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Mixed Convective MHD Flow of Second Grade Fluid with Viscous Dissipation and Joule Heating Past a Vertical Infinite Plate with Mass Transfer

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Abstract

The boundary layer flow of an unsteady, two-dimensional, laminar, mixed convective and mass transfer of a viscoelastic, incompressible and electrically conducting fluid past an infinite vertical flat plate with suction in the presence of transverse magnetic field has been analysed by taking into account the effect of viscous dissipation and Joule heating. The momentum and energy equations are reduced to couple non-linear partial differential equations along with the boundary conditions by using a suitable similarity transformation. These partial differential equations are transformed to a system of coupled non-linear ordinary differential equations by employing a perturbation technique. The system is then solved by developing a suitable numerical procedure such as fourth order Runge-Kutta method along with shooting technique. The expressions for velocity field, temperature field, concentration field and rate of heat transfer have been obtained. The results are discussed in detailed with the help of graphs and table to observe the effect of different pertinent parameters.

Keywords: Second grade fluid, Joule heating, Eckert number

1. Introduction

The mixed convection boundary layer flow of a non-Newtonian fluid in the presence of strong magnetic field has wide range of applications in nuclear engineering and industries. In astrophysical and geophysical studies, the MHD boundary layer flows of an electrically conducting fluid through porous media have also enormous applications. These studies are used for modelling and simulation. Many researchers have studied the transient laminar natural convection flow past a vertical porous plate for the application in the branch of science and technology such as in the field of agriculture engineering and chemical engineering. With the advancement of science and technology, MHD study on any fluid flow phenomenon exhibits some results which have constructive application for the design of devices. MHD heat transfer has great importance in the liquid metal flows, ionized gas flow in a nuclear reactor and electrolytes. The steady flow of a non-Newtonian fluid past a porous plate with suction has been examined by Mansutti et.al. (1993). Sattar (1994) has investigated the free convection and mass transfer flow past an infinite vertical porous plate with time dependent temperature and concentration. Choudhury et al. (2000) have studied the MHD boundary layer flow of a non-Newtonian fluid past a flat plate. The mass transfer effects on unsteady flow past an accelerated vertical porous plate have been discussed by Das et.al (2006).

Considering the application in underground water resource and seepage of water under dam, Raptis et al. (1981) examined the free convection flow through a vertical porous media. In the other context Raptis et al. (1981, 1982) studied the similar problem by introducing mass transfer, constant suction and constant heat flux. Lai et al. (1990) have shown an interesting problem regarding free convective flow of Newtonian fluid through a vertical porous medium with varying permeability. Several authors have studied the radiation effect on the flow past a parallel plate (e.g., Sharma (2005), Raptis et al. (1998)). Rahman et al. (2008) presented an interesting result on a natural convection flow past a vertical plate considering the temperature dependent thermal conductivity and heat conduction. Using implicit finite difference scheme Laganathan et al. (2010) solved a coupled non-linear momentum and energy equations for the investigation of the effect of thermal conductivity on the free convective flow over a semi-infinite vertical plate under the influence of transverse magnetic field. In polymer processing industries, researchers deals with non-Newtonian fluids in which stress strain relations are non-linear. The differential types of

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fluid flow exhibits a little effect of deformation gradient on the stress. In the constitutive relations stress is just a function of velocity gradient and its higher power and the stress relaxation time is very small. Rivlin-Ericksen (Rivlin et al. (1955, 1955), Dunn et al. (1995)) fluid shows small relaxation time factor in addition to a convective time derivative in its constitutive relation. The boundary layer flow of non-Newtonian Rivlin-Ericksen fluid past a wedge is investigated by Massoudi and Ramezan (1989). They presented results for velocity and shear stress at the wall for different suction/injection rates and different wedge angle. The flow and heat transfer in a second grade fluid have been studied by many researchers in different contexts. For example, Parida et al. (2011) have examined the magnetic effect on the flow and heat transfer of second grade fluids in a channel with porous wall and later Bhargava et al. (2012) have numerically analysed the flow and heat transfer of a second grade fluid over an oscillatory stretching sheet including viscous dissipation and Joule heating. A boundary layer analysis for Newtonian conducting fluid over an infinite vertical porous plate has been carried out by Singh et al. (2009). In this investigation they studied the effect of viscous dissipation, Joule heating, thermal diffusion and Hall current. The study of heat dissipation on the flow of viscoelastic fluid past an infinite vertical plate is made by Uwanta et al. (2011). The analysis of mixed convective MHD flow of second grade fluid past a vertical infinite plate with mass transfer has been reported by Mahanta et al. (2012). Poonia et al. (2010) have investigated MHD free convection and mass transfer flow of Newtonian fluid over an infinite vertical porous plate with viscous dissipation.

