Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.2, No.10, 2012
www.iiste.org

DERIVATION OF A CLASS OF HYBRID ADAMS MOULTON METHOD WITH CONTINOUS COEFFICIENTS

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Abstract

This research work is focused on the derivation of both the continuous and discrete models of the hybrid Adams Moulton method for step number k =1 and k = 2. These formulations incorporate both the off – grid interpolation and off- grid collocation schemes. The convergence analysis reveals that derived schemes are zero stable, of good order and error constants which by implication shows that the schemes are consistent.

Keywords: Hybrids Schemes, Adams Methods, Linear K–Step Method, Consistency, Zero Stable

1.0 Introduction

The derivation of hybrid Adams Moulton Schemes of both the continuous and discrete forms for the off-grid interpolation and the off – grid collocation system of polynomials is our primary focus in this research work. Sequel to this would be to ascertain the zero stability of each of the discrete forms.

Performances of these schemes on solving some non stiff initial value problems shall be affirmed in the second phase of this work which will focus on the application of these derived scheme and comparism of the discrete schemes with the single Adams-Moulton Methods and its alternative in Awe (1997) and Alagbe (1999)

In this paper we regard the Linear Multi-step Method and Trapezoidal (Adams – Moulton Method) as extrapolation and substitution methods respectively.

We also define k –Step hybrid schemes as follows:

$$\sum_{i=0}^{k}\alpha_i y_{n+i} = hk\sum_{i=0}\beta_i f_{n+i} + h\beta_r f_{n+v} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.1)$$

where $\alpha_k = \pm 1, \alpha_0$ and β_0 are not both zero and $v \notin \{0,1,\ldots k\}$ and of course $f_{n++v} = f(x_{n+v}, y_{n+v})$.

Gregg & Stetter (1964) satisfied the co-essential condition of zero-stability, the aim of which to reduce some of the difficult inherent in the LMM (i.e poor stability) was achieved.

Hybrid scheme was as a result of the desire to increase the order without increasing the step number and then without reducing the stability interval. However, the hybrid methods have not yet gained the popularity deserved due to the presence of off-grid point which requires special predicator which will not alternate the accuracy of the corrector to estimate them.

1.1 Adams Methods

This is an important class of linear multi-step method of the form:

$$y_{n+1} = y_n + h\left[1 + \frac{1}{2}\nabla + \frac{5}{12}\nabla^2 + \frac{3}{8}\nabla^3 + \frac{251}{720}\nabla^4 + \ldots\right] f_n \quad (1.2)$$

The corrected form of Adams-Moulton form is expressed as:

$$y_{n+1} = y_n + h\left[1 - \frac{1}{2}\nabla + \frac{1}{12}\nabla^2 - \frac{1}{24}\nabla^3 + \frac{19}{120}\nabla^4 - \ldots\right] f_{n+1} \quad (1.3)$$

The trapezoidal scheme is a special case of the Adams –Moulton method, in which only the first two terms in the bracket is retained. This method is of the highest order among the single –step method. It is being expressed as

$$y_{n+1} = \frac{h}{2}\left[f_{n+1} + f_n\right]$$

2.0 Derivation of Continuous and Discrete Hybrid Adams-Moulton Scheme

Our concern primarily is to derive the continuous and discrete form of both one-step and two-step Adams-Moulton scheme and consequently carried out the error and zero-stable analysis of the discrete forms. Matrix inversion technique was the tool implored to achieve the derivation of the continuous form.

2.1 Derivation of Multi-Step Collocation Method

In order to derive the continuous and discrete one and two step hybrid Adams-Moulton schemes, we employed the approach used by Sirisena (1997) where a k-step multi-step collocation method point was obtained as:

$$\bar{y}(x) = \sum_{j=0}^{k-1} \alpha_j(x) y(x_{n+j}) + h \sum_{j=0}^{m-1} \beta_j(x) f\left(\bar{x}_j, y\left(\bar{x}_j\right)\right) \quad (2.1)$$

Where $\alpha_j(x)$ and $\beta_j(x)$ are the continuous coefficients

We defined $\alpha_j(x)$ and $\beta_j(x)$ respectively as:

$$\alpha_j(x) = \sum_{i=0}^{t+m-1} \alpha_{j,i+1} x^i$$

$$h\beta_j(x) = \sum_{i=0}^{t+m-1} h\beta_{j,i+1} x^i \quad \ldots\ldots(2.2)$$

To get $\alpha_j(x)$ and $\beta_j(x)$, we arrived at matrix equation of the form DC=I………… (2.3)

where I is the identity matrix of dimension (t + m) (t + m), D and C are defined as =

$$D = \begin{bmatrix} 1 & x_n & x_n^2 & \cdots & x_n^t & \cdots & x_n^{t+m-1} \\ 1 & x_{n+1} & x_{n+1}^2 & \cdots & x_{n+1}^t & \cdots & x_{n+1}^{t+m-1} \\ \vdots & \vdots & \vdots & \cdots & \cdots & \cdots & \vdots \\ 1 & x_{n+k-1} & x_{n+k-1}^2 & \cdots & x_{n+k-1}^t & \cdots & x_{n+k-1}^{t+m-1} \\ 0 & 1 & 2\bar{x}_o & \cdots & t\bar{x}_0^{(t-1)} & \cdots & (t+m-1)\bar{x}_0^{(t+m-2)} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ 1 & 0 & 2\bar{x}_{m-1} & \cdots & t\bar{x}_{m-1}^{(t-1)} & \cdots & (t+m-1)\bar{x}_{m-1}^{(t+m-2)} \end{bmatrix} \quad \ldots\ldots(2.4)$$

Thus, matrix (2.4) is the multi-step collocation matrix of dimension (t+m) (t+m) while matrix C of the same dimension whose columns give the continuous coefficients given as:

$$C = \begin{bmatrix} \alpha_{01} & \alpha_{11} & \cdots & \alpha_{t-1,1} & h\beta_{01} & \cdots & h\beta_{m-1,1} \\ \alpha_{02} & \alpha_{12} & \cdots & \alpha_{t-1,2} & h\beta_{02} & \cdots & h\beta_{m-1,2} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ \alpha_{0j+m} & \alpha_{1j+m} & \cdots & \alpha_{t-1,j+m} & h\beta_{0j+m} & \cdots & h\beta_{m-1,m+j} \end{bmatrix} \quad \ldots\ldots(2.5)$$

