Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.3, No.2, 2013
www.iiste.org

Weather Control - Astrobilliard and Mathematical Tsunami

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Abstract

In this paper we give mathematical model to generalize mathematically the concept of billiard game to be played into a universe region, which leads to the ionosphere’s Plasma heating, thus allow one to get a tsunami waves where he need.

Keyword: Riemannian manifold, Energy conservation, electro- magnetic fields, Plasma heating**.**

1. Introduction

Suppose a Riemannian manifold M with a smooth boundary [6]. Then the billiard dynamical system in regain M is generated by the free motion of a mass-point with elastic reflection in the boundary, which means that the point moves along a geodesic line in M with a constant speed. At a smooth boundary point the billiard ball reflects so that the tangential component of its velocity remains the same, while the normal component changes its sign. In two dimensions this collision is described by a well-known law of geometrical optics, [the angle of incidence equals the angle of reflection]. Thus lead to say that the billiards theory and the geometrical optics theory have many features in common**.**

Since universe contain plasmas, which consist of charged particles, so description of a gas as a system of elastic balls moving inside a container goes back to Boltzmann and Poincaré [1]. For an ideal gas then balls representing its molecules are replaced by mass points (particles). The probability of interaction of two arbitrary particles is zero; so the particles move independently from each other.

Behavior of such a system is described by means of billiards that had been introduced by Birkhoff [2], a particle moves linearly and uniformly inside a closed domain M , and bounces off the boundary in such a way that the normal component of its velocity changes its direction to the opposite, while the tangential component of its velocity is constant in time. Since the energy of the particle is preserved in time the description of a gas by Boltzmann’s model has a physical sense only when the gas is in equilibrium state.

2. Behavior of Particles motions and energy conservation

Below we will generalize concept of particle motions to say that the motions of particles are always geodesies with respect to a metric on the surface conformal to the original one, since geodesies should be curves of shortest arclength [7]. We took the definition of geodesic to be a curve whose tangential component of acceleration vanishes, however, because this characterized lines in 3-space and seemed a more directly verifiable condition.

Theorem 1: Let $M : x(u,v)$ be a surface with metric E and G . Then , extremals for the arclength integral

$$J = \int \sqrt{E\dot{u}^2 + G\dot{v}^2}\,dt$$

are geodesies on the surface M .

Proof. We calculate the two variable Euler-Lagrange equations to be

$$\frac{E\dot{u}^2 + G\dot{v}^2}{2\sqrt{E\dot{u}^2 + G\dot{v}^2}} - \frac{d}{dt}\left(\frac{E\dot{u}}{\sqrt{E\dot{u}^2 + G\dot{v}^2}}\right) = 0$$

$$\frac{Eu'^2 + Gv'^2}{2\sqrt{E\dot{u}^2 + G\dot{v}^2}} - \frac{d}{dt}\left(\frac{Gv'}{\sqrt{E\dot{u}^2 + G\dot{v}^2}}\right) = 0$$

Now we introduce the arclength parameter $s(t)$ since we know geodesies must be parametrized to have constant speed. So we have the relations:

$$\frac{d}{dt} = \frac{d}{ds}.\sqrt{E\dot{u}^2 + G\dot{v}^2}$$

$$u' = \frac{du}{ds} = \frac{du}{dt}\frac{dt}{ds} = \frac{\dot{u}}{\sqrt{E\dot{u}^2 + G\dot{v}^2}}$$

$$v' = \frac{dv}{ds} = \frac{dv}{dt}\frac{dt}{ds} = \frac{\dot{v}}{\sqrt{E\dot{u}^2 + G\dot{v}^2}}$$

Putting these into the equations above gives:

$$\frac{E_u}{2}u'^2 + \frac{G_u}{2}v'^2 - \frac{d}{ds}(Eu') = 0$$

$$\frac{E_v}{2}u'^2 + \frac{G_v}{2}v'^2 - \frac{d}{ds}(Gv') = 0$$

Carrying out the differentiation and simplifying produces

$$u'' + \frac{E_u}{2E}u'^2 + \frac{E_v}{E}u'v' - \frac{G_u}{2E}v'^2 = 0$$

and

$$v'' - \frac{E_v}{2G}u'^2 + \frac{G_u}{G}u'v' + \frac{G_v}{2G}v'^2 = 0$$

These are the geodesic equations.