The objective of the present study is to extend the work of Poonia et al. (2010) by considering viscoelastic effect characterized by second grade fluid incorporating the effects of viscous dissipation and Joule heating. These governing equations of motion are solved numerically to a better converging solution. The flow phenomena has been characterized with the help of flow parameters and the effect of these parameters on the velocity field, temperature, concentration and skin friction have been analysed and the results are presented graphically and discussed qualitatively.

2. Problem Formulation

Let us consider an unsteady two-dimensional mixed convective boundary layer flow of an electrically conducting non-Newtonian second grade viscoelastic fluid through an infinite vertical heated plate in the presence of thermal and concentration buoyancy effects. The effect of viscous dissipation and Joule heating are taken into account. The x-axis is taken in the upward direction of the plate and y-axis is normal to it. A constant magnetic field of strength B_0 is applied in the direction perpendicular to the plate and the effect of the induced magnetic field is neglected. All the fluid properties are assumed to be isotropic and constant.
The problem under consideration is treated under boundary layer and Boussinesq approximation frame work. With the underlying assumptions the conservation of mass, momentum, energy and species concentration lead to the following partial differential equations:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} + \frac{k_0}{\rho}\left(\frac{\partial^3 u}{\partial t \partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right) + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) - \frac{\sigma B_0{}^2}{\rho}u \tag{2}$$

$$\rho c_p\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right) = K\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma B_0^2 u^2 \tag{3}$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

The symbols u and v denote the fluid velocity in the x- and y- directions respectively. Here, T and C represent the temperature and concentration fields respectively, ρ is the density, μ is the coefficient of viscosity, D is the mass diffusivity, g is the acceleration due to gravity, β_T is the coefficient of thermal expansion, β_C is the coefficient of volumetric expansion, t is the time, σ is the electrical conductivity, c_p is the specific heat at constant pressure and K is the thermal conductivity.

From Eq. (1), it is clear that v is constant or functions of time only, so we consider it in the following form

$$v = -v_0\left(1+\varepsilon A e^{i\omega t}\right) \tag{5}$$

where A is the suction parameter (real positive constant), ε and εA are small less than unity, ω-the frequency of the suction velocity and v_0 is a suction velocity which is non-zero positive constant. The negative sign indicates that the suction is towards the plate.

The boundary conditions of Eqs. (2)- (4) are:

$$\left.\begin{array}{l} \mathrm{u}=0,\ \mathrm{T}=\mathrm{T}_\infty+\mathrm{T}_0(\mathrm{t})\left(\mathrm{T}_\mathrm{w}-\mathrm{T}_\infty\right),\quad \mathrm{C}=\mathrm{C}_\infty+\mathrm{C}_0(\mathrm{t})\left(\mathrm{C}_\mathrm{w}-\mathrm{C}_\infty\right),\quad \text{at y=0} \\ \mathrm{u}\to 0,\ \dfrac{\partial \mathrm{u}}{\partial \mathrm{y}}\to 0,\ \mathrm{T}\to T_\infty,\quad C\to C_\infty,\ \text{as } \mathrm{y}\to\infty \end{array}\right\} \tag{6}$$

The subscripts w and ∞ refer to the conditions at wall and far away from the plate respectively. Here, $T_0 = C_0 = \left(1+\varepsilon e^{i\omega t}\right)$. Following Fosdick et al. (1980) an extra stress free boundary condition is augmented at far away from the plate. Introducing the following change of variables

$$\left.\begin{array}{l} y=\left(\dfrac{\upsilon}{v_0}\right)\tilde{y},\quad t=\left(\dfrac{4\upsilon}{v_0^2}\right)\tilde{t},\quad u=v_0\tilde{u},\quad w=\left(\dfrac{v_0^2}{4\upsilon}\right)\tilde{w} \\ T=T_\infty+\theta\left(T_w-T_\infty\right),\quad C=C_\infty+\phi\left(C_w-C_\infty\right) \end{array}\right\} \tag{7}$$

where $\upsilon=\dfrac{\mu}{\rho}$ is the dynamic viscosity.

