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MHD Flow of A Third Grade Fluid with Heat Transfer And Slip Boundary Condition Down An Inclined Plane

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Abstract

In this work, we consider the combine effects of slip boundary ohmic heating on MHD flow of a third grade fluid down an inclined plane. The couple non-linear ordinary differential equations arising from the model were solved using both the regular and homotopy perturbation. Effects of the various thermo physical parameters are studied and depicted graphically.

Keywords: Slip boundary, MHD, Third grade fluid, Ohmic heating, inclined plane.

1. Introduction

Non-Newtonian fluid flow play an important role's in several industrial manufacturing processes. An example of such non-Newtonian fluids includes drilling mud, polymer solutions or melts, certain oils and greases and many other emulsions. Some of the typical applications of non-Newtonian fluid flow are noticed in the drilling of oil and gas wells, polymer sheet extrusion from a dye, glass fibber and paper production, drawing of plastic films, waste fluids e.t.c.

Because of the complexity of non-Newtonian fluids, it is very difficulty to analyse a single model that exhibits all its properties. The normal stress differences is describe in second grade of non-Newtonian fluid, but it can not predict shear thinning or thickening properties due to its constant apparent viscosity. The third grade fluid model attempt to include such characteristics of visco-elastics fluids.

Several researchers discussed the slip effect on fluid flow. Asghar et. al. (2006) examines the effects of partial slip on the rotating flow of an incompressible third grade fluid past a uniformly porous plate. Miccal and James (2008) discuss the effect of replacing the standard no slip boundary condition of fluid mechanics applying for the so called Falkner-Skan solutions, with a boundary condition that allows some degree of tangential fluid slip. Ellahi (2009) discuss the slip condition of an Oldroyd 8 – constant fluid and Sajid (2008) investigate the effect of slip condition on thin film flow.

In this paper, we examine the effects of slip boundary condition on a thin film flow of an MHD third grade fluid down an inclined plane. The heat transfer analysis is also carried out.

Many analytical and numerical techniques have been proposed by various authors for the solution of governing nonlinear differential equations of non-Newtonian fluids. We have solved the governing nonlinear equation of present problem using the traditional perturbation method (Nayfeh,1979) and homotopy perturbation method (He 2003 & 2009). We noticed that the solution obtain from the two methods are in a complete agreement. It is also observed that homotopy perturbation method is powerful analytical technique that is simple and straight forward and does not require the existence of any small or large parameter as does in traditional perturbation method. The plan of the paper is as follow: Section 2 contains the basic equations. Section 3 deals with the formulation of the problem with slip condition and also include solutions of the problem by traditional perturbation method and homotopy perturbation method. Results and discussion are presented in section 4 while section 5 concludes the paper.

2. Basic Equation

The fundamental equations governing the MHD flow of an incompressible electrically conducting fluid are the field equation:

$$\nabla \cdot v = 0 \qquad (1)$$

$$\rho \frac{Dv}{Dt} = -\nabla p + div\,\tau + J \times B + \rho f \qquad (2)$$

where ρ is the density f the fluid, v is the fluid velocity, B is the magnetic induction so that

$$B = B_0 + b \qquad (3)$$

and

$$J = \sigma(E + v \times B) \qquad (4)$$

is the current density, σ is the electrical conductivity, E is the electrical filed which is not considered (i.e $E = 0$), B_0 and b are applied and induced magnetic filed respectively, D/Dt denote the material derivative, p is the pressure, f is the external body force and τ is the Cauchy stress tensor which for a third grade fluid satisfies the constitutive equation

$$\tau = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) + \beta_3 (trA_1^2) A_1 \qquad (5)$$

$$A_n = \frac{DA_{n-1}}{Dt} + A_{n-1} \nabla v + (\nabla v)^{\perp} A_{n-1}, \qquad n \geq 1 \qquad (6)$$

where pI is the isotropic stress due to constraint incompressibility, μ is the dynamics viscosity, $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3$ are the material constants; $\perp$ indicate the matrix transpose, A_1, A_2, A_3 are the first three Rivlin-Ericken tensors and $A_0 = I$ is the identity tensor.

3. Problem Formulation

We consider a thin film of an incompressible MHD fluid of a third grade flowing in an inclined plane. The ambient air is assumed stationary so that the flow is due to gravity alone.