We define t as the number of interpolation points and m is the number of collocation points. From (2.3), we notice that $C = D^{-1}$. If we define

$$C = (c_{ij}), \; i,j = 1,2,3 \ldots, n$$

$$D = (d_{ij}), \; i,j = 1,2,3, \ldots, n$$

$$E = (e_{i,j}), \; i,j = 1,2,3, \ldots, n$$

This implies that a suitable algorithm for getting the elements of C is the following:

$$C_{ij} = e_{ij} - \sum u_{i,k} c_{k,j}, i = j = 1,2,\ldots n, \ldots\ldots (2.6)$$

$$e_{i,j} = \frac{e_{i,j}}{l_{i,j}}, j = 1,2,\ldots n.$$

$$e^1_{ij} = \left(e_{ij} - \sum_{k=1}^{i-1} l_{ik} e_{kj} \right) \Big/ l_{ij}, i = 1,2,\ldots n., j = 1,2,\ldots n. \ldots\ldots (2.7)$$

$$l_{ij} = d_{ij} - \sum_{k=1}^{i-1} l_{ik} u_{kj,} j \le i, j = 1,2,\ldots n.; i = 1,2,\ldots, n$$

$$l_{ij} = d_{ij,} i = 1,2,\ldots, n. \ldots\ldots (2.8)$$

$$u_{ij} = \left(d_{ij} - \sum_{k=1}^{i-1} l_{ik} u_{kj} \right) \Big/ l_{ii}, i, < j, i = 1,2,..n.$$

$$u_{ij} = d_{ij} / l_{ii}; j = 1,2,\ldots n, \ldots\ldots (2.9)$$

Provided $l_{ii} \neq 0$.

2.2 Derivation of Continuous and Discrete Hybrid Adams-Moulton Scheme

Case K=1

The matrix for this case is given below as

$$D = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 \\ 1 & x_{n+u} & x_{n+u}^2 & x_{n+u}^3 \\ 0 & 1 & 2x_n & 3x_n \\ 0 & 1 & 2x_{n+1} & 3x_{n+1} \end{bmatrix}$$ and its equation (2.1) equivalence is

$$\bar{y}(x) = \alpha_0(x) y_n + \alpha_u(x) y_{n+u} + h\{\beta_0(x) f_n + \beta_1(x) f_{n+1}\}$$

By applying the set of formulae (2.6) - (2.9, we have

$$l_{i1} = d_{i1}, i = 1,2,3,4, \ldots\ldots (2.10)$$

$$l_{11} = d_{11} = 1$$

$$l_{21} = d_{21} = 1$$

$$l_{31} = d_{31} = 0$$

$$l_{41} = d_{41} = 0$$

Now using

$$u_{ij} = d_{ij} / l_{ij},\ j = 1,2,3,4,\ i = 1$$
$$j = 2$$
$$u_{12} = \frac{d_{12}}{l_{11}} = x_n$$
$$j = 3$$
$$u_{13} = \frac{d_{13}}{l_{11}} = x_n^2$$
$$j = 4$$
$$u_{14} = \frac{d_{14}}{l_{11}} = x_n^3$$

Using

$$l_{ij} = d_{ij} - \sum_{k=1}^{j-1} l_{ik} u_{kj},\ ij$$
$$i = 2,\ j = 2$$
$$l_{22} = d_{22} - \sum_{k=1} l_{ik} u_{k2}$$
$$l_{22} = d_{22} - l_{21} u_{12}$$
$$l_{22} = x_n + uh - x_n = uh$$
$$i = 3,\ j = 2$$
$$l_{32} = d_{32} - l_{31} u_{12}$$
$$l_{32} = 1 - 0.x_n = 1,\ i = 4,\ j = 2$$
$$l_{42} = d_{42} - l_{41} u_{12} = 1 - 0,\ x_n = 1$$

Using

$$u_{ij} = \left(d_{ij} - \sum_{k=1}^{i-1} l_{ik} u_{kj} \right) \Big/ l_{ii},\ i \le j,\ i = 1,2,$$

Mathematical Theory and Modeling
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$$i = 1, j = 4$$
$$u_{23} = \frac{(d_{23} - l_{21}u_{13})}{l_{22}}$$
$$= \frac{\left((x_n + uh)^2 - x_n.1\right)}{uh}$$
$$= 2x_n + uh$$
$$i = 2, j = 4$$
$$u_{24} = \frac{(d_{24} - l_{21}u_{14})}{l_{22}}$$
$$= \frac{((x_n + uh)^3 - x^3.1)}{uh}$$
$$= 3x_n^2 + 3x_n uh + u^2h^2$$

Again using

$$l_{ij} = d_{ij} - \sum_{k=1}^{i-1} l_{ik}u_{kj,} i \geq j$$
$$i = 3, j = 3$$
$$l_{33} = d_{33} - (l_{31}u_{13} + l_{32}u_{23})$$
$$= 2x_n - 2x_n - uh = -uh$$
$$i = 4, j = 3$$
$$l_{43} = d_{43} - (l_{41}u_{13} + l_{32}u_{23})$$
$$= 2x_n + 2h - 2x_n - uh$$
$$= (2 - u)h$$

Using

$$u_{ij} = \left(d_{ij} - \sum_{k=1}^{i-1} l_{ik}u_{kj}\right) \Big/ l_{ii}, i \leq j, i = 3, j = 4,$$
$$u_{34} = \frac{(d_{34} - (l_{31}u_{14} + l_{32}u_{24}))}{l_{33}}$$
$$= \frac{(3x_n^2 - 3x_n^2 - 3x_n uh - u^2h^2)}{-uh}$$
$$= 3x_n + uh$$

Using

$$l_{ij} = d_{ij} - \sum_{k=1}^{i-1} l_{ik} u_{kj,}\ j \le i, i = 4, j = 1$$
$$l_{44} = d_{44} - \left(l_{41}u_{14} + l_{42}u_{24} + l_{43}u_{34}\right)$$
$$= 3(x_n + h)^2 - 3x_n^2 - 3x_n uh - u^2h^2 - (2h - uh)(3x_n + uh)$$
$$= 3h^2 - 2uh^2 = (3 - 2u)h^2$$