Using scaling relation Eq. (7) in the governing Eqs. (2)-(4) and after dropping the tilde(~), we get the following equations in non-dimensional form.

$$\frac{1}{4}\frac{\partial u}{\partial t}-(1+\varepsilon A e^{i\omega t})\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+\alpha\left\{\frac{1}{4}\frac{\partial^3 u}{\partial t\partial y^2}-(1+\varepsilon A e^{i\omega t})\frac{\partial^3 u}{\partial y^3}\right\}+G_r\theta+\mathrm{G_m}\phi-Mu \tag{8}$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t}-\left(1+\varepsilon A e^{i\omega t}\right)\frac{\partial \theta}{\partial y}=\frac{1}{\mathrm{P}_r}\frac{\partial^2 \theta}{\partial y^2}+E_c\left(\frac{\partial u}{\partial y}\right)^2+E_cMu^2 \tag{9}$$

$$\frac{1}{4}\frac{\partial \phi}{\partial t}-(1+\varepsilon A e^{i\omega t})\frac{\partial \phi}{\partial y}=\frac{1}{S_c}\frac{\partial^2 \phi}{\partial y^2} \tag{10}$$

The dimensionless numbers are:

- The Prandtl number $\mathrm{P_r}=\mu c_p/K$ that represents the ratio of momentum to thermal diffusivity,
- The Schmidt number $S_c=\upsilon/D$ that represents the ratio of momentum to mass diffusivity,
- The dimensionless number $G_r=g\beta_T\upsilon(T_w-T_\infty)/v_0^3$ is the Grashof number, $\mathrm{G_m}=g\beta_C\upsilon(C_w-C_\infty)/v_0^3$ is the modified Grashof number, $E_c=v_0^2/c_p(T_w-T_\infty)$ is the Eckert number, $M=\sigma B_0^2\upsilon/\rho v_0^2$ is the magnetic field parameter and $\alpha=k_0v_0^2/\rho\upsilon^2$ is the viscoelastic parameter.

The corresponding boundary conditions Eq. (6) in view of scaling relation Eq. (7) reduces to

$$\left.\begin{array}{l} u=0,\quad \theta=T_0,\quad \phi=C_0\quad at\ \ y=0 \\ u\to 0,\quad \dfrac{\partial u}{\partial y}\to 0,\quad \theta\to 0,\quad \phi\to 0\ \ as\ \ y\to\infty \end{array}\right\} \tag{11}$$

3. Method of Solution

In order to obtain solutions of Eqs. (8)--(10) together with boundary conditions Eq. (11), we superimposed the unsteady flow on the mean steady flow i.e.

$$\left.\begin{aligned} u(t,y) &= u_0(y) + \varepsilon e^{i\omega t} u_1(y) \\ \theta(t,y) &= (1-\theta_0(y)) + \varepsilon e^{i\omega t}(1-\theta_1(y)) \\ \phi(t,y) &= (1-\phi_0(y)) + \varepsilon e^{i\omega t}(1-\phi_1(y)) \end{aligned}\right\}$$

(12) Substituting Eqs. (12) into Eqs. (8)-(10) and equating the coefficient of the powers of ε, we obtain the following relations:

Zeroth Order:

$$\alpha u_0''' - u_0'' - u_0' + M u_0 - G_r(1-\theta_0) - G_m(1-\phi_0) = 0 \quad (13)$$

$$\theta_0'' + \mathrm{P_r}\,\theta_0' - E_c\,\mathrm{P_r}\left(u_0'^2 + M u_0^2\right) = 0 \quad (14)$$

$$\phi_0'' + S_c\phi_0' = 0$$ (15) with boundary conditions:

$$\left.\begin{aligned} &u_0 = 0,\ \theta_0 = 0,\ \phi_0 = 0,\ at\ y = 0 \\ &u_0 \to 0,\ u_0' \to 0,\ \theta_0 \to 1,\ \phi_0 \to 1,\ as\ y \to \infty \end{aligned}\right\} \quad (16)$$

