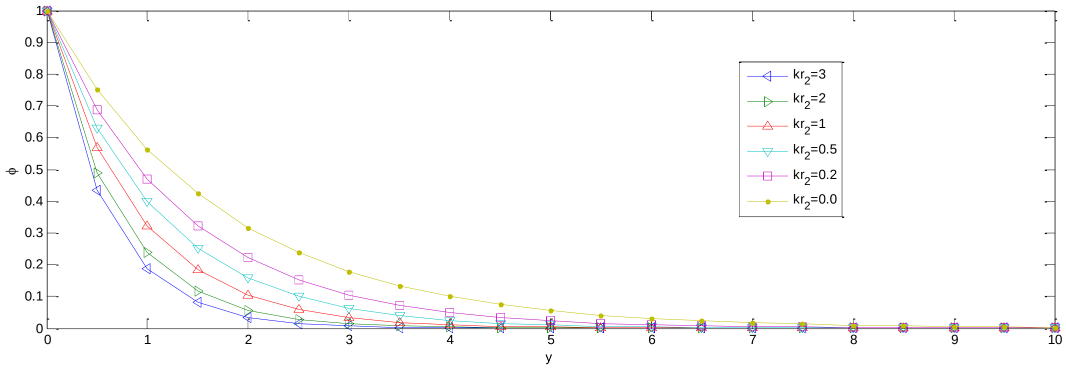

Fig.5: The effect of η on velocity

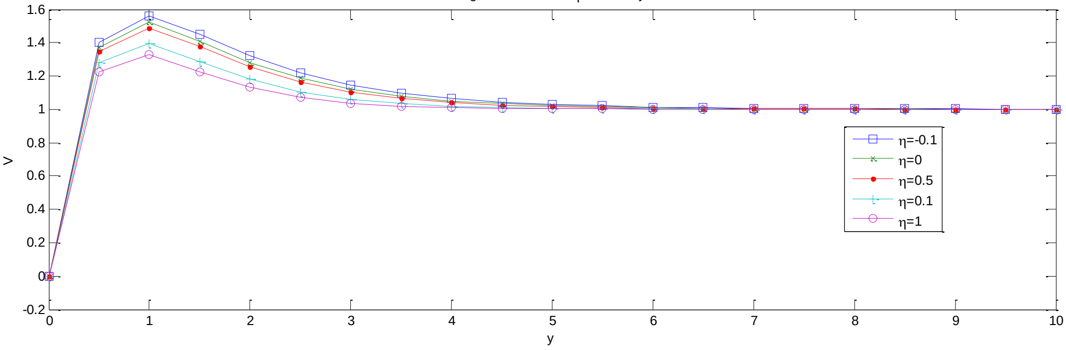

Fig.6: The effect of η on temperature

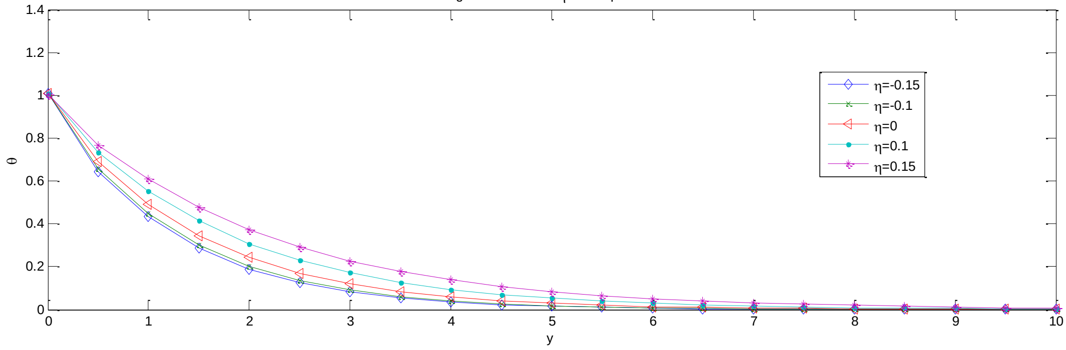

Fig.7: The effect of Gc on velocity

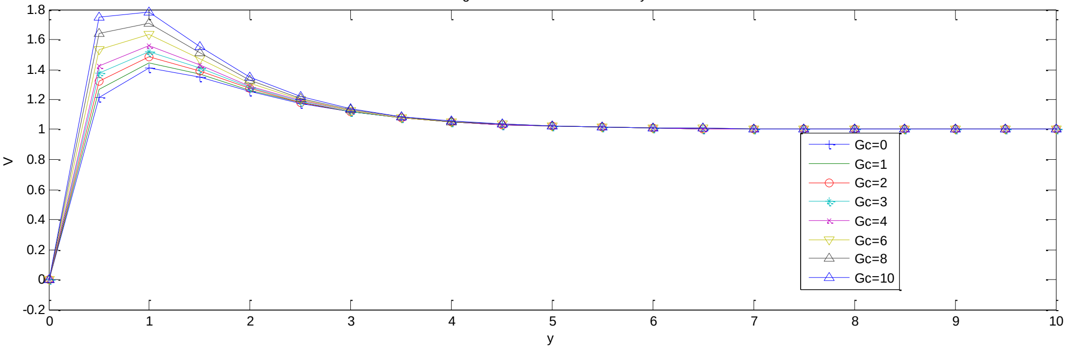

