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Natural Frequencies of Magnetoelastic Longitudinal Wave Propagation in an Orthotropic Circular Cylinder

Abo-el-nour N. Abd-alla [1,2*] and Aishah Raizah [3]

[1] Department of Mathematics, Faculty of Science, Jazan University, Jazan, Saudi Arabia

[2] Department of Mathematics, Faculty of Science, Sohag University, Sohag, Egypt.

[3] Department of Mathematics, Science College, King Khalid University, Abha, Saudi Arabia

[*] E-mail address: aboelnourabdalla@yahoo.com

Abstract

In this paper, we study the longitudinal wave propagation in a perfectly conducting elastic circular cylinder in the presence of an axial initial magnetic field. The elastic cylinder is assumed to be made of an orthotropic material. The problem is represented by the equations of elasticity taking into account of the effect of the magnetic field as given by Maxwell's equations in the quasi-static approximation. The stress free conditions on the inner and outer surfaces of the hollow circular cylinder are used to form a frequency equation in terms of the wavelength, the cylinder radii and the material constants. Numerical calculations are obtained and the results are represented graphically. It is observed that the longitudinal elastic waves in a solid body propagating under the influence of a superimposed magnetic field can be different significantly from that of those propagating in the absence of a magnetic field. Also, elastic waves may convey information on electromagnetic properties of the material: for example through a precise measurement of the surface current induced by the presence of the magnetic field. Finally, some of the earlier results are deduced as particular cases.

Keywords: Natural frequencies, Magnetoelasticity, Longitudinal wave, Orthotropic materials,

1. Introduction

Longitudinal waves are waves that have vibrations along or parallel to their direction of travel; that is, waves in which the motion of the medium is in the same direction as the motion of the wave. The study of wave propagation over a continuous media is of practical importance in the field of engineering, medicine, optics science, seismology, acoustics and in space science.

With the advancement of space research, it has become necessary to obtain a deep insight in the behavior of materials, especially of the anisotropic ones that are so frequently used in the missiles and other allied systems. Without taking the consideration of the effect of the magnetic field, the analysis of longitudinal wave propagation in anisotropic and homogeneous circular cylindrical shell, according to the theory of elasticity, have been done by many authors: [1, 2, 3, 4, 5]. Moreover, the propagation of harmonic waves, in circular cylinders which are made of isotropic or anisotropic materials, have been investigated and evaluated numerically, on the basis of the theory of elasticity, by Mirsky [6], Tsai [7] and White and Tongtaow [8].

Among many important problems which are considered in such studies, the problems of elastic wave propagation in the presence of a steady magnetic field have investigated when the material was isotropic homogeneous by Andreou et al. [9], Das et al. [10], Gourakishwar [11], Paria [12], Suhubi [13]. Some of the analogous results on magnetoelastic waves propagation problems, but in an anisotropic medium, were obtained by Abd-alla [14, 15], Datta [16]. General details and many references on these subjects may be found in monographs published by: Eringen et al. [17-18], Auld [19], Moon [20] and Nowacki [21].

Recently, the interaction of electromagnetic fields with the motion of a deformable solid is being receiving greater attention by many investigators. Therefore, many researchers investigated the effect of the magnetic field on the wave propagation in anisotropic cylindrical materials such as: Barakati and Zhupanska [22] studied the effects of pulsed electromagnetic fields on the dynamic mechanical response of electrically conductive anisotropic plates. Dinzart and Sabar [23] presented numerical investigations into magneto-electro-elastic moduli responsible for the magneto-electric coupling as functions of the volume reaction and characteristics of the coated inclusions. Akbarovet al. [24] studied torsional wave dispersion in a three-layered (sandwich) hollow cylinder with finite initial strains. Chattopadhyay et al. [25] studied the propagation of horizontally polarized shear waves in an internal magnetoelastic monoclinic stratum with irregularity in lower interface. Tang and Xu [26] employed the method of eigenfunction expansion to solve the problems of transient torsional vibration responses of finite, semi-infinite and infinite hollow cylinders. Acharya et al. [27] investigated the effect of the transverse isotropy and magnetic field on the interface waves in a conducting medium subject to the initial state of stress of the form of hydrostatic tension or compression. Petrov et al. [28] focused on the nature of ferromagnetic resonance (FMR) under the influence of acoustic oscillations with the same frequency as FMR. Mol'chenko et al. [29] constructed a two-dimensional nonlinear magnetoelastic model of a current-carrying orthotropic shell of revolution taking into account of finite orthotropic conductivity, permeability and permittivity. Abd-Alla and Abo-Dahab [30] studied the influence of the viscosity on reflection and refraction of plane shear elastic waves in two magnetized semi-infinite media. Selim [31] showed the effect of damping on the propagation of torsional waves in an initially stressed, dissipative, incompressible cylinder of infinite length. Dai and Wang [32] illustrated an analytical method to solve magneto-elastic wave propagation and perturbation of the magnetic field vector in an orthotropic laminated hollow cylinder with arbitrary thickness. Liu and Chang [33] investigated the interactive behaviors among transverse magnetic fields, axial loads and external force of a magneto-elastic beam with general boundary conditions.