First Order:

$$\alpha u_1''' - \left(1 + \frac{i\omega\alpha}{4}\right)u_1'' - u_1' + \left(M + \frac{i\omega}{4}\right)u_1 - G_r(1-\theta_1) - G_m(1-\phi_1) - A u_0' + \alpha A u_0''' = 0 \quad (17)$$

$$\theta_1'' + \mathrm{P_r}\,\theta_1' - \frac{i\omega}{4}\mathrm{P_r}\,\theta_1 + \frac{i\omega}{4}\mathrm{P}_r + \mathrm{P}_r\,A\theta_0' - 2E_c\,\mathrm{P_r}\left(u_0'u_1' + M u_0 u_1\right) = 0 \quad (18)$$

$$\phi_1'' + S_c\phi_1' - \frac{i\omega}{4}S_c\phi_1 + \frac{i\omega}{4}S_c + S_c A\phi_0' = 0 \quad (19)$$

with boundary conditions:

$$\left.\begin{aligned} &\mathrm{u}_1 = 0,\ \theta_1 = 0,\ \phi_1 = 0, \quad \text{at} \quad \mathrm{y} = 0 \\ &\mathrm{u}_1 \to 0,\ \mathrm{u}_1' \to 0,\ \theta_1 \to 1,\ \phi_1 \to 1,\ as\ \mathrm{y} \to \infty \end{aligned}\right\} \quad (20)$$

4. Simulation and Results

4.1 Note on the numerical method

The perturbed equations for each separate power of ε are independent decoupled higher order differential equations which allows a sequential solution to the problem. However the equations of each order are coupled differential equations. The numerical method used to solve these coupled differential equations together with boundary conditions is the Runge-Kutta fourth-order method. Firstly, these equations are reduced to first order differential equations. Since the equations are of higher order and the starting values at *y=0* are not available, therefore the shooting method is used to solve the boundary value problem. The physical domain of the problem is unbounded and the computational domain is bounded and even the far field boundary conditions depend upon the physical parameters of the problem. Therefore the computational domain is chosen sufficiently large so that the numerical approximation solutions meet the boundary conditions at the infinity.

4.2 Results and discussion

The effect of different flow parameters like viscous, viscoelastic and magnetic strength on the velocity of the fluid as well as the effects of viscous dissipation and Joule effects on temperature field are analysed. The distribution of velocity for various values of the second grade parameter is described in Fig. 1. The values of the other flow parameters are kept constant and mentioned in the figure caption. The effect of the second grade parameter α is to decrease the velocity throughout the flow field which is quite obvious and demonstrate the implementation of the proposed numerical scheme. It is also clear that the velocity approaches to zero at the far away from the plate. The influence of the viscoelastic or the second grade parameter on the temperature and concentration fields is very small and therefore the results are not shown here. The effect of magnetic strength on the motion of the fluid and the temperature distribution is analysed in Fig. 2. It is clearly seen that the magnetic field acts like drag force (Lorenz force) and decelerates the motion of the fluid in the boundary layer and then finally it approaches to zero. The heat generated due to viscous dissipation and Joule heating is characterized in the right panels of Figs. 2 and 3. Due to higher value of magnetic parameter, there is a significant generation of heat (Right panel Fig. 2) near the plate and it decreases the thermal boundary layer thickness.

The influence of Eckert number is shown in Fig. 3. It is observed that the velocity and temperature fields are the increasing function of E_c. The higher Eckert number implies greater viscous dissipative heat and causes an increase in the velocity as well as temperature (Fig. 3).

The effect of Schmidt number on the velocity and concentration fields is explained in Fig. 4. Concentration reduces with an increase of Schmidt number and as results a large buoyancy force develops and it decreases the fluid velocity (left panel of Fig. 4). On the other hand, the high Schmidt number reduces temperature distribution throughout the domain and it is more visible in the presence of Joule heating.

The rate of heat transfer between plate and the fluid is studied through non-dimensional Nusselt number. The rate of heat transfer in terms of Nusselt number is given by

$$N_u = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

Nusselt numbers (N_u) at phase $\omega t = 0$ are given in Table-1. It can be clearly seen that the Nusselt number increases with an increase of magnetic parameter. This is due to the fact that the applied magnetic field induces a Lorenz force and a Joule heating and is responsible for the increase of the rate of heat transfer and the decrease of the thermal boundary layer thickness. It is further observed that an increase in the Prandtl number results a increase in the Nusselt number. However the reverse effect is observed when the Eckert number increases.

5. Conclusion

The mixed convective MHD second grade flow and heat and mass transfer past a vertical infinite plate with viscous dissipation and Joule heating is studied numerically. The results of the investigation highlights that the second grade parameter reduces the velocity field and its effect on temperature and concentration fields is negligible. It is observed in the presence of uniform magnetic field the velocity and temperature decreases with increase of magnetic parameter. The high Eckert number flow enhances the velocity and temperature fields. The numerical solution further reveals that the Schmidt number controls the velocity boundary layer thickness. The large Schmidt number decreases the concentration field faster and thereby high buoyancy develops and decelerates the fluid flow. On the other hand the large Prandtl number enhances the heat transfer at the plate.