Fig.8: The effect of Sc on velocity

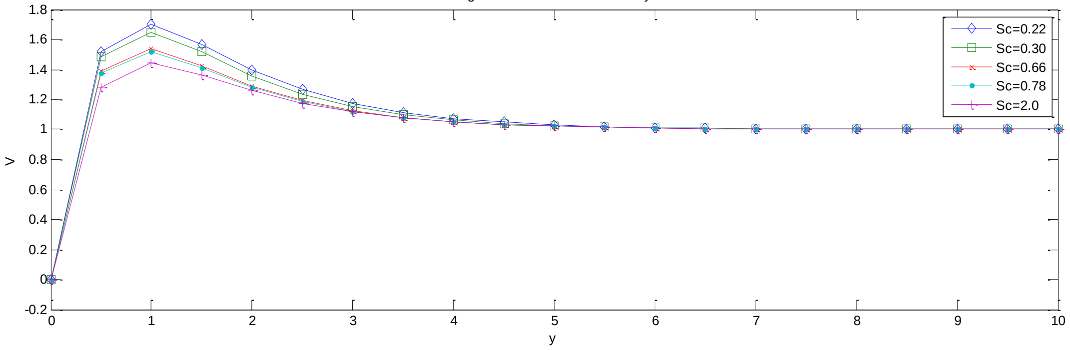

Fig.9: The effect of Sc on concentration

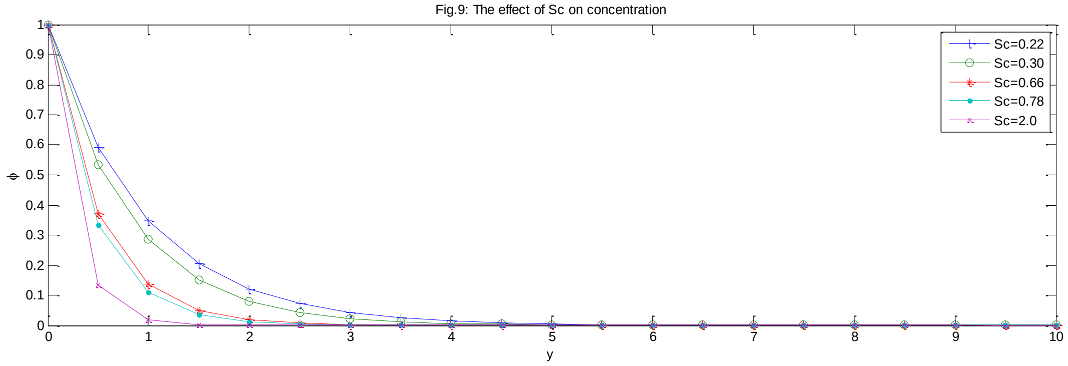

Fig.10: The effect of Gr on velocity

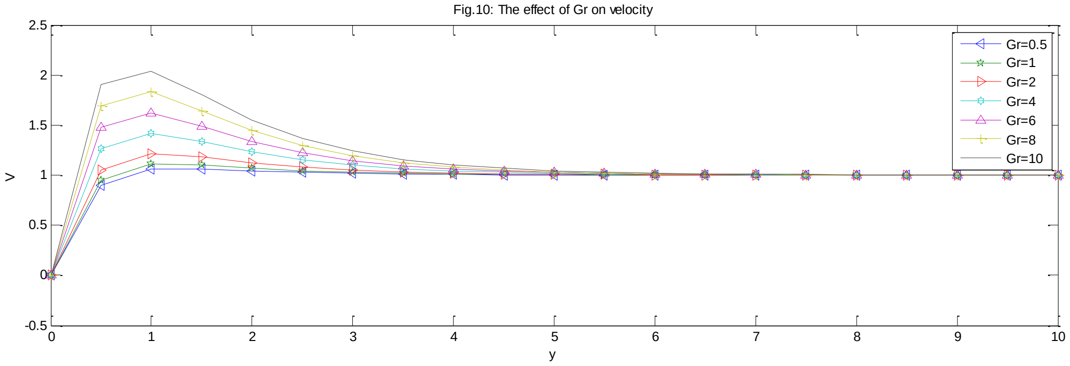

Fig.11: The effect of M on velocity

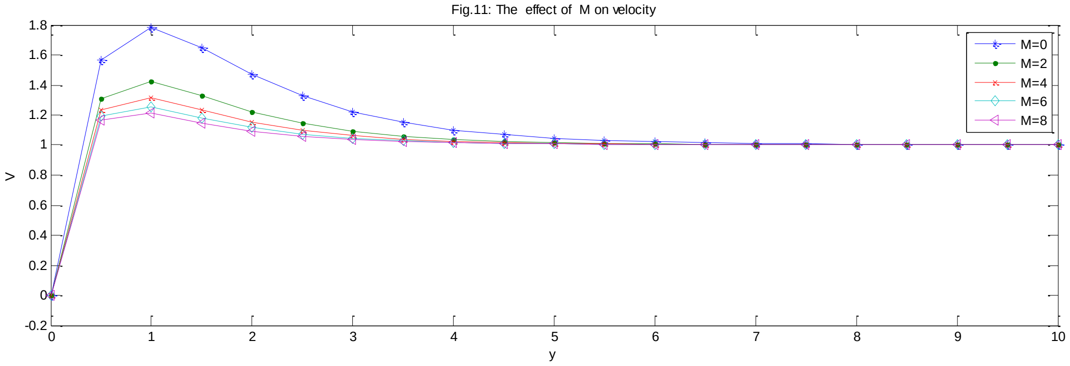