Using (2.7) with

$$i = j = 1$$
$$e_{ij}^1 = e_{11}^1 = \frac{e_{11}}{l_{11}} = 1$$
$$j = 2$$
$$e_{12}^1 = \frac{e_{12}}{l_{11}} = 0$$
$$j = 3$$
$$e_{13}^1 = \frac{e_{13}}{l_{11}} = 0$$
$$j = 4$$
$$e_{14}^1 = \frac{e_{14}}{l_{11}} = 0$$

$$e_{ij}^{1} = \left(e_{ij} - \sum_{k=1}^{i-1} l_{ik} e_{kj} \right) \Big/ l_{ij}, i = 1,2,3,4;\ j = 1,2,3,4.$$

$$i = 2,\ j = 1$$

$$e_{21}^{1} = \frac{(e_{21} - l_{21} e_{11})}{l_{22}}$$

$$= \frac{(0-1)}{uh} = \frac{-1}{uh}$$

$$e_{31}^{1} = \frac{\{e_{31} - (l_{31} e_{11}^{1} + l_{32} e_{21}^{1})\}}{l_{33}}$$

$$= \frac{1}{uh}\left(\frac{-1}{uh}\right) = \frac{-1}{u^2 h^2}$$

$$i = 4,\ j = 1$$

$$e_{41}^{1} = \frac{\{e_{41} - (l_{41} e_{11}^{1} + l_{42} e_{21}^{1} + l_{43} e_{31}^{1})\}}{l_{44}}$$

$$= \frac{\left\{\frac{1}{uh} + (2h + uh)\left(\frac{1}{u^2 h^2}\right)\right\}}{(3-2u)h^2}$$

$$= \frac{2}{(3-2u)u^2 h^3}$$

i = 2 = j

$$e_{22}^{1} = \frac{(e_{22} - l_{21} e_{12}^{1})}{l_{22}}$$

$$= \frac{1}{uh}$$

$$i = 3,\ j = 2$$

$$e_{32}^{1} = \frac{\{e_{32} - (l_{31} e_{12}^{1} + l_{32} e_{22}^{1})\}}{l_{33}}$$

$$=\left\{\frac{-1}{uh}\times\frac{-1}{uh}\right\}=\frac{1}{u^2h^2}$$

$i=4,\ j=2$

$$e_{42}^1=\frac{\left\{e_{42}-\left(l_{41}e_{12}^1+l_{42}e_{22}^1+l_{43}e_{32}^1\right)\right\}}{l_{44}}$$

$$=\frac{\left\{\frac{-1}{uh}-\frac{(2-u)h}{u^2h^2}\right\}}{(3-2u)h^2}$$

$$=\frac{-2}{(3-2u)u^2h^2}$$

$i=2,\ j=3$

$$e_{23}^1=\frac{\left\{e_{23}-\left(l_{21}e_{13}^1\right)\right\}}{l_{22}}$$

$$=\frac{(0-1.0)}{l_{22}}=0$$

$i=2,\ j=4$

$$e_{24}^1=\frac{\left(e_{24}-l_{21}e_{14}^1\right)}{l_{22}}$$

$$=\frac{(0-1.0)}{uh}=0$$

$i=j=3$

$$e_{33}^1=\frac{\left\{e_{33}-\left(l_{31}e_{13}^1+l_{32}e_{23}^1\right)\right\}}{l_{33}}$$

$$=\frac{-1}{uh}$$

$i=4,\ j=3$

$$e_{43}^1=\frac{\left\{e_{43}-\left(l_{41}e_{13}^1+l_{42}e_{23}^1+l_{43}e_{33}^1\right)\right\}}{l_{33}}$$

$$=\frac{\left\{-\frac{(2-u)}{uh}\right\}}{(3-2u)h^2}$$

$$=\frac{(2-u)}{(3-2u)uh^2}$$

$i=3,\ j=4$

$$e_{34}^{1} = \frac{\{e_{34} - (l_{31}e_{14}^{1} + l_{32}e_{24}^{1})\}}{l_{33}}$$

$$= \frac{0}{-uh} = 0$$

$$i = j = 4$$

$$e_{44}^{1} = \frac{\{e_{44} - (l_{41}e_{14}^{1} + l_{42}e_{24}^{1} + l_{43}e_{34}^{1})\}}{l_{44}}$$

$$= \frac{1}{(3-2u)h^2}$$

C values are now computed thus;

$$c_{41} = e_{41}^{1} = \frac{2}{u^2h^2(3-2u)}$$

$$i = 4,\ j = 2$$

$$c_{42} = e_{42}^{1} = \frac{-2}{u^2h^2(3-2u)}$$

$$i = 4,\ j = 3$$

$$c_{43} = e_{43}^{1} = \frac{(2-u)}{uh^2(3-2u)}$$

$$i = j = 4$$

$$c_{44} = e_{44}^{1} = \frac{1}{(3-2u)h^2}$$

Using

$$c_{ij} = e_{ij}^{1} - \sum_{k=4} u_{ik}c_{kj}$$

$$i = 3,\ j = 1$$

$$c_{31} = e_{31}^{1} - (u_{34}c_{41})$$

$$= \frac{-1}{u^2h^2} - 3x_n - uh\left(\frac{2}{u^2h^2(3-2u)}\right)$$

$$= \frac{(-6x_n - 3h)}{u^2h^3(3-2u)}$$

$$i = 3,\ j = 2$$

$$c_{32} = e_{32}^{1} - (u_{34}c_{42})$$

$$= \frac{1}{u^2h^2} + \frac{2(3x_n + uh)}{u^2h^3(3-2u)}$$
$$= \frac{3(h + 2x_n)}{u^2h^3(3-2u)}$$
$$i = j = 3$$
$$c_{33} = e_{33}^1 - (u_{34}c_{42})$$
$$= \frac{-1}{uh} - (3x_n + uh)\left\{\frac{(2-u)}{uh^2(3-2u)}\right\}$$
$$= \frac{\{(u^2-3)h + 3(u-2)x_n\}}{uh^2(3-2u)}$$
$$i = 3,\ j = 4$$
$$c_{34} = e_{34}^1 - (u_{34}c_{44})$$
$$= \frac{-(3x_n + uh)}{(3-2u)h^2}$$