In this study an attempt has been made to investigate the longitudinal wave propagation in an orthotropic circular cylinder permeated by a magnetic field. The frequency equations have been derived in the form of a determinant involving Bessel functions and its roots give the values of the characteristic circular frequency parameters of the first three modes for various geometries. These roots, which correspond to various mode, have been verified numerically and represented graphically in different values for the magnetic field. Finally, some of the earlier results are deduced as particular cases.

2. Basic Equations

The equations of motion for a perfect conducting elastic solid in uniform magnetic field are [10]:

$$\tau_{ji,j} + f_i = \rho \ddot{u}_i \qquad i,j=1,2,3 \tag{1}$$

where τ_{ij} is the mechanical stress tensor, ρ is the mass density of the material, f_i is Lorentz force and given as follows:

$$\vec{f} = \frac{\mu_o}{4\pi}[\vec{\nabla} \times \vec{h}] \times \vec{H}_0 \tag{2}$$

where

$$\vec{h} = \vec{\nabla} \times \left(\vec{u} \times \vec{H}_o \right) \tag{3}$$

$$\vec{H}_o = (0,0,H_o) \tag{4}$$

and H_o is the intensity of the uniform axial magnetic field , $\vec{h}$ small perturbation of the magnetic field, μ_o is the magnetic permeability in the medium. From (3) and (4) $\vec{h}$ may be written as:

$$\vec{h} = H_o \frac{\partial u}{\partial z} \vec{e}_r - H_o \left[\frac{\partial u}{\partial r} + \frac{u}{r} \right] \vec{e}_z \tag{5}$$

Using (4) and (5) in (2), Lorentz force becomes:

$$\vec{f} = \rho \alpha^2 \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} \right] \vec{e}_r \tag{6}$$

where $$\alpha^2 = \frac{\mu_o H_o{}^2}{4\pi\rho}$$

Maxwell's equations in this study may be written as (in Gaussian units):

$$\vec{\nabla} \times \vec{H} = \frac{4\pi}{c}\vec{J}, \qquad \vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \qquad \vec{\nabla} \cdot \vec{B} = 0, \qquad \vec{\nabla} \cdot \vec{D} = \rho_e \tag{7}$$

where $\vec{H}, \vec{B}, \vec{E}, \vec{J}$ denote, respectively, the magnetic field intensity, magnetic induction, electric field intensity and current density vectors, c is the velocity of light in vacuum, and the electric field intensity is given as the form

$$E_\theta = \frac{\mu_o H_o}{c}[\frac{\partial u}{\partial t}] \tag{8}$$

Electromagnetic equations in vacuum are:

$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \right)\left(\vec{h}^*, \vec{E}^*\right) = 0, \quad curl\left(\vec{h}^*, \vec{E}^*\right) = \frac{1}{c}\frac{\partial}{\partial t}\left(\vec{E}^*, -\vec{h}^*\right) \tag{9}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

The strain components are given in terms of the displacements by:

$$e_{rr} = \frac{\partial u}{\partial r}, \qquad e_{\theta\theta} = \frac{u}{r}, \qquad e_{zz} = \frac{\partial w}{\partial z},$$

$$e_{rz} = \frac{1}{2}[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}], \qquad e_{r\theta} = 0, \qquad e_{\theta z} = 0. \tag{10}$$

where e_{ij} denote the strain components.