Fig.12: The effect of Pr on velocity

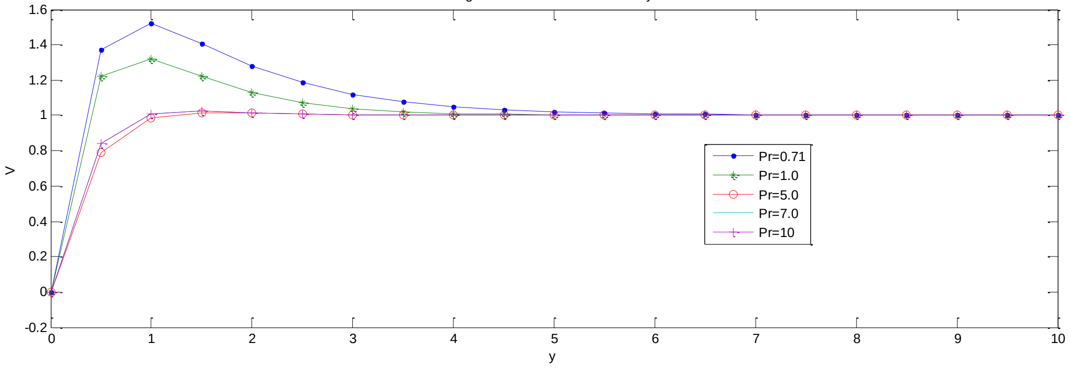

Fig.13: The effect of Pr on temperature

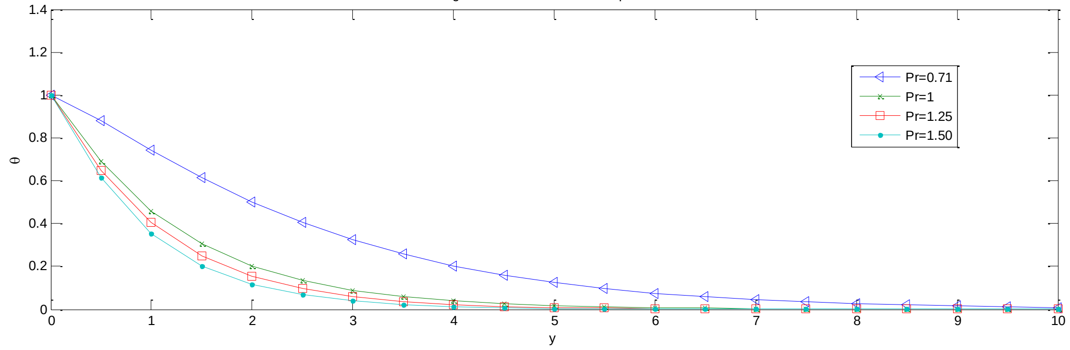

Fig .15:The effect of K on velocity

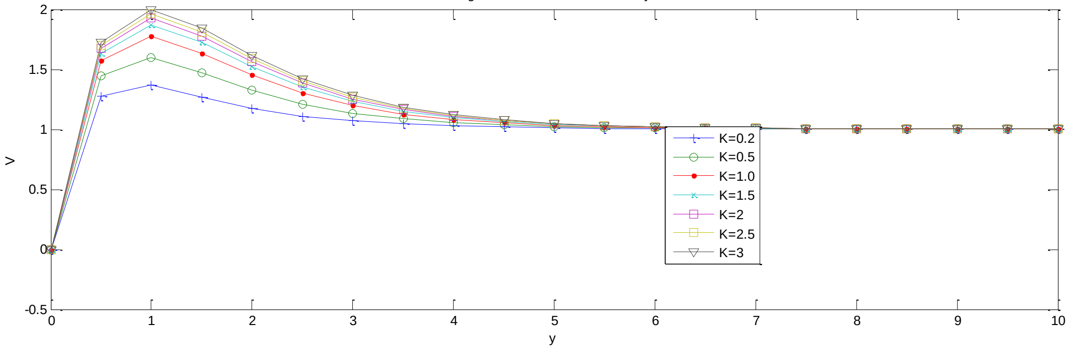

Fig.16: The effect of Ec on temperature