Now with

$$c_{ij} = e_{ij}^1 - \sum_{k=3}^{k+1} u_{ik}c_{kj}$$
$$i = 2,\ j = 1$$
$$c_{21} = e_{21}^1 - (u_{23}c_{21} + u_{24}c_{41})$$
$$= -\frac{1}{uh} - \frac{\{2x_n + uh(-6x_n - 3h)\}}{u^2h^3(3-2u)} - \frac{\{3x_n^2 + 3x_nuh + 2u^2h^2\}}{u^2h^3(3-2u)}$$
$$= \frac{6x_nx_{n+1}}{u^2h^3(3-2u)}$$
$$i = 2 = j$$

$$c_{22} = e_{21}^1 - u_{23}c_{32} - u_{24}c_{42}$$
$$= \frac{1}{uh} - 2x_n - uh\left\{\frac{3(h+2x_n)}{(3-2x_n)u^2h^3}\right\} + \frac{2}{u^2h^3(3-2u)} + 3x_nx_{n+u} + u^2h^2$$
$$= -\frac{6x_nx_{n+1}}{u^2h^2(3-2u)}$$
$$i = 2,\ j = 3$$
$$c_{23} = e_{23}^1 - u_{23}c_{33} - c_{43}u_{24}$$

$$= -2x_n - uh\left\{\frac{\left(u^2h + 3ux_n - 6x_n - 3h\right)}{uh^2(3-2u)}\right\} - 3x_n^2 - 3x_n uh - u^2h^2\left\{\frac{(2-u)}{uh^2(3-2u)}\right\}$$

$$= \frac{\left\{6x_n x_{n+1} - 3u\left(x_n^2 - h^2\right) - u^2h\left(x_n + 2h\right)\right\}}{uh^2(3-2u)}$$

$i = 2,\ j = 4$

$$c_{24} = e_{24}^1 - u_{23}c_{34} - u_{24}c_{44}$$

$$= -\left(2x_n + uh\right) - \frac{\left(3x_n + uh\right)}{(3-2u)h^2} - 3x_n uh - u^2h^2\left(\frac{1}{(3-2u)h^2}\right)$$

$$= \frac{x_n\left(3x_n + 2uh\right)}{h^2(3-2u)}$$

Now consider

$$C_{ij} = e_{ij}^1 - \sum_{k=2}^{k+2} u_{ik}c_{kj}$$

$i = 1 = j$

$$c_{11} = e_{11}^1 - \left\{u_{12}c_{21} + u_{13}c_{31} + u_{14}c_{41}\right\}$$

$$= 1 - x_n\left\{\frac{6x_n x_{n+1}}{u^2h^3(3-2u)}\right\} - x_n^2\left\{\frac{-3\left(2x_n - h\right)}{u^2h^3(3-2u)}\right\} - x_n^3\left\{\frac{2}{u^2h^3(3-2u)}\right\}$$

$$= \frac{\left\{3u^2h^3 - 2u^3h^3 - 3x_n^2h - 2x_n^3\right\}}{u^2h^3(3-2u)}$$

$i = 1,\ j = 2$

$$c_{12} = e_{12}^1 - \left\{u_{12}c_{22} + u_{13}c_{34} + u_{14}c_{42}\right\}$$

$$= -x_n\left\{\frac{-6x_n x_{n+1}}{u^2h^3(3-2u)}\right\} - \frac{x_n^2\left(6x_{n+\frac{1}{2}}\right)}{u^2h^3(3-2u)} - x_n^3\left\{\frac{-2}{u^2h^3(3-2u)}\right\}$$

$$= \frac{\left(3x_n^2h + 2x_n^3\right)}{u^2h^3(3-2u)}$$

$i = 1,\ j = 3$

$$c_{13} = e_{13}^1 - \left\{u_{12}c_{23} + u_{13}c_{33} + u_{14}c_{43}\right\}$$

$$= -x_n\frac{\left\{6x_n^2 - 3ux_n^2 + 6x_nh + u^2x_nh - 2u^2h^2 + 3uh^2\right\}}{uh^2(3-2u)} - \frac{x_n\left\{u^2h + 3ux_n - 6x_n - 3h\right\}}{uh^2(3-2u)} - x_n^3\left\{\frac{(2-u)}{uh^2(3-2u)}\right\}$$

$$= \frac{\{-3x_n^2 h + u^2 x_n^2 h + u x_n^3 + 2u^2 x_n h^2 - 3u x_n h^2 - 2x_n^3\}}{uh^2(3-2u)}$$

$i = 1, j = 4$

$$c_{14} = e_{14}^1 - \{u_{12}c_{24} + u_{13}c_{34} + u_{14}c_{44}\}$$
$$= \frac{-x_n\{(3x_n^2 + 2u x_n h)\}}{h^2(3-2u)} - \frac{x_n^2(-3x_n - uh)}{h^2(3-2u)} - \frac{x_n^3}{h^2(3-2u)}$$
$$= \frac{-x_n^2 x_{n+u}}{h^2(3-2u)}$$

The continuous scheme coefficients column by column for c values are now computed. Substituting these values into the general form (2.10) yield the desired continuous schemes.