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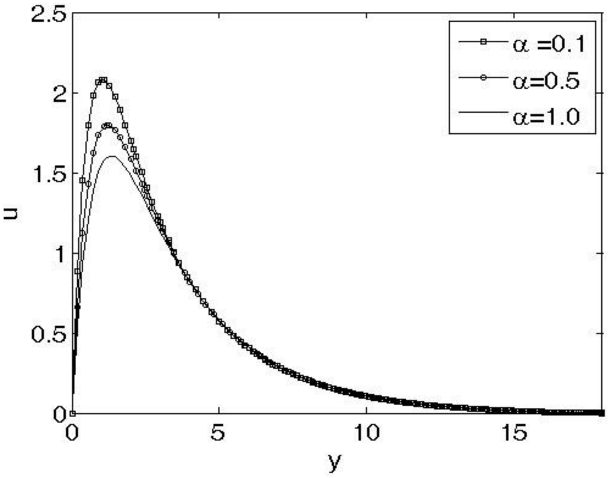

Figure 1: Velocity profile for different values of α with G_r=G_m=5, M=2, P_r=0.7, E_c=0.001, S_c=0.3, ω=10, ωt=0, A=0.5 and ε=0.01

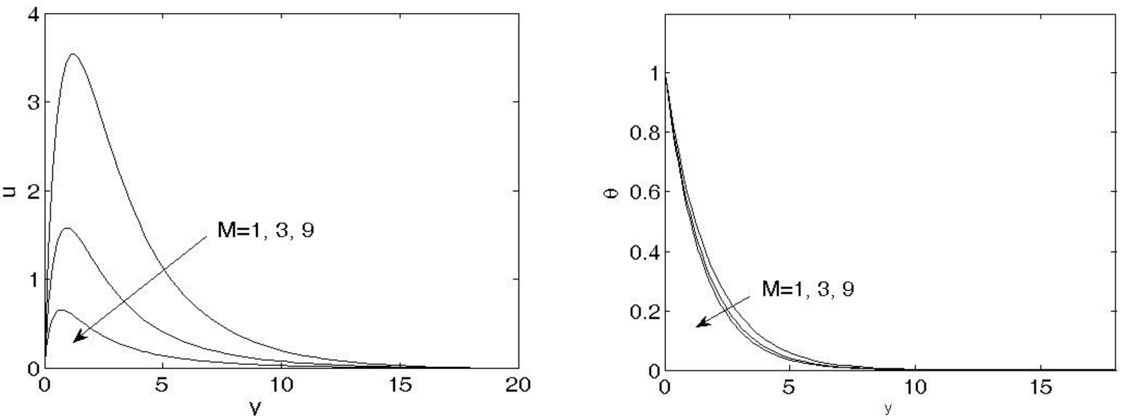

Figure 2: Effects of magnetic parameter (M) on the velocity and temperature profiles with G_r=G_m=5, P_r=0.7, E_c=0.001, S_c=0.3, ω=10, ωt=0, A=0.5 and ε=0.01. left: Velocity, right: Temperature