For an orthotropic elastic body, the Cauchy stress components are given in terms of independent elastic constants c_{ij} as follows

$$\begin{aligned}
\tau_{rr} &= c_{11}\frac{\partial u}{\partial r} + c_{12}\frac{u}{r} + c_{13}\frac{\partial w}{\partial z}, & \tau_{\theta\theta} &= c_{12}\frac{\partial u}{\partial r} + c_{11}\frac{u}{r} + c_{13}\frac{\partial w}{\partial z}, \\
\tau_{zz} &= c_{13}\frac{\partial u}{\partial r} + c_{13}\frac{u}{r} + c_{33}\frac{\partial w}{\partial z}, & \tau_{rz} &= c_{44}(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}), \\
\tau_{z\theta} &= 0, & \tau_{r\theta} &= 0.
\end{aligned} \tag{11}$$

Substituting (6) and (11) into (1), one may get the equations of motion in terms of the displacements components as:

$$c_{11}\left[\frac{\partial^2 u}{\partial r^2} - \frac{u}{r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right] + c_{13}\left[\frac{\partial^2 w}{\partial r \partial z}\right] + c_{44}\left[\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial r \partial z}\right] + \rho\alpha^2\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right] = \rho\frac{\partial^2 u}{\partial t^2} \tag{12}$$

$$[c_{44} + c_{13}]\left[\frac{\partial^2 u}{\partial r \partial z} + \frac{1}{r}\frac{\partial u}{\partial z}\right] + c_{33}\left[\frac{\partial^2 w}{\partial z^2}\right] + c_{44}\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right] = \rho\frac{\partial^2 w}{\partial t^2} \tag{13}$$

3. Formulation of the Problem

Longitudinal wave propagation in a circular cylinder of tetragonal elastic material of inner and outer radii, a and b, subjected to an axial magnetic field is considered. The cylinder is treated as a perfect conductor and the regions inside and outside the elastic material are assumed to be vacuum.

We assume that waves are characterized by the displacement components in the radial and axial directions only. The displacement field, in this case, in cylindrical coordinates *(r,* θ*, z)*, is given by

$$u = u(r,z,t), \quad v = 0, \quad w = w(r,z,t), \tag{14}$$

where u, v, w are the displacement components in the radial, circumferential, and axial directions, respectively, and all other quantities involved are functions of r, z and t only, where t denotes the time.

4. Solution of the Problem

4.1. Harmonic solutions:

We now consider the propagation of an infinite strain of sinusoidal waves along a hollow circular cylinder of infinite extent such that the displacement at each point is a sample harmonic function of z and t. Therefore, we shall seek the solution of the equations of motion and follow the same procedure as in Mirsky [6]:

$$u(r,z,t) = \frac{d\phi}{dr}\cos(\lambda t + qz), \quad w(r,z,t) = \eta\phi\sin(\lambda t + qz) \tag{15}$$

where $\phi=\phi(r)$, $q=\dfrac{2\pi}{l}$ is the wave number, l is the wavelength, λ is the angular frequency and η is an arbitrary constant to be determined later in the analysis. Putting Eq. (15) in (13) and (14), one obtains:

$$\left(\frac{d^2\phi}{dr^2}+\frac{1}{r}\frac{d\phi}{dr}\right)+\left[\frac{\rho\lambda^2-\left(c_{44}+\rho\alpha^2\right)q^2+\eta q\left(c_{44}+c_{13}\right)}{c_{11}+\rho\alpha^2}\right]\phi=0 \quad (16)$$

$$\left(\frac{d^2\phi}{dr^2}+\frac{1}{r}\frac{d\phi}{dr}\right)+\left[\frac{\eta\left(\rho\lambda^2-q^2c_{33}\right)}{\eta c_{44}-q\left(c_{44}+c_{13}\right)}\right]\phi=0 \quad (17)$$