$$\alpha_0(x) = c_{11} + c_{21}x + c_{31}x^2 + c_{41}x^3$$
$$= \frac{(3u^2h^2 - 2u^3h^3 - 3x_n^2 h - 2x_n^3)}{u^2h^3(3-2u)} + \frac{(6x_n^2 + 6x_n h)x}{u^2h^3(3-2u)} + \frac{(-6x_n - 3h)x^2}{u^2h^3(3-2u)} + \frac{2x^3}{u^2h^3(3-2u)}$$
$$= \frac{\{2(x-x_n)^3 - 3h(x-x_n)^2 + u^2h^3(3-2u)\}}{u^2h^3(3-2u)}$$

$$\alpha_1(x) = c_{12} + c_{22}x + c_{23}x^2 + c_{24}x^3$$
$$= \frac{(3x_n^2 h + 2x_n^3)}{u^2h^3} - \frac{(6x_n h + 6x_n^2)x}{u^2h^3(3-2u)} + \frac{(6x_n + 3h)x^2}{u^2h^3(3-2u)} - \frac{2x^2}{u^2h^3(3-2u)}$$
$$= -2(x-x_n)^3 + \frac{3h(x-x_n)^2}{u^2h^3(3-2u)}$$

$$\beta_0(x) = c_{13} + c_{23}x + c_{33}x^2 + c_{34}x^3$$
$$= \frac{\{-3x_n^2 h - 2u^2x_n^2 h + 3u^2 x_n^2 h^2 + 2u^2 x_n^2 h - 3u x_n h - 2x_n^3\}}{uh(3-2u)}$$
$$+ \frac{\{x_n^3 + 6x_n^2 - 3u x_n^2 + 6x_n h - 3u^2 x_n h + u^2 x_n h - 2u^2 h^2 - 3uh\}x}{uh^2(3-2u)}$$
$$+ \frac{\{u^2h^2 + 3u x_n - 6x_n - 3h\}}{uh^2(3-2u)} + \frac{(2-u)x^3}{uh^2(3-2u)}$$
$$= \frac{\{(2-u)(x-x_n)^3 - (3h - u^2h)(x-x_n)^2 - (2u^2 - 3u)(x-x_n)h\}}{uh^2(3-2u)}$$

$$\beta_1(x) = c_{14} + c_{24}x + c_{34}x^2 + c_{44}x^3$$
$$= \frac{\left(-ux_n^2h - x_n^3\right)}{h^2(3-2u)} + \frac{\left(3x_n^2 + 2ux_nh\right)x}{h^2(3-2u)} + \frac{\left(3x_n + uh\right)x^2}{h^2(3-2u)} + \frac{x^3}{h^2(3-2u)}$$
$$= (x-x_n)^3 - \frac{uh(x-x_n)^2}{h^2(3-2u)}$$

Hence forth,

$$\bar{y}(x) = \left\{\frac{2(x-x_n)^3 - 3h(x-x_n)^2 + u^2h^3(3-2u)}{u^2h^3(3-2u)}\right\}y_n + \left\{\frac{-2(x-x_n)^3 + 3h(x-x_n)^2}{u^2h^3(3-2u)}\right\}y_{n+u}$$
$$+ \left\{\frac{(2-u)(x-x_n)^3 - (3-u^2)h(x-x_n)^2 - (2u^2-3u)h^2(x-x_n)}{uh^2(3-2u)}\right\}f_n + \left\{\frac{(x-x_n)^3 - uh(x-x_n)^2}{h^2(3-2u)}\right\}f_{n+1} \ldots (2.11)$$

Where (2.11) is the continuous form of one step Adams-Moulton scheme for k=1. If (2.11) is evaluated at x_{n+1} and a substitution $u = \frac{1}{2}$ is made, the result is

$$\bar{y}(x_{n+1}) = -y_n + 2y_{n+\frac{1}{2}} - \frac{h}{4}f_n + \frac{h}{4}f_{n+1}, \textit{hence}$$

$$y_{n+1} + y_n - 2y_{n+\frac{1}{2}} = \frac{h}{4}\left[f_{n+1} - f_n\right] \ldots\ldots (2.12)$$

If on the other hand, equation (2.11) is differentiated with respect to x and then evaluated at $x = x_{n+\frac{1}{2}}$, the result obtained is

$$y^1\left(x_{n+\frac{1}{2}}\right) = \frac{-3}{h}y_n + \frac{3}{h}y_{n+\frac{1}{2}} - \frac{5}{8}f_n + \frac{1}{8}f_{n+1},$$
$$y_n - y_{n+\frac{1}{2}} = \frac{h}{24}\left[f_{n+1} - 5f_n - 8f_{n+\frac{1}{2}}\right] \ldots\ldots (2.12b)$$

The schemes (2.12a) and (2.12b) can be referred to as (2.12) and are indeed of interpolation polynomial of case k =1

We consider another system of matrix of the same case k=1 where the schemes shall be derived at the interpolation points

$$D = \begin{pmatrix} 1 & x_n & x_n^2 & x_n^3 \\ 0 & 1 & 2x_n & 3x_n^2 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 \\ 0 & 1 & 2x_{n+u} & 3x_{n+u}^2 \end{pmatrix}$$

The general forms of this system of matrix is given as :

$$\bar{y}(x) = \alpha_0(x)y_n + h\{\beta_0(x)f_n + \beta_1(x)f_{n+1} + \beta_n(x)f_{n+u}\}\ldots\ldots(2.13)$$

As seen in the first case using the same set of formulae (2.6) – (2.9), we have the values of c given as follows:

$$c_{41} = 0;$$
$$c_{42} = \frac{(u-1)}{(3uh^2(u-1))}$$
$$c_{43} = \frac{-u}{3uh^2(u-1)}$$
$$c_{44} = \frac{-1}{3uh^2(u-1)}$$
$$c_{31} = 0$$
$$c_{32} = \frac{-(2x_n + uh + h)}{2uh^2}$$
$$c_{33} = \frac{(uh + 2x_n)}{2h^2(u-1)}$$
$$c_{34} = \frac{-2x_{n+1/2}}{2uh^2(u-1)}$$
$$c_{21} = 0$$
$$c_{22} = \frac{\{uh^2 + x_n^2 + ux_nh + x_nh\}}{uh^2}$$
$$c_{23} = \frac{-(ux_nh + x_n^2)}{h^2(u-1)}$$
$$c_{24} = \frac{x_nx_{n+1}}{uh^2(u-1)}$$
$$c_{11} = 1$$

$$c_{12} = \frac{-\left\{6ux_n h^2 + 3ux_n^2 h + 3x_n^2 h + 2x_n^3\right\}}{6uh^2}$$

$$c_{13} = \frac{-\left(3x_n^2 h + 2x_n^3\right)}{6uh^2(x-1)}$$

Following the same procedure as in the first case, the coefficients for the continuous schemes, $\alpha_j(x)\, and \beta_j(x)$ are obtained and by similar substitution into equation (2.13), the desired continuous scheme is obtained as follows;

$$\bar{y}(x) = [1]y_n + \left\{\frac{2(u-1)(x-x_n)^3 - 3h(u^2-1)(x-x_n)^2 + 6uh^2(u-1)(x-x_n)}{6uh^2(u-1)}\right\} f_n$$

$$+ \left\{\frac{-2u(x-x_n)^3 + 3u^2 h(x-x_n)^2}{6uh^2(u-1)}\right\} f_{n+1} + \left\{\frac{2(x-x_n)^3 - 3h(x-x_n)^2}{6uh^2(u-1)}\right\} f_{n+u} \ldots\ldots\ldots\ldots(2.14)$$