Now that we know that geodesies are extremals of the arclength integral, Recall that kinetic and potential energies have the form:

$$T = \frac{1}{2}\left|\frac{d\alpha}{dt}\right|^2 = \frac{1}{2}\left(E\dot{u}^2 + G\dot{v}^2\right) \quad and \quad V = (u, v)$$

where α a is a particle trajectory. The action integral $\int (T ? \quad V)dt$ is independent of t, which implies that energy $H = T + V$ is a constant along paths of motion. Specifically,

$$
\begin{aligned}
& T - V - \dot{u}\frac{\partial (T - V)}{\partial \dot{u}} - \dot{v}\frac{\partial (T - V)}{\partial \dot{v}} = c \\
& \frac{1}{2}\left(E\dot{u}^2 + G\dot{v}^2\right) - V - \dot{u}(E\dot{u}) - \dot{v}(G\dot{v}) = c \\
& \frac{1}{2}\left(E\dot{u}^2 + G\dot{v}^2\right) - V - E\dot{u}^2 - G\dot{v}^2 = c \\
& -\frac{1}{2}\left(E\dot{u}^2 + G\dot{v}^2\right) - V = c \\
& T + V = -c
\end{aligned}
$$

Now let's take a curve $\alpha : I \rightarrow M$ representing the motion of a particle determined by Hamilton's principle on a constraint surface M. We take the metric of the surface to be E, $F = 0$ and G. Because we know that the path of the particle must conserve energy and because this condition is incompatible with the unit speed condition, we must explicitly reparametrize the curve to have constant energy. To do this, let H_0 denote the constant energy along the path of the particle and define

$$\tau = \int \frac{1}{\sqrt{2(H_0 - V)}}\sqrt{E\dot{u}^2 + G\dot{v}^2}\,dt$$

Lemma 1. The reparametrized curve $\alpha(\tau)$ has constant energy.

Proof. The fundamental theorem of calculus implies

$$\frac{d\tau}{dt} = \frac{1}{\sqrt{2(H_0 - V)}}\sqrt{E\dot{u}^2 + G\dot{v}^2}$$

so that the chain rule then give:

$$
\begin{aligned}
\frac{du}{d\tau} &= \frac{du}{dt}\frac{dt}{d\tau} = \frac{du}{dt}\frac{\sqrt{2(H_0 - V)}}{\sqrt{E\dot{u}^2 + G\dot{v}^2}} \\
\frac{dv}{d\tau} &= \frac{dv}{dt}\frac{dt}{d\tau} = \frac{dv}{dt}\frac{\sqrt{2(H_0 - V)}}{\sqrt{E\dot{u}^2 + G\dot{v}^2}}
\end{aligned}
$$

Using this, the following calculation then shows that this reparametrization conserves energy at the value H_0 along the curve.

$$\begin{aligned}
T(\tau)+V(\tau) &= \frac{1}{2}\left(E(\tau)\left(\frac{du}{d\tau}\right)^2 + G(\tau)\left(\frac{dv}{d\tau}\right)^2\right)+V(\tau)\\
&= \frac{1}{2}\left(E\left(\frac{du}{d\tau}\right)^2 \frac{2(H_0-V)}{E\dot{u}^2+G\dot{v}^2} + G\left(\frac{dv}{d\tau}\right)^2 \frac{2(H_0-V)}{E\dot{u}^2+G\dot{v}^2}\right)+V(\tau)\\
&= 2(H_0-V)\frac{1}{2}\left(\frac{E\dot{u}^2+G\dot{v}^2}{E\dot{u}^2+G\dot{v}^2}\right)+V\\
&= H_0 - V + V\\
&= H_0
\end{aligned}$$

which shows that $T(\tau) = H_0 ? V$. Also, the general energy equation $T+V=H$ gives $2T-H = 2T-(T+V) = T-V$. We shall use both of these relations in the following to determine the equations of motion of a particle as extremals of

$$\int T(t) ? V(r)d\tau$$

According to conservation of energy, these extremals must be found within the class of all curves with constant energy H_0, so:

$$\int T(\tau) - V(\tau)\, d\tau = \int 2T(\tau) - H_0(\tau)\, d\tau$$

Therefore :

$$\begin{aligned}
\int 2T(\tau)\, d\tau &= \int 2\,(H_0(\tau)-V)d\tau\\
&= \int 2\,(H_0-V)\frac{d\tau}{dt}dt\\
&= \int 2\,(H_0-V)\frac{1}{\sqrt{2(H_0-V)}}\sqrt{E\dot{u}^2+G\dot{v}^2}\,dt\\
&= \int \sqrt{2\,(H_0-V)E\dot{u}^2 + 2\,(H_0-V)G\dot{v}^2}\,dt\\
&= \int \sqrt{\bar{E}\dot{u}^2+\bar{G}\dot{v}^2}\,dt
\end{aligned}$$