By neglect the surface tension of the fluid and the film is of uniform thickness δ, we seek a velocity field of the form

$$v = [u(y), 0, 0,] \qquad (7)$$

In the absence of modified presence gradient, equation (1)-(4) along with equation (5)-(7) yields

$$\frac{d^2u}{dy} + 6\beta \left(\frac{du}{dy}\right)^2 \frac{d^2u}{dy^2} + K - Mu = 0 \qquad (8)$$

Subject to the boundary condition

$$u - \Lambda \left[\mu \left(\frac{du}{dy}\right) + 2\beta \left(\frac{d^2u}{dy^2}\right)^3 \right] = 0 \quad \text{at} \quad y = 0 \qquad (9)$$

$$\frac{du}{dy} = 0 \quad \text{at} \quad y = 1 \qquad (10)$$

where

$\beta = \frac{(\beta_2 + \beta_3)\mu}{\delta^4}$ is third grade fluid parameter

$K = f_1 \sin\alpha$ while $f_1 = \frac{\delta^3 \rho g}{\mu}$ is the gravitational parameter.

$M = \frac{\delta^2 \sigma B_0^2}{\mu}$ is the magnetic parameter.

Equation (9) is the slip conditions where Λ is coefficient of slip and equation (10) comes from $\tau_{yx} = 0$ at $y = \delta$.
In the sequence, we take $\varepsilon = \beta$ and solve the system of equation (8)-(10) by the traditional perturbation method and also by the homotopy perturbation.

Heat Transfer Analysis

The thermal boundary layer equation for the thermodynamically compatible third grade fluid with viscous dissipation, work done due to deformation and joule heating is given as

$$k\frac{d^2T}{dy^2} + \mu\left(\frac{du}{dy}\right)^2 + 2(\beta_2 + \beta_3)\left(\frac{du}{dy}\right)^4 + \sigma B_0^2 u^2 = 0 \tag{11}$$

with boundary condition

$$T(y) = T_w \quad \text{at} \quad y = 0 \tag{12}$$

$$T(y) = T_\delta \quad \text{at} \quad y = \delta \tag{13}$$

where k is the thermal conductivity, T is the temperature, and T_δ is the temperature of the ambient fluid.
Introducing the following dimensionless variable

$$\bar{u} = \frac{u}{u'} \quad , \quad \bar{T} = \frac{T - T_1}{T_2 - T_1} \tag{14}$$

where $T_1 = T_w$ and $T_2 = T_\delta$

The system of equation (11)-(13) and (14) after dropping the caps take the following form:

$$\frac{d^2T}{dy^2} + B_r\left(\frac{du}{dy}\right)^2 + 2B_r\beta\left(\frac{du}{dy}\right)^4 + B_r M u^2 = 0 \tag{15}$$

$$T(y) = 0 \quad \text{at} \quad y = 0 \tag{16}$$

$$T(y) = 1 \quad \text{at} \quad y = 1 \tag{17}$$

where $B_r = \dfrac{\mu^3}{k\delta^2(T_2 - T_1)}$ is the Brinkman number

$M = \dfrac{\sigma B_0^2 \delta^2}{\mu}$ is the magnetic parameter

$\beta = \dfrac{(\beta_2 + \beta_3)\mu}{\delta^4}$ is third grade fluid parameter

Again, in the sequence, we take $\varepsilon = \beta$ and solve the system of equation (15)-(17) by the traditional perturbation method and also by the homotopy perturbation.

(A) Solution of the problem by Regular perturbation method

Let us assume ε as a small parameter in order to solve equation (8) by this method, we expand

$$u(y,\varepsilon)=u_0(y)+\varepsilon u_1(y)+\varepsilon^2 u_2(y)+\cdots\cdots \qquad (18)$$

Substituting equation (18) into equation (8) and rearranging based on powers of $\varepsilon-$terms.
We obtain the following problems of different order with their boundary conditions:
Zeroth – order problem and its solution

$$\frac{d^2u_0}{dy^2}-Mu+K=0 \qquad (19)$$

$$u_0-\Lambda\mu\left(\frac{du_0}{dy}\right)=0 \text{ at } y=0 \qquad (20)$$

$$\frac{du_0}{dy}=0 \quad \text{at} \quad y=1 \qquad (21)$$

The solution of system of equation (19)-(21) is given by

$$u_0=c_1e^{y\sqrt{M}}+c_2e^{-y\sqrt{M}}+\frac{K}{M} \qquad (22)$$

First order problem and its solution

$$\frac{d^2u_1}{dy^2}+6\frac{d^2u_0}{dy^2}\left(\frac{du_0}{dy}\right)^2-Mu_1=0 \qquad (23)$$

$$u_1-\Lambda\mu\left(\frac{du_1}{dy}\right)-2\Lambda\mu\left(\frac{du_0}{dy}\right)^3=0 \quad \text{at} \quad y=0 \qquad (24)$$

$$\frac{du_1}{dy}=0 \quad \text{at} \quad y=1 \qquad (25)$$

The solution of system of equation (23)-(25) is given by

$$u_1=(c_7+c_8y)e^{y\sqrt{M}}+(c_9+c_{10}y)e^{-y\sqrt{M}}+c_{11}e^{3y\sqrt{M}}+c_{12}e^{-3y\sqrt{M}} \qquad (26)$$