Putting u = ½ and proceeding to get our discrete forms as did earlier by evaluating (2.14) at these two different points $x = x_{n+1}$ and $x_n = x_{n+1}$, we have:

a. at $x = x_n = x_{n+1}$, the result is

$$y_{n+1} = y_n + \frac{h}{6} f_n + \frac{h}{6} f_{n+1} + \frac{4}{6} h f_{n+\frac{1}{2}}$$

$$y_{n+1} - y_n = \frac{h}{6}\left[f_n + f_{n+1} + 4 f_{n+\frac{1}{2}}\right] \ldots\ldots\ldots\ldots(2.15a)$$

b. at $x = x + x_{n+\frac{1}{2}}$, the result is

$$y_{n+\frac{1}{2}} - y_n = \frac{h}{24}\left[5 f_n - f_{n+1} + 8 f_{n+\frac{1}{2}}\right] \ldots\ldots\ldots\ldots(2.15b)$$

The collocation schemes of D yields just the same equation in (2.15b) . These are the hybrid collocation schemes of the case k=1

Equation (2.15a) and (2.15b) can be referred to as (2.15)

We also considered two different systems of matrices D_3 and D_4 on the derivation of both hybrid interpolation and the collocation where k = 2

$$D_3 = \begin{bmatrix} 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 \\ 0 & 1 & 2x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 \\ 0 & 1 & 2x_{n+u} & 3x_{n+u}^2 & 4x_{n+u}^3 \end{bmatrix}$$

The general forms of D_3 is given as

$$\bar{y}(x) = \alpha_1(x)y_{n+1} + h[\beta_0(x)f_n + \beta_1(x)f_{n+1} + \beta_2(x)f_{n+2} + \beta_n(x)f_{n+u}]$$

Appling the set of formulae (2.6)-(2.9) on this system gives the c-entries as follows

$$c_{51} = c_{41} = c_{31} = 0, c_{11} = 1$$

$$c_{52} = \frac{-1}{8uh^3}$$

$$c_{53} = \frac{1}{\{4uh^3(u-2)(u-1)\}}$$

$$c_{42} = \frac{\{4uh + 12x_n + 12h\}}{24uh^3}$$

$$c_{43} = \frac{-\{12x_n + 4(2+u)h\}}{12h^3(u-1)}$$

$$c_{44} = \frac{(3x_n + uh + h)}{6h^3(u-2)}$$

$$c_{45} = \frac{-x_{n+1}}{uh^3(u-2)(u-1)}$$

$$c_{32} = \frac{-\{2uh^2 + 2uh^2 + 6x_n h + 2h^2 + 2ux_n h\}}{4uh^3}$$

$$c_{33} = \frac{\{2uh^2 + 2ux_n h + 3x_n^2 + 4x_n h\}}{2h^3(u-1)}$$

$$c_{34} = \frac{-\{3x_n^2 + 2x_n h + 3x_n^2 + 4x_n h\}}{4h^3(u-2)}$$

$$c_{35} = \frac{(3x_n^2 + 6x_n h + 2h^2)}{2uh^3(u-2)(u-1)}$$

$$c_{22} = \frac{(2uh^3 + x_n^3 + 3x_n^2 h + 3ux_n h^2 + ux_n^2 h + 2x)}{2uh^3}$$

$$c_{23} = \frac{-\left\{2ux_n h^2 + ux_n h + 2x_n^2 h + x_n^3\right\}}{h^3(u-1)}$$

$$c_{25} = \frac{-\left(3x_n^2 h + 2x_n h^2 + x_n^3\right)}{uh^3(u-1)}$$

$$c_{12} = \frac{-\left\{24ux_n h^3 + 18ux_n^2 h^2 + 4ux_n^3 h + 3x_n^4 + 12x_n^3 h^2 + 12x_n^2 h^2 + 10uxh^4 + 3h^4\right\}}{24uh^3}$$

$$c_{13} = \frac{-\left\{3x_n + 5h^4 - 8uxh^4 + 12ux_n^2 h^2 + 4ux_n^3 h + 8x_n^3 h\right\}}{12(u-1)h^3}$$

$$c_{14} = \frac{-\left\{4x_n^3 h + 4ux_n^3 h + 6ux_n^2 h^2 + 3x_n^4 + 2uh^4 + h^4\right\}}{24h^3(u-2)}$$

$$c_{15} = \frac{-\left\{4x_n^3 h + 4x_n^2 h^2 + x_n^4 - h^4\right\}}{4uh^3(u-2)(u-1)}$$

As in the previous cases, continuous schemes coefficients $\alpha_i(x)'s$ and the $\beta_i(x)'s$ are obtained and substituted into the general form (2.16) to obtain the desired continuous schemes as follows;

$$\bar{y}(x) = [1]y_{n+1} + \frac{\left\{-3(x-x_n)^4 + (4u-12)(x-x_n)^3 h - (18u+12)(x-x_n)^2 h^2 + 24uh^3(x-x_n) + (3-10u)h^4\right\}f_n}{24uh^3}$$
$$+ \frac{\left\{3(x-x_n)^4 - 4(2-u)h(x-x_n)^3 + 12uh^2(x-x_n)^2 + (5-8u)h^4\right\}f_{n+1}}{12h^3(u-1)}$$
$$+ \frac{\left\{-3(x-x_n)^4 + 4h(u+1)(x-x_n)^3 - 6uh^2(x-x_n)^2 + (2u-1)h^4\right\}f_{n+2}}{24h^3(u-2)}$$
$$+ \frac{\left\{(x-x_n)^4 - 4(x-x_n)^3 h + 4(x-x_n)^2 h^2 - h^4\right\}f_{n+u}}{4uh^3(u-1)(u-2)} \quad \ldots\ldots (2.17)$$