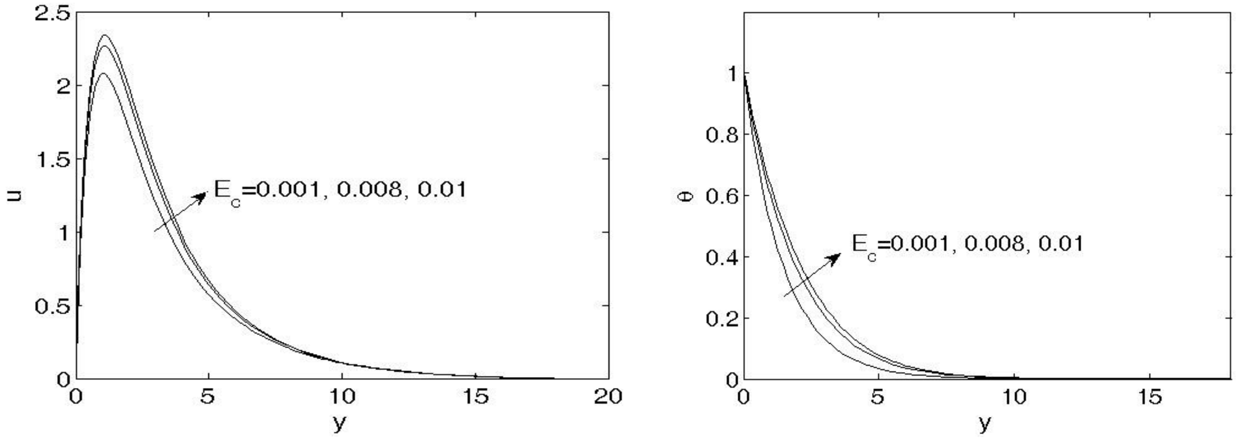

Figure 3: Effects of Eckert number (E_c) on the velocity and temperature profiles with G_r=G_m=5, P_r=0.7, M=2, S_c=0.3, ω=10, ωt=0, A=0.5 and ε=0.01. left: Velocity, right: Temperature

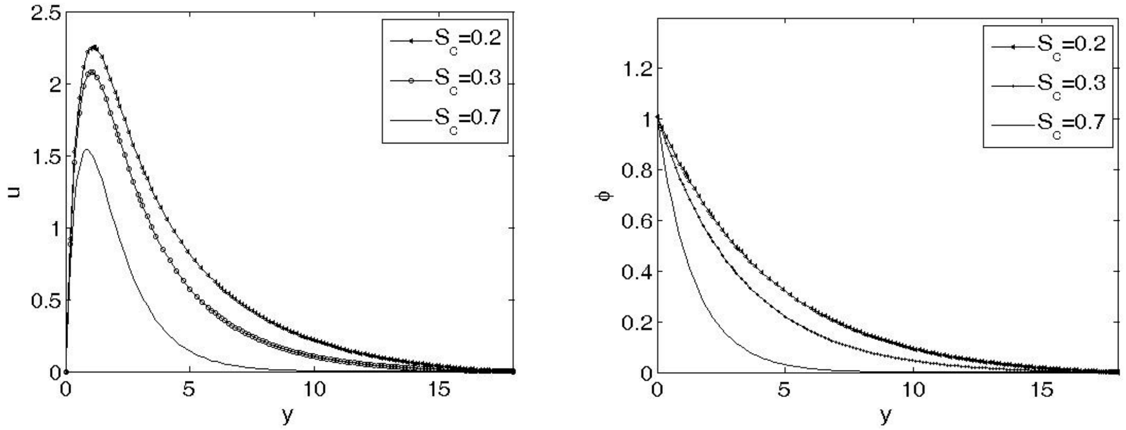

Figure 4: Effects of Schmidt number (S_c) on the velocity and concentration profiles with G_r=G_m=5, M=2, P_r=0.7, E_c=0.001, ω=10, ωt=0, A=0.5 and ε=0.01. left: Velocity, right: Concentration.

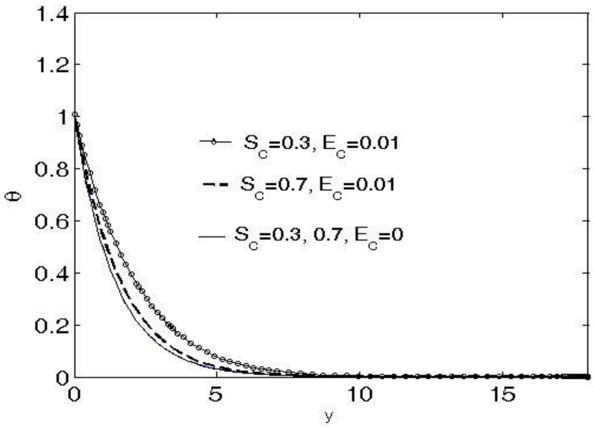

Figure 5: Effect of Schmidt number (S_c) on temperature profile with G_r=G_m=5, M=2, P_r=0.7, ω=10, ωt=0, A=0.5 and ε=0.01.

Table 1: Nusselt number data for ωt=0, ω=10, A=0.5, ε=0.01, α=0.1, G_r=G_m=5 and S_c=0.3

M	P_r	E_c	N_u
1	0.7	0.001	0.116963
2	0.7	0.001	0.118819
3	0.7	0.001	0.119624
3	1.0	0.001	0.166763
3	2.0	0.001	0.239854
3	5.0	0.001	0.367016
3	5.0	0.005	0.352350
3	5.0	0.008	0.340166
3	5.0	0.010	0.331399