Second order problem and its solution

$$\frac{d^2u_2}{dy^2}+6\frac{d^2u_1}{dy^2}\left(\frac{du_0}{dy}\right)^2+12\frac{du_0}{dy}\frac{du_1}{dy}\frac{d^2u_0}{dy^2}-Mu_2=0 \qquad (27)$$

$$u_2-\Lambda\mu\left(\frac{du_2}{dy}\right)-6\Lambda\mu\left(\frac{du_0}{dy}\right)^2\left(\frac{du_1}{dy}\right)=0 \quad \text{at} \quad y=0 \qquad (28)$$

$$\frac{du_2}{dy}=0 \quad \text{at} \quad y=1 \qquad (29)$$

The solution of system of equation (27)-(29) is given by

$$u_2 = \left(c_{25} + c_{26}y + c_{27}y^2\right)e^{y\sqrt{M}} + \left(c_{28} + c_{29}y + c_{30}y^2\right)e^{-y\sqrt{M}} + \left(c_{31} + c_{32}y\right)e^{3y\sqrt{M}}$$
$$+ \left(c_{33} + c_{34}y\right)e^{-3y\sqrt{M}} + c_{35}e^{5y\sqrt{M}} + c_{36}e^{-5y\sqrt{M}} \tag{30}$$

Next, we find the approximate solution for temperature distribution, for which we write

$$T(y,\varepsilon) = T_0(y) + \varepsilon T_1(y) + \varepsilon^2 T_2(y) + \cdots\cdots \tag{31}$$

Substituting equation (31) into equation (15) –(17) and collecting the same power of ε, yields different order problems.

Zeroth –order problem with boundary conditions:

$$\frac{d^2T_0}{dy^2} + B_r\left[\left(\frac{du_0}{dy}\right)^2 + Mu_0\right] = 0 \tag{32}$$

$$T_0(0) = 0 \quad \text{at} \quad y = 0 \tag{33}$$

$$T_0(1) = 1 \quad \text{at} \quad y = 1 \tag{34}$$

with the solution

$$T_0 = c_{41}e^{y\sqrt{M}} + c_{42}e^{-y\sqrt{M}} + c_{43}e^{2y\sqrt{M}} + c_{44}e^{-2y\sqrt{M}} + c_{45}e^{4y\sqrt{M}} + c_{46}e^{-4y\sqrt{M}}$$
$$+ c_{47}y^2 + c_{39}y + c_{40} \tag{35}$$

First –order problem with boundary conditions:

$$\frac{d^2T_1}{dy^2} + 2B_r\left[\frac{du_0}{dy}\frac{du_1}{dy} + \left(\frac{du_0}{dy}\right)^4 + Mu_0u_1\right] = 0 \tag{36}$$

$$T_1(0) = 0 \quad \text{at} \quad y = 0 \tag{37}$$

$$T_1(1) = 1 \quad \text{at} \quad y = 1 \tag{38}$$

with the solution

$$T_1 = \left(c_{50} + c_{51}y\right)e^{y\sqrt{M}} + \left(c_{52} + c_{53}y\right)e^{-y\sqrt{M}} + \left(c_{54} + c_{55}y\right)e^{2y\sqrt{M}}$$
$$+ \left(c_{56} + c_{57}y\right)e^{-2y\sqrt{M}} + c_{58}e^{3y\sqrt{M}} + c_{59}e^{-3y\sqrt{M}} + c_{60}e^{4y\sqrt{M}}$$
$$+ c_{61}e^{-4y\sqrt{M}} + c_{62}y^2 + c_{48}y + c_{49} \tag{39}$$