Eq. (16) is consistent with (17) provided that η is chosen to satisfy the equations

$$\frac{\eta\left(\rho\lambda^2-q^2c_{33}\right)}{\eta c_{44}-q\left(c_{44}+c_{13}\right)}=\frac{\rho\lambda^2-\left(c_{44}+\rho\alpha^2\right)q^2+\eta q\left(c_{44}+c_{13}\right)}{c_{11}+\rho\alpha^2}=P^2 \quad (18)$$

Eliminating η from (18), we find that p^2 satisfies the equation:

$$A(p^2)^2+Bp^2+C=0 \quad (19)$$

where

$$\begin{aligned}
A&=c_{44}\left(c_{11}+\rho\alpha^2\right),\\
B&=-[\rho\lambda^2\left(c_{11}+\rho\alpha^2+c_{44}\right)+q^2\left(c_{13}^2+c_{44}\left(2c_{13}-\rho\alpha^2\right)-c_{33}c_{11}-\rho\alpha^2c_{33}\right)\\
C&=\left(\rho\lambda^2-q^2c_{33}\right)\left(\rho\lambda^2-\rho\alpha^2q^2-c_{44}q^2\right)
\end{aligned} \quad (20)$$

If p_1^2 and p_2^2 are the roots of this equation, the corresponding functions $\phi_1=\phi_1(r),\phi_2=\phi_2(r)$ satisfy the equations:

$$[\frac{d^2\phi_1}{dr^2}+\frac{1}{r}\frac{d\phi_1}{dr}]+p_1^2\phi_1=0,\quad [\frac{d^2\phi_2}{dr^2}+\frac{1}{r}\frac{d\phi_2}{dr}]+p_2^2\phi_2=0, \quad (21)$$

where

$$P_1^2=\frac{-B-D}{2A},\qquad P_2^2=\frac{-B+D}{2A};\quad D=\sqrt{B^2-4AC}$$

The general solutions of Eqs. (21) are

$$\phi_1(r)=A_1Z_0(P_1r)+B_1W_0(P_1r),\quad \phi_2(r)=A_2Z_0(P_2r)+B_2W_0(P_2r), \quad (22)$$

where A_1,B_1,A_2 *and* B_2 are constants of integration and for brevity *Z* denote the Bessel function *J* or *I* and *W* denote the Bessel function *Y* or *K,* according to the signs of p_1^2 and p_2^2.

The displacement field may now be written as

$$u=[\frac{d\phi_1}{dr}+\frac{d\phi_2}{dr}]\cos(\lambda t+qz), \qquad w=[\eta_1\phi_1+\eta_2\phi_2]\sin(\lambda t+qz) \tag{23}$$

where

$$\eta_1=\frac{\left(c_{44}+\rho\alpha^2\right)q^2-\rho\lambda^2+P_1^2\left(c_{11}+\rho\alpha^2\right)}{q\left(c_{44}+c_{13}\right)}, \quad \eta_2=\frac{q\left(c_{44}+c_{13}\right)P_2^2}{c_{44}P_2^2-\rho\lambda^2+q^2c_{33}} \tag{24}$$

4.2. Solution of Electric field intensity in vacuum

The general solution of E_θ^* from $(10)_4$ take the form

$$E^*=\begin{cases}A_3Z_0(kr)\sin(\lambda t+qz), & r\le a\\ B_3W_0(kr)\sin(\lambda t+qz), & r\ge b\end{cases} \tag{25}$$

where $k=\sqrt{(\lambda^2/c^2)-q^2}$, A_3 *and* B_3 are arbitrary constants and for brevity W denotes the Bessel function Y or K, according to the signs of k^2 .

4.3. Boundary conditions:

For free motion, the boundary conditions are required for the total stress to be vanished and the continuity of the electric field on the surfaces $r = a, b$, i.e.