where $\bar{E} = 2(H_0 - V)E$ and $\bar{G} = 2(H_0 ? V)G$ defines a new metric on M conformal with respect to the the original E and G . (assume that $H_0 > V$ in this region of the parameter domain.) It explain that finding an extremal for $\int T - V \, d\tau$ corresponds to finding an extremal for $\int \sqrt{\bar{E}\dot{u}^2 + \bar{G}\dot{v}^2}\,dt$. But, by Theorem (1), $\int \sqrt{\bar{E}\dot{u}^2 + \bar{G}\dot{v}^2}\,dt$ has extremals which are geodesies on M with respect to the metric $\bar{E} = 2(H_0 - V)E$ and $\bar{G} = 2(H_0 \ ? \ V)G$. Therefore, we have the following special case of a result of Jacobi.

Theorem 2: Let α denote the path of a particle with constant energy H_0 under the influence of a potential V constrained to lie on a surface M with metric E , $F = 0$ and G . Then, for $H_0 > V$, a is a geodesic on M with respect to a conformal metric

$$E = 2(H_0\text{-}V)E \quad ; \quad F = 0 \quad ; \quad G = 2(H_0\text{-}V)G.$$

Now, most important questions about a mechanical system is whether or not there is a periodic orbit ?, by above theorem, this question is equivalent to asking whether a surface M has a closed geodesic. Since any compact (i.e. closed and bounded) surface does indeed have a closed geodesic. Applying this to a surface in conjunction with Jacobi's theorem gives:

Corollary 1: If M is a compact surface and $V(u, v)$ is a potential function with $H_0 > V(u, v)$ for all (u, v) in the parameter domain of M , then there exists a periodic solution to the equations of motion of a particle constrained to move on the surface under the influence of V .

Note that if $V = -1/\sqrt{u^2 + v^2}$ the Newtonian potential in a plane, then

$$V_u^2 + V_v^2 = \frac{1}{(u^2 + v^2)^2}$$

and

$$V_{uu} + V_{vv} = -\frac{1}{(u^2 + v^2)^{\frac{3}{2}}}$$

which gives:

$$K = -\frac{H_0}{4(r.H_0 + 1)^3}$$

where $r = \sqrt{u^2 + v^2}$ is the radial distance from the origin. Now, the constant energy equation $\frac{v^2}{2} - \frac{1}{r} = H_0$ gives $v^2 - 2(H_0 r + 1)/r > 0$, hence, the denominator of Gauss curvature K is positive. Then:

$$\begin{aligned} H_0 > 0 &\Leftrightarrow K < 0 \\ H_0 = 0 &\Leftrightarrow K = 0 \\ H_0 < 0 &\Leftrightarrow K > 0. \end{aligned}$$

Therefore, orbits are characterized in terms of their Gauss curvatures. In particular, a point where $K > 0$ guarantees a periodic orbit.

3. Charged Particles Dynamics in Electric and Magnetic Fields

Understand the plasmas behavior within universe region, implies that first necessary understanding the behavior of individual charged particles under the influence of electric and magnetic fields [4].

The motion of a particle with electric charge *q* and mass *m* in electric and magnetic fields can be determined from the combined electrostatic and Lorentz force:

$$F = q(E + v \times B)$$

For $E = 0$ and a homogeneous magnetic field, the kinetic particle energy remains constant because the Lorentz force is always perpendicular to the velocity and can thus change only its direction, but not its magnitude. In a uniform magnetic field B the motion of a charged particle has two parts. Firstly, it has a circular motion perpendicular to the magnetic field, the radius of the circle being called the Larmor radius.

$$\rho_L = \frac{m_q v_\perp}{|q| B}$$

This radius increases with the energy of the particle and decreases with the strength of the magnetic field. For a typical ion in a *JET* plasma the Larmor radius is a few millimeters. For an electron the Larmor radius is smaller by the square root of the electron-ion mass ratio and is typically a tenth of a millimeter. Because of the opposite signs of their charges the electrons and ions circulate in opposite directions [3]. The other part of the motion is that along the magnetic field. In a uniform magnetic field the charged particle’s motion parallel to the field is unaffected by the field, and the particle’s “parallel velocity” is constant. When the two parts of the motion are combined we have a helical trajectory

4. Particle Heating Acceleration via Fermi Processes

The mechanisms of particle acceleration and heating can be classified as *dynamic*, *hydrodynamic* and *electromagnetic*; we will now consider a standard astrophysical particle acceleration process which called *Fermi acceleration*. Fermi made the first serious attempt at explaining the power law nature of the cosmic ray spectrum. He noted that if cosmic rays are injected steadily into a localized acceleration region where they gain energy at a rate that is proportional to their energy while at the same time their escape from the region is an energy-independent Poisson process [8], then the stationary particle distribution will always be a power law. Fermi also argued that the most efficient acceleration mechanism would be a stochastic one.