Second–order problem with boundary conditions:

$$\frac{d^2T_2}{dy^2} + B_r\left[2\frac{du_0}{dy}\frac{du_2}{dy} + \left(\frac{du_1}{dy}\right)^2 + 8\left(\frac{du_0}{dy}\right)^3\frac{du_1}{dy} + 2Mu_0u_2 + Mu_1^2\right] = 0 \tag{40}$$

$$T_2(0) = 0 \quad \text{at} \quad y = 0 \tag{41}$$

$$T_2(1) = 1 \quad \text{at} \quad y = 1 \tag{42}$$

with the solution

$$\begin{aligned} T_2 = {} & \left(c_{65} + c_{66}y + c_{67}y^2\right)e^{y\sqrt{M}} + \left(c_{68} + c_{69}y + c_{70}y^2\right)e^{-y\sqrt{M}} \\ & + \left(c_{71} + c_{72}y + c_{73}y^2\right)e^{2y\sqrt{M}} + \left(c_{74} + c_{75}y + c_{76}y^2\right)e^{-2y\sqrt{M}} \\ & + \left(c_{77} + c_{78}y\right)e^{3y\sqrt{M}} + \left(c_{79} + c_{80}y\right)e^{-3y\sqrt{M}} + \left(c_{81} + c_{82}y\right)e^{4y\sqrt{M}} \\ & + \left(c_{83} + c_{84}y\right)e^{-4y\sqrt{M}} + c_{85}e^{5y\sqrt{M}} + c_{86}e^{-5y\sqrt{M}} + c_{87}e^{6y\sqrt{M}} + c_{88}e^{-6y\sqrt{M}} \\ & + c_{89}y^4 + c_{90}y^3 + c_{90}y^2 + c_{63}y + c_{64} \end{aligned} \tag{43}$$

(B) Solution by Homotopy perturbation method

The problem under consideration i.e equations (8)-(10) can be written as

$$L(v) - L(u_0) + qL(u_0) + q\left[6\frac{\beta}{\mu}\left(\frac{dv}{dy}\right)^2 \frac{d^2v}{dy} - Mv + K\right] = 0 \tag{44}$$

where $L = \dfrac{d^2}{dy^2}$ and equation (28) is the initial guess approximation.

Let $v = v_0 + qv_1 + q^2v_2 + \cdots$ (45)

Substitute equation (45) into (44) and equating the coefficient of like powers of q, we have

Zeroth –order problem with boundary conditions:

$$\frac{d^2v_0}{dy^2} - \frac{d^2u_0}{dy^2} = 0 \tag{46}$$

$$v_0 - \Lambda\mu\left(\frac{dv_0}{dy}\right) = 0 \quad \text{at} \quad y = 0 \tag{47}$$

$$\frac{dv_0}{dy} = 0 \quad \text{at} \quad y = 1 \tag{48}$$

with the solution

$$v_0 = c_1e^{y\sqrt{M}} + c_2e^{-y\sqrt{M}} + c_3^*y + c_4^* \tag{49}$$

First –order problem with boundary conditions:

$$\frac{d^2v_1}{dy^2} + \frac{d^2u_0}{dy^2} + 6\frac{\beta}{\mu}\frac{d^2v_0}{dy^2}\left(\frac{dv_0}{dy}\right)^2 - Mv_0 + K = 0 \tag{50}$$

$$v_1 - \Lambda\mu\left(\frac{dv_0}{dy}\right) - 2\Lambda\mu\left(\frac{dv_0}{dy}\right)^3 = 0 \quad \text{at} \quad y = 0 \tag{51}$$

$$\frac{dv_1}{dy}=0 \qquad \text{at} \qquad y=1 \tag{52}$$

with the solution

$$v_1=c_8^* e^{y\sqrt{M}}+c_9^* e^{-y\sqrt{M}}+c_{10}^* e^{2y\sqrt{M}}+c_{11}^* e^{-2y\sqrt{M}}+c_{12}^* e^{3y\sqrt{M}}+c_{13}^* e^{-3y\sqrt{M}} \\ +c_{14}^* y^2+c_6^* y+c_7^* \tag{53}$$