$$\left.\begin{array}{l}\tau_{rr}+M_{rr}-M_{rr}^*=0\\ \tau_{rz}+M_{rz}-M_{rz}^*=0\\ E=E^*\end{array}\right\} \quad on \quad r=a,b \tag{26}$$

where τ_{rr},τ_{rz} are the components of the mechanical stresses, M_{rr},M_{rz} are the components of Maxwell's stresses in the medium and M_{rr}^*,M_{rz}^* are Maxwell's stresses in vacuum. Eliminating A_1,A_2,B_1,B_2,A_3,B_3 after applying the boundary conditions (26), we get the determinant must be vanished leading to the following frequency equation (dispersion relation) as:

$$\Delta=\left|X_{ij}\right|=0, \quad i,j=1,2,....6 \tag{27}$$

where

$$X_{11} = (c_{11} + \rho\alpha^2)P_1^2 Z_2(P_1 a) - (c_{11} + c_{12} + 2\rho\alpha^2)\frac{P_1\delta}{a} Z_1(P_1 a) + qc_{13}\eta_1 Z_0(P_1 a),$$

$$X_{12} = (c_{11} + \rho\alpha^2)P_1^2 W_2(P_1 a) - (c_{11} + c_{12} + 2\rho\alpha^2)\frac{P_1}{a} W_1(P_1 a) + qc_{13}\eta_1 W_0(P_1 a),$$

$$X_{13} = (c_{11} + \rho\alpha^2)P_2^2 Z_2(P_2 a) - (c_{11} + c_{12} + 2\rho\alpha^2)\frac{P_2\delta}{a} Z_1(P_2 a) + qc_{13}\eta_1 Z_0(P_2 a),$$

$$X_{14} = (c_{11} + \rho\alpha^2)P_2^2 W_2(P_2 a) - (c_{11} + c_{12} + 2\rho\alpha^2)\frac{P_2}{a} W_1(P_2 a) + qc_{13}\eta_1 W_0(P_2 a),$$

$$X_{15} = \frac{cH_0}{4\pi\lambda}\left[\frac{1}{a} Z_0(ka) - \delta k Z_1(ka)\right], \qquad X_{16} = 0,$$

$$X_{21} = (c_{11} + \rho\alpha^2)P_1^2 Z_2(P_1 b) - (c_{11} + c_{12} + 2\rho\alpha^2)\frac{P_1\delta}{b} Z_1(P_1 b) + qc_{13}\eta_1 Z_0(P_1 b),$$

$$X_{22} = (c_{11} + \rho\alpha^2)P_1^2 W_2(P_1 b) - (c_{11} + c_{12} + 2\rho\alpha^2)\frac{P_1}{b} W_1(P_1 b) + qc_{13}\eta_1 W_0(P_1 b),$$

$$X_{23} = (c_{11} + \rho\alpha^2)P_2^2 Z_2(P_2 b) - (c_{11} + c_{12} + 2\rho\alpha^2)\frac{P_2\delta}{b} Z_1(P_2 b) + qc_{13}\eta_1 Z_0(P_2 b), \tag{28}$$

$$X_{24} = (c_{11} + \rho\alpha^2)P_2^2 W_2(P_2 b) - (c_{11} + c_{12} + 2\rho\alpha^2)\frac{P_2}{b} W_1(P_2 b) + qc_{13}\eta_1 W_0(P_2 b),$$

$$X_{25} = 0, \qquad X_{26} = \frac{H_0 c}{4\pi\lambda}\left[\frac{1}{b} W_0(kb) - k W_1(kb)\right],$$

$$X_{31} = \left[\frac{-c_{44}\eta_1 + q(c_{44} + \rho\alpha^2)}{-c_{44}\eta_2 + q(c_{44} + \rho\alpha^2)}\right]\delta P_1 Z_1(P_1 a), \qquad X_{32} = \left[\frac{-c_{44}\eta_1 + q(c_{44} + \rho\alpha^2)}{-c_{44}\eta_2 + q(c_{44} + \rho\alpha^2)}\right] P_1 W_1(P_1 a),$$

$$X_{33} = \delta P_2 Z_1(P_2 a), \qquad X_{34} = P_2 W_1(P_2 a),$$

$$X_{35} = \frac{qH_0 c}{4\pi\lambda}\left[\frac{Z_0(ka)}{-c_{44}\eta_2 + q(c_{44} + \rho\alpha^2)}\right], \qquad X_{36} = 0,$$

$$X_{41} = \left[\frac{-c_{44}\eta_1 + q(c_{44} + \rho\alpha^2)}{-c_{44}\eta_2 + q(c_{44} + \rho\alpha^2)}\right]\delta P_1 Z_1(P_1 b), \qquad X_{42} = \left[\frac{-c_{44}\eta_1 + q(c_{44} + \rho\alpha^2)}{-c_{44}\eta_2 + q(c_{44} + \rho\alpha^2)}\right] P_1 W_1(P_1 b),$$