Evaluating (2.17) at $x = x_{n+2}$ $u = 3/2$, the resulting scheme is given as:

$$y_{n+2} - y_{n+1} = \frac{h}{6}\left\{f_{n+1} + f_{n+2} + 4f_{n+3/2}\right\} \quad \ldots\ldots (2.18a)$$

If (2.17) is evaluated at points $x = x_{n+u}$ and $u = 3/2$, we have

$$y_{n+3/2} - y_{n+1} = \frac{h}{192}\left\{-f_n + 46f_{n+1} - 5f_{n+2} + 56f_{n+3/2}\right\} \quad \ldots\ldots (2.18b)$$

However, if the point $u = \frac{3}{2}$ is maintained and (2.17) is evaluated at $x = x_n$, the result is another discrete scheme of the form:

$$y_{n+1} - y_n = \frac{h}{6}\left\{2f_n + 7f_{n+1} + f_{n+2} - 4f_{n+3/2}\right\} \quad (2.18c)$$

Equations (2.18a), (2.18b) and (2.18c) are referred to as (2.18)

Taking on another system of matrix D_4 of an interpolation scheme, where k = 2:

$$D_4 = \begin{bmatrix} 1 & x_{n+1} & x_{n+!}^2 & x_{n+1}^3 & x_{n+1}^4 \\ 1 & x_{n+u} & x_{n+u}^2 & x_{n+u}^3 & x_{n+u}^4 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 \\ 0 & 1 & 2x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 \end{bmatrix}$$

the general form of the matrix is given as:

$$\bar{y}(x) = \alpha_1(x)y_{n+1} + \alpha_u(x)y_{n+u} + h\{\beta_0(x)f_n + \beta_n(x)f_{n+1} + \beta_2(x)f_{n+2}\} \quad (2.19)$$

As in the previous cases, matrix C is determined with the columns simplification yielding the continuous schemes:

$$\begin{aligned} y(x) &= \frac{\left\{-(x-x_{n+1})^4 + 2(x-x_{n+1})^2h^2 + (u-1)^2(u^2-2u-1)h^4\right\}y_{n+u}}{(u-1)^2(u^2-2u-1)h^4} \\ &+ \frac{\left\{(x-x_{n+1})^4 - 2(x-x_{n+1})^2h^2\right\}y_{n+1}}{(u-1)^2(u^2-2u-1)h^4} + \left\{-(2u-5)(x-x_{n+1})^4 + 2(u^2-2u-1)h^3\right\}f_n \\ &+ \frac{\left\{(u^2-2u-2)(x-x_{n+1})^4 - (u-1)(u^2-2u-1)(x-x_{n+1})3h - 2(u^2-2u-2)h^3\right\}f_{n+1}}{3(u-1)(u^2-2u-1)h^3} \\ &+ \frac{\left\{-(2u+1)(x-x_{n+1})^4 + 2(u^2-2u-1)(x-x_{n+1})^3h + (u-1)(3u+1)(x-x_{n+1})^2h^2\right\}f_{n+2}}{12(u^2-2u-1)h^3} \quad (2.20) \end{aligned}$$

Desired hybrid point of elevation remains $U=\frac{3}{2}$ and so (2.20) is evaluated to yield

$$7y_{n+2}+9y_{n+1}-16y_{n+3/2}=\frac{h}{12}\left[f_n-32f_{n+1}+19f_{n+2}\right]\ldots\ldots(2.21a)$$

at the point $x=x_{n+2}$

If consideration at the point $x=x_n$ is made, we have:

$$9y_{n+1}+7y_n-16y_{n+3/2}=\frac{h}{4}\left[-9f_n-48f_{n+1}-3f_{n+3}\right]\ldots\ldots(2.21b)$$

Finding the differential expression with respect to x for (2.20) and evaluating the differential coefficient at $x=x_{n+1}$, the result becomes:

$$y_{n+1}+y_{n+3/2}=\frac{h}{192}\left[f_n-46f_{n+1}+5f_{n+2}-56f_{n+3/2}\right]\ldots\ldots(2.21c)$$

As usual, equation (2.21a),(2.21b) and (2.21c) are referred to as (2.21)

3.0 Convergence Analysis

This section shows the validity and consistency of the derived scheme in section two . The tools for the assignment would be the familiar investigation of the zero stability by finding the order and error constant of the each of the schemes.

3.**1 Definitions**

(a) The scheme (1.1) is said to be zero stable if no root of the polynomial $p(\xi)=\sum_{i=0}^{k}d_i\xi^i$ has modulo greater than one and every root with modulo one must be distinct or simple.

(b) The order p and error constant c_{p+1} for (1.1) could be defined thus:
If $c_0=c_1=\ldots c_p=0, c_{p+1}=0$, then the principal local term error at x_{n+k} is $c_{p+1}h^{p+1}y^{p+1}(x_n)$.

$$c_q = \frac{1}{q!}\left[(-t)^q \alpha_0 + (1-t)^q \alpha_1 + (2-t)^q \alpha_2 + \ldots + (k-t)^q\right]$$
$$-\frac{1}{(q-1)!}\left[(-t)^{q-1}\beta_0 + (1-t)^{q-1}\beta_1 + (2-t)^{q-1}\beta_2 + \ldots + (k-t)^{q-1}\beta_k + (v-t)^{q-1}\beta_v\right] \ldots\ldots\ldots\ldots (3.1)$$

(c) A numerical method (1.1) is said to be consistent if $p \geq 1$, where p is the order of the method.

3.1 Example

We take on the derived schemes in section 2 above, showing that each of the schemes is in conformity to the definition above or otherwise.