Second –order problem with boundary conditions:

$$\frac{d^2v_1}{dy^2}+6\frac{\beta}{\mu}\left[2\frac{d^2v_0}{dy^2}\frac{dv_0}{dy}\frac{dv_1}{dy}+\left(\frac{dv_0}{dy}\right)^2\frac{d^2v_1}{dy^2}\right]-Mv_1=0 \tag{54}$$

$$v_2-\Lambda\mu\left(\frac{dv_1}{dy}\right)-6\Lambda\mu\left(\frac{dv_0}{dy}\right)^2\left(\frac{dv_1}{dy}\right)=0 \qquad \text{at} \qquad y=0 \tag{55}$$

$$\frac{dv_2}{dy}=0 \qquad \text{at} \qquad y=1 \tag{56}$$

with the solution

$$v_2=\left(c_{17}^*+c_{18}^* y\right)e^{y\sqrt{M}}+\left(c_{19}^*+c_{20}^* y\right)e^{-y\sqrt{M}}+\left(c_{21}^*+c_{22}^* y\right)e^{2y\sqrt{M}}+\left(c_{23}^*+c_{24}^* y\right)e^{-2y\sqrt{M}} \\ +c_{25}^* e^{3y\sqrt{M}}+c_{26}^* e^{-3y\sqrt{M}}+c_{27}^* e^{4y\sqrt{M}}+c_{28}^* e^{-4y\sqrt{M}}+c_{29}^* e^{5y\sqrt{M}} \\ +c_{30}^* e^{-5y\sqrt{M}}+c_{31}^* y^4+c_{32}^* y^3+c_{33}^* y^2+c_{15}^* y+c_{16} \tag{57}$$

Next, we find approximate solution of temperature profile using homotopy perturbation by written equation (15) as

$$L(\overline{T})-L(\theta_0)+qL(\theta_0)+q\left[B_r\left(\frac{dv}{dy}\right)^2+2B_r\beta\left(\frac{dv}{dy}\right)^4+B_rMv^2\right]=0 \tag{58}$$

Let $\overline{T}=\overline{T}_0+q\overline{T}_1+q^2\overline{T}_2+\cdots\cdots$ (59)

Substitute equation (55) into (54) and equating the coefficient of like powers of q, we have

Zeroth –order problem with boundary conditions:

$$\frac{d^2\overline{T}_0}{dy^2}-\frac{d^2\theta_0}{dy^2}=0 \tag{60}$$

$$\overline{T}_0(0)=0 \qquad \text{at} \qquad y=0 \tag{61}$$

$$\overline{T}_0(1)=1 \qquad \text{at} \qquad y=1 \tag{62}$$

Using equation (35) as θ_0 to serve as initial guess approximation, we have

$$\overline{T}_0 = c_{34}^{*}e^{y\sqrt{M}} + c_{35}^{*}e^{-y\sqrt{M}} + c_{36}^{*}e^{2y\sqrt{M}} + c_{37}^{*}e^{-2y\sqrt{M}} + c_{38}^{*}e^{4y\sqrt{M}} + c_{39}^{*}e^{-4y\sqrt{M}} + c_{40}^{*}y^2 + c_{41}^{*}y + c_{42}^{*} \tag{63}$$

First-order problem with boundary conditions:

$$\frac{d^2\overline{T}_1}{dy^2} - \frac{d^2\theta_0}{dy^2} + B_r\left(\frac{dv_0}{dy}\right)^2 + B_r M v_0^2 = 0 \tag{64}$$

$$\overline{T}_1(0) = 0 \quad \text{at} \quad y = 0 \tag{65}$$

$$\overline{T}_1(1) = 1 \quad \text{at} \quad y = 1 \tag{66}$$

with the solution

$$\overline{T}_1 = \left(c_{45}^{*} + c_{46}^{*}y\right)e^{y\sqrt{M}} + \left(c_{47}^{*} + c_{48}^{*}y\right)e^{-y\sqrt{M}} + c_{49}^{*}e^{2y\sqrt{M}} + c_{50}^{*}e^{-2y\sqrt{M}} + c_{51}^{*}e^{4y\sqrt{M}} + c_{52}^{*}e^{-4y\sqrt{M}} + c_{53}^{*}y^4 + c_{54}^{*}y^3 + c_{55}^{*}y^2 + c_{56}^{*}y + c_{57}^{*} \tag{67}$$