Ionosphere contain plasmas, which consist of charged particles. These charged particles can have a distribution of momenta *p* and this distribution can be a function of 3D space x and time t . The distribution function is normally given by the parameter

$$f\,(x\,,\,p\,,\,t\,)$$

This is also sometimes referred to as a *phase space density* because f gives the number of particles in the 6-dimensional phase space defined by p and x at a given t. The *number density* of particles is the number of particles per unit volume at a particular location x and is defined by

$$n(x,\,t) = \int f(x,\,p,\,t)\,dv$$

where $dv = dv_x dv_y dv_z$. Particle conservation requires that the rate of change of the number of particles per unit time per unit volume is equal to the flux of particles across the surface of the volume This conservation law is given by Vlasov equation:

$$\frac{\partial f}{\partial t} + v.\nabla f + \frac{\partial}{\partial P}\left(\frac{dP}{dt}f\right) = 0$$

which is sometimes also referred to as the *collisionless Boltzmann equation* because Coulomb collisions are assumed to be negligible. If electrostatic interactions between particles are important, then inclusion of a Coulomb collision term gives the Boltzmann equation. Under steady-state conditions, the Boltzmann equation gives the Maxwellian distribution:

$$f\,(P,x) = n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} exp\left(\frac{\left|P - \langle P\rangle\right|^2}{2\pi kT}\right)$$

Which is well-known solution to the particle distribution function?

This is the distribution function for particles in thermal equilibrium. The mean kinetic energy is $\varepsilon = \frac{1}{2}kT$ per particle per degree of freedom.

In much powerful astrophysical process, large amounts of energy appear to be dissipated into the ambient medium over a relatively short timescale. If the gas density is high (cold state), then Coulomb collisions between electrons and ions will efficiently thermalize the plasma. The dissipated energy ends up distributed equally amongst the particles, forming a Maxwellian distribution. This process is referred to as plasma *heating*.

Particle collisions in the plasma play an important role for all collective effects like e.g. resistivity in the plasma. When an electron collides with a neutral atom, no force is felt until the electron is close to the atom; these collisions are similar to billiard-ball collisions. However, when an electron collides with an ion, the electron is gradually deflected by the long-range Coulomb field of the ion. This force falls off comparatively slowly with distance, in fact with the inverse square of the distance between the particles. As a result of this long range interaction any given particle is colliding simultaneously with a large number of the particles. In a plasma such as that in JET each particle is simultaneously "in collision" with millions of other particles [5, 8]. An effective collision time can be defined for each particle species as the time for the multiple collisions to produce a deflection through a large angle. The collision times depend sensitively on the plasma temperature, but taking typical JET plasma the collision time of the electrons is a few hundred microseconds, and of the ions is tens of milliseconds. The distance travelled in this time gives a mean free path of hundreds of meters for both ions and electrons.

5. Plasma Heating Systems

Now note that ions and electrons as the plasma are confined to rotate around the magnetic field lines, electromagnetic waves of a frequency matched to the ions or electrons gyrofrequency are able to resonate or damp their wave power into the plasma particles.

Microwave heating is actually a sub-category of dielectric heating in that insulating materials are heated primarily by dielectric loss. The difference is that of frequency. An electromagnetic wave (at frequencies above 100 MHz) can be launched from a small dimension emitter and conveyed through space. So heating (of material) can takes place simply by placed it in the path of the waves, but note that it is a non-contact process.

High-frequency electromagnetic waves are generated by oscillators If the waves have the correct frequency (or wavelength) and polarization, then energy can be transferred to the plasma, and thus increasing the temperature of the bulk plasma according to collide with other plasma particles

6. Conclusion

We believe that the relations derived above give mathematical model which allow reasonable order of magnitude estimates for many cases of interest, thus naturally lead to the presence of Plasma heating. So if one have system of microwave generators (say radar field) one can heat the plasma of ionosphere on any region over oceans to high degree and put off the radar which oscillate the ionosphere (when Upper atmosphere layers be so cold) and this oscillate the water under region (according to Air pressure), so this process allow person to control the weather or to make waves at any volume.

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