Example 3.1.1

One of the derived schemes for step number k=1 is given as:

$$y_{n+1} + y_n - 2y_{n+\frac{1}{2}} = \frac{h}{4}\left[f_{n+1} - f_n\right] \ldots\ldots\ldots\ldots (2.12a)$$

From equation (2.12a)

$$\alpha_0 = 1, \alpha_1 = 1, \alpha_{\frac{1}{2}} = -2, \beta_0 = \frac{-1}{4}, \beta_1 = \frac{1}{4}$$

$$c_0 = \alpha_0 + \alpha_1 + \alpha_{1/2} = 1 + 1 - 2 = 0$$

$$c_1 = \left(\alpha_1 + \frac{1}{2}\alpha_{1/2}\right) - (\beta_0 + \beta_1)$$

$$= \left(1 - 2\left(\frac{1}{2}\right)\right) - \frac{1}{4}(1-1) = 0$$

$$c_2 = \frac{1}{2!}\left(\alpha_1 + \left(\frac{1}{2}\right)^2 \alpha_{1/2}\right) - \beta_1$$

$$= \frac{1}{2}\left(1 + \frac{1}{2}(-2)\right) - \frac{1}{4} = 0$$

$$c_3 = \frac{1}{3!}\left(\alpha_1 + \left(\frac{1}{2}\right)^3 \alpha_{1/2}\right) - \frac{1}{2!}\beta_1$$

$$= \frac{1}{6}\left(1 + \frac{1}{8}(-2)\right) - \frac{1}{8} = 0$$

$$c_4 = \frac{1}{4!}\left(1 + \left(\frac{1}{2}\right)^4 \alpha_{1/2}\right) - \frac{1}{3!}\beta_1$$

$$= \frac{1}{24}\left(1 + \frac{1}{6}(-2)\right) - \frac{1}{6}.\frac{1}{4}$$

$$= -\frac{1}{192}$$

Therefore the order p = 3 and

$$p(\xi) = \alpha_1 \{+\alpha_0 - \alpha_{1/2}\}^{1/2}$$

$$= \{+1 - 2\}^{1/2} = 0$$

$$\Rightarrow (\xi^{1/2} - 1) = 0$$

$$i.e |\xi| = 1$$

Hence, the schemes is zero stable.

Example 3.1.2

Another scheme of k = 1 was given as:

$$y_n - y_{n+1/2} = \frac{h}{24}\left[f_{n+1} - 5f_n - 8f_{n+1/2}\right]$$

Here, $\alpha_0 = 1, \alpha_{1/2} = -1, \beta_0 = 5/24, \beta_1 = 1/24, and, \beta_{1/2} = -1/3$ and so

$$c_0 = \alpha + \alpha_{\frac{1}{2}} = 1 - 1 = 0;$$

$$c_1 = \frac{1}{2}\alpha_{\frac{1}{2}} - \left(\beta_0 + \beta_1 + \beta_{\frac{1}{2}}\right) = -\frac{1}{2} + \frac{12}{24} = 0;$$

$$c_2 = \frac{1}{2}!\left(\frac{1}{2}\right)^2 - \left(\beta_1 + \frac{1}{2}\beta_{\frac{1}{2}}\right) = -\frac{1}{8} - \left(\frac{1}{24} - \frac{1}{3}(\frac{1}{2})\right) = 0$$

$$c_3 = \frac{1}{3}!\left(\frac{1}{2}\right)^3 \alpha_{\frac{1}{2}} - \frac{1}{2}\left(\beta_1 + (\frac{1}{2})^2 \beta_{\frac{1}{2}}\right) = -\frac{1}{48} - \frac{1}{2}\left(\frac{1}{24} - \frac{1}{12}\right) = -\frac{1}{48} - \frac{1}{2}\left(-\frac{1}{24}\right) = -\frac{1}{48} + \frac{1}{48} = 0$$

$$c_4 = \frac{1}{4}!\left(\frac{1}{2}\right)^4 \alpha_{\frac{1}{2}} - \frac{1}{3!}\left(\beta_1 + \left(\frac{1}{2}\right)^3 \beta_{\frac{1}{2}}\right) = -\frac{1}{384} - \frac{1}{144} + \frac{1}{144} = -\frac{1}{384}$$

So that the order p = 3 and $c_{p+1} = -\dfrac{1}{384}$; we seek the root of the characteristics polynomial as follows:

$$p(\xi) = \alpha_0 - \alpha_{\frac{1}{2}}\xi = 1 - \xi^{\frac{1}{2}} = 0$$
$$|\xi| = 1$$

Hence, by definitions 3.1, we concluded that the method is zero stable.

We consider the scheme $y_{n+\frac{3}{2}} - y_{n+1} = \dfrac{h}{192}\left[-f_n + 46f_{n+1} - 5f_{n+2} + 56f_{n+\frac{3}{2}}\right]$

Here, $\alpha_1 = -1, \alpha_{\frac{3}{2}} = 1, \beta_0 = -\dfrac{1}{192}, \beta_2 = -\dfrac{5}{192}, \beta_1 = \dfrac{23}{96}, and, \beta_{\frac{3}{2}} = \dfrac{7}{24}$

Then observe that $c_0 = c_1 = \ldots\ldots\ldots\ldots\ldots\ldots c_4 = 0$

However,

$$c_5 = \frac{1}{5!}\left(\alpha_1 + (\tfrac{3}{2})^5 \alpha_{\frac{3}{2}}\right) - \frac{1}{4!}\left(\beta_1 + 2^4\beta_2 - (\tfrac{3}{2})^4 \beta_{\frac{3}{2}}\right)$$
$$= \frac{1}{120}\left(-1 + \frac{243}{32}\right) - \frac{1}{24}\left(\frac{46}{192} - \frac{80}{192} - \frac{4236}{3072}\right)$$
$$= \frac{1}{120}\left(\frac{211}{32}\right) - \frac{3692}{24}(3072)$$
$$= 4.87 \times 10^{-3}$$

$\Rightarrow$ The order p=4 and $c_{p+1} = 4.87 \times 10^{-3}$. the absolute root for the polynomial is thus computed;

$$p(\xi) = \alpha_{3/2}\xi^{3/2} - \alpha_1\xi = \xi^{3/2} - \xi = 0$$
$$i.e\,\xi\left(\xi^{1/2} - 1\right) = 0$$
$$\Rightarrow \xi = 0, or, \xi = 1, and, so, |\xi| = 1$$

Hence, the method is zero stable.

4.0 Conclusion

There is no doubt that the schemes are consistent and zero stable and could also be used by Numerical Analysts to solve differential equations experimentally. The obtained result in comparism with the theoretical result would also be of great importance in future research work.

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