Second order problem with boundary conditions:

$$\frac{d^2\overline{T}}{dy^2} + 2B_r\frac{dv_0}{dy}\frac{dv_1}{dy} + 4B_r\beta\left(\frac{dv_0}{dy}\right)^3\frac{dv_1}{dy} + 2B_r M v_0 v_1 = 0 \tag{68}$$

$$\overline{T}_2(0) = 0 \quad \text{at} \quad y = 0 \tag{69}$$

$$\overline{T}_2(1) = 1 \quad \text{at} \quad y = 1 \tag{70}$$

with solution

$$\overline{T}_2 = \left(c_{58}^{*} + c_{59}^{*}y + c_{60}^{*}y^2\right)e^{y\sqrt{M}} + \left(c_{61}^{*} + c_{62}^{*}y + c_{63}^{*}y^2\right)e^{-y\sqrt{M}} + \left(c_{64}^{*} + c_{65}^{*}y\right)e^{2y\sqrt{M}} + \left(c_{66}^{*} + c_{67}^{*}y\right)e^{-2y\sqrt{M}} + \left(c_{68}^{*} + c_{69}^{*}y\right)e^{3y\sqrt{M}} + \left(c_{70}^{*} + c_{71}^{*}y\right)e^{-3y\sqrt{M}} + c_{72}^{*}e^{4y\sqrt{M}} + c_{73}^{*}{}^{-4y\sqrt{M}} + c_{74}^{*}e^{5y\sqrt{M}} + c_{75}^{*}e^{-5y\sqrt{M}} + c_{76}^{*}e^{6y\sqrt{M}} + c_{77}^{*}e^{-6y\sqrt{M}} + c_{78}^{*}y^5 + c_{79}^{*}y^4 + c_{80}^{*}y^3 + c_{81}^{*}y^2 + c_{82}^{*}y + c_{83}^{*} \tag{71}$$

4. Results and Discussion

Approximate analytical solutions for velocity and temperature distribution of a thin film flow of an MHD third grade fluid down an inclined plane have been found. The governing nonlinear ordinary differential equations are solved using traditional regular perturbation method as well as the recently introduced homotopy perturbation technique and the results are compared.

Figure 1-6 shows that the solution obtained by two method are the same for identical value of ε and β. Therefore, we discuss only solution obtained by homotopy perturbation method. Figure 1 and 2 illustrates the effect of magnetic parameter M on the velocity and temperature distribution of the fluid down an inclined plane. Increase in the value of M reduces the velocity distribution and increases the temperature distribution as slip parameter Λ is constant. Figure 3 and 4 shows the effect of parameter K on the velocity and temperature distribution. Clearly, increase in value of parameter K increases the velocity and temperature distribution due to fact that increase in parameter K correspond to the increasing in angle of inclination. In figure 5 and 6, we find out that increase in slip parameter Λ, the velocity and temperature distribution increases.

5. Conclusion

A thin film flow of an MHD third grade fluid down an inclined plane with slip boundary condition has been discussed. It was found that the solutions obtained by traditional perturbation and homotopy perturbation technique are identical at the same values of ε and β, this is illustrated in Figures 1-6. The effect of slip parameter, Magnetic parameter and other parameters involved in the problem are discussed and results are displayed in graph to see their effects.

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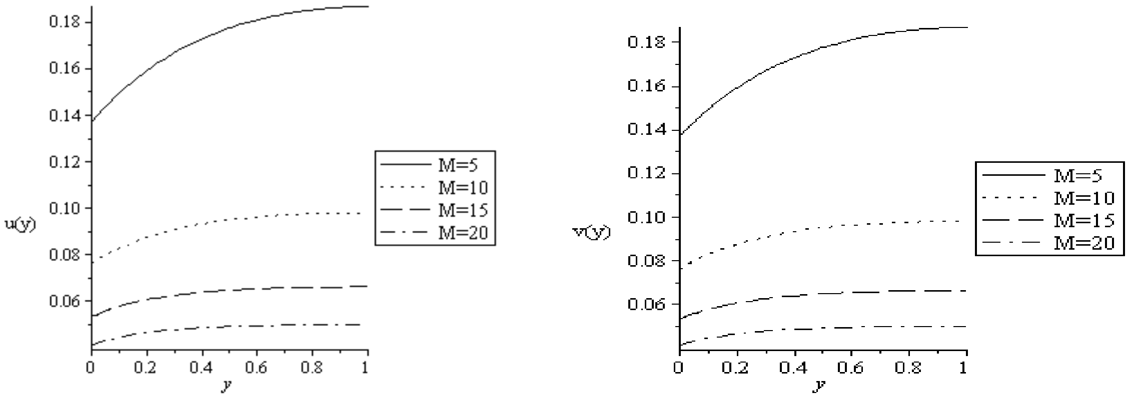
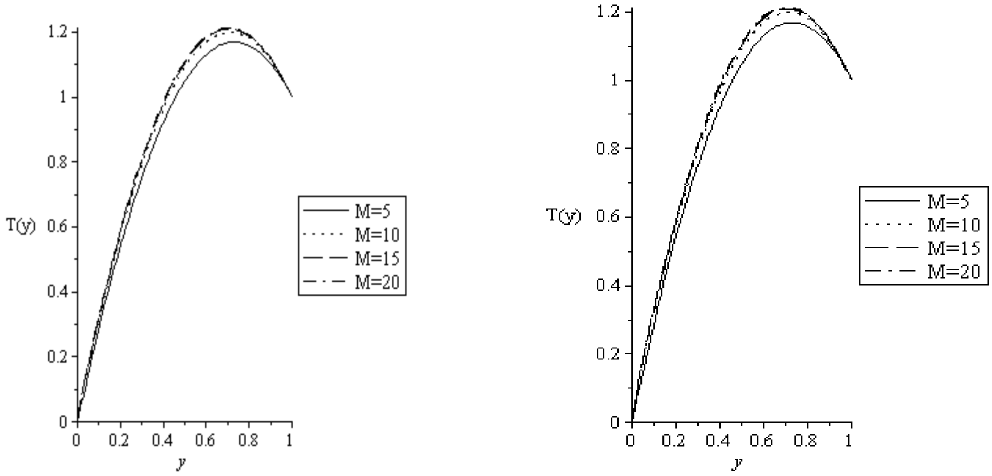


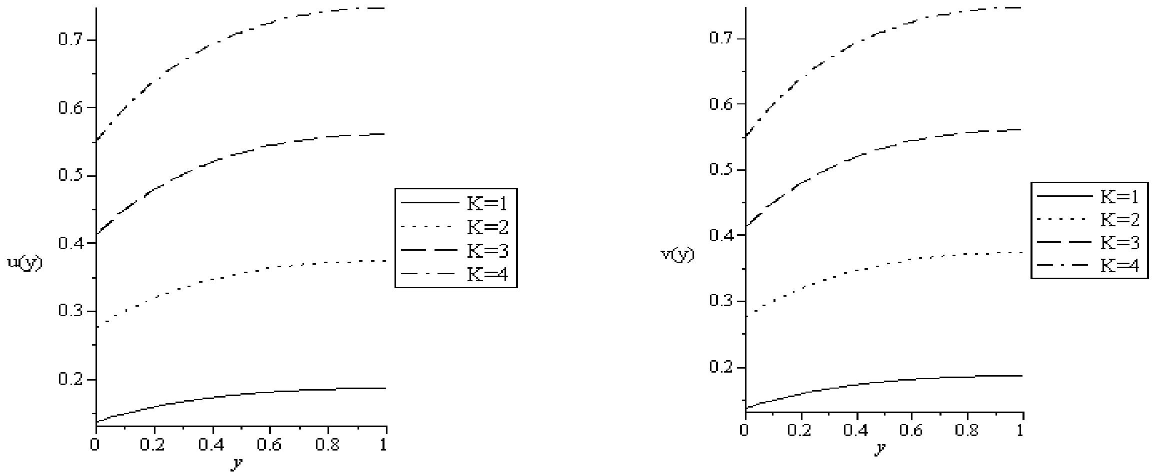
(a) Traditional Perturbation (b) Homotopy Perturbation

Figure 1: Velocity profile for different values of magnetic parameter M when $\delta = 1$, $K = +1$, $\beta = 0.001$, $\Lambda = 1$



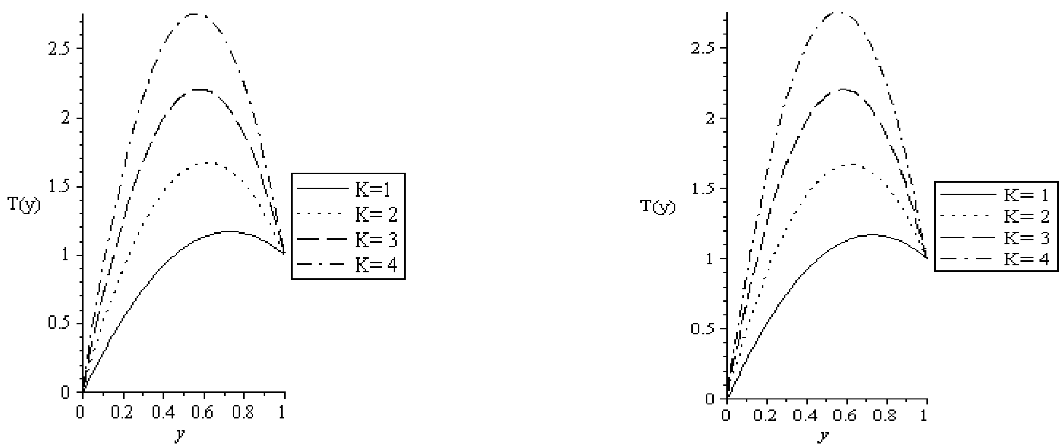
(a) Traditional Perturbation (b) Homotopy Perturbation

Figure 2: Temperature profile for different values of magnetic parameter M when $\delta = 1$, $K = +1$, $\beta = 0.001$, $\Lambda = 1$



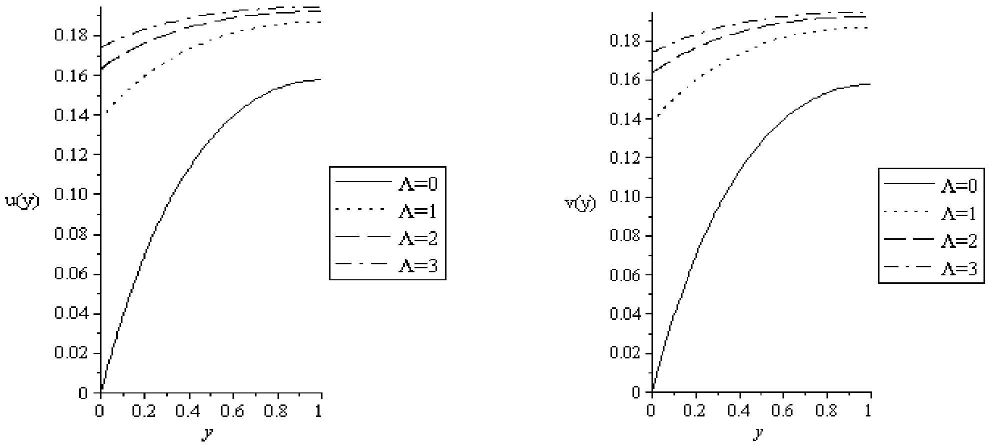
(a) Traditional Perturbation (b) Homotopy Perturbation

Figure 3: Velocity profile for different values of parameter K when $\delta = 1$, $M = 5$, $\beta = 0.001$, $\Lambda = 1$



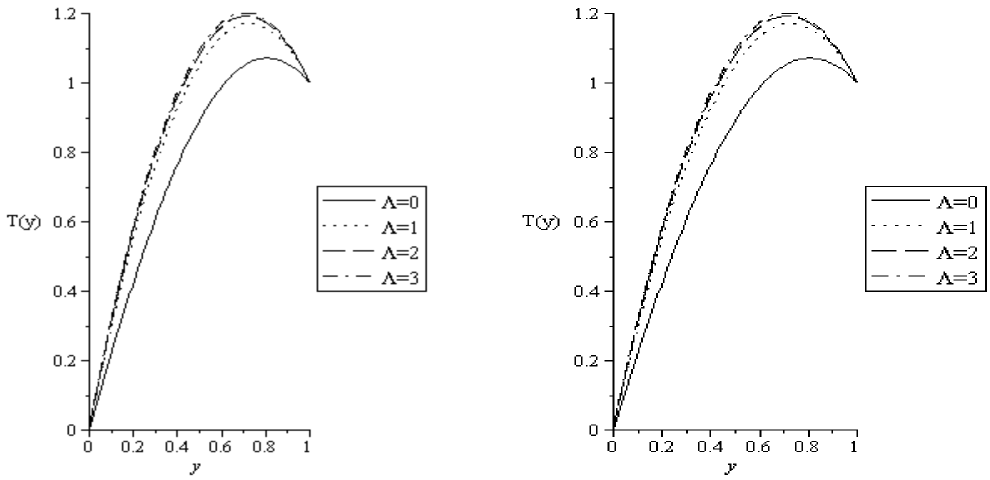
(a) Traditional Perturbation (b) Homotopy Perturbation

Figure 4: Temperature profile for different values of parameter K when $\delta = 1$, $M = 5$, $\beta = 0.001$, $\Lambda = 1$



(a) Traditional Perturbation (b) Homotopy Perturbation

Figure 5: Velocity profile for different values of slip parameter Λ when $\delta = 1$, $M = 5$, $\beta = 0.001$, $K = +1$



(a) Traditional Perturbation (b) Homotopy Perturbation

Figure 6: Temperature profile for different values of slip parameter Λ when $\delta = 1$, $M = 5$, $\beta = 0.001$, $K = +1$