$$X_{43} = \delta P_2 Z_1(P_2 b), \qquad X_{44} = P_2 W_1(P_2 b),$$

$$X_{45} = 0, \qquad X_{46} = \frac{qH_0 c}{4\pi\lambda}\left[\frac{W_0(kb)}{-c_{44}\eta_2 + q(c_{44} + \rho\alpha^2)}\right],$$

$$X_{51}=\frac{\lambda H_0\delta}{c}P_1Z_1(P_1a),\qquad X_{52}=\frac{\lambda H_0}{c}P_1W_1(P_1a),$$

$$X_{53}=\frac{\lambda H_0\delta}{c}P_2Z_1(P_2a),\qquad X_{54}=\frac{\lambda H_0}{c}P_2W_1(P_2a),$$

$$X_{55}=Z_0(ka),\qquad X_{56}=0,$$

$$X_{61}=\frac{\lambda H_0\delta}{c}P_1Z_1(P_1b),\qquad X_{62}=\frac{\lambda H_0}{c}P_1W_1(P_1b),$$

$$X_{63}=\frac{\lambda H_0\delta}{c}P_2Z_1(P_2b),\qquad X_{64}=\frac{\lambda H_0}{c}P_2W_1(P_2b),$$

$$X_{65}=0,\qquad X_{66}=W_0(kb).$$

where $\delta=1$ at $Z=J$ and $\delta=-1$ at $Z=I$.

5. Radial and Axial Vibrations

As the wave number $q\to 0$ (i.e., for infinite wavelength), the following simplifications have been made by using the result of [6]:

$$q^2=0,\qquad P_1\to\sqrt{\frac{\rho\lambda^2}{c_{11}+\rho\alpha^2}}$$

$$k\to\sqrt{\frac{\lambda^2}{c^2}},\qquad P_2\to\sqrt{\frac{\rho\lambda^2}{c_{44}}},$$

$$q\eta_1\to\frac{\left(c_{11}-c_{44}+\rho\alpha^2\right)\rho\lambda^2}{c_{44}\left(c_{44}+c_{13}\right)},\qquad q\eta_2\to\frac{q^2(c_{44}+c_{13})}{c_{44}-c_{11}-\rho\alpha^2},$$

$$\frac{q}{\left[-c_{44}\eta_2+q(c_{44}+\rho\alpha^2)\right]}\to 0,\qquad \frac{\left[-c_{44}\eta_1+q(c_{44}+\rho\alpha^2)\right]}{\left[-c_{44}\eta_2+q(c_{44}+\rho\alpha^2)\right]}\to 0.$$

and the characteristic equation (27) may be written as the product of two determinants

$$\Delta_1\cdot\Delta_2=0 \qquad (29)$$

where

$$\Delta_1 = \begin{vmatrix} \overline{X}_{11} & \overline{X}_{12} & \overline{X}_{15} & 0 \\ \overline{X}_{21} & \overline{X}_{22} & 0 & \overline{X}_{26} \\ \overline{X}_{51} & \overline{X}_{52} & \overline{X}_{55} & 0 \\ \overline{X}_{61} & \overline{X}_{62} & 0 & \overline{X}_{66} \end{vmatrix} = 0, \qquad \Delta_2 = \begin{vmatrix} \overline{X}_{33} & \overline{X}_{34} \\ \overline{X}_{43} & \overline{X}_{44} \end{vmatrix} = 0. \tag{30}$$

The elements $\overline{X}_{ij}$ are given by (28) with $q \to 0$. The equation $\Delta_1 = 0$ represents a motion involving the radial displacement u only, corresponding to the radial vibrations [15]. $\Delta_2 = 0$ represents a motion involving the axial displacement w only, corresponding to the axial-shear vibrations [6] and [13].

6. The Numerical Calculations

For numerical calculations, we consider the following transformations:

$$\Omega = \frac{\lambda}{\lambda_i}, \qquad \lambda_i = \frac{\gamma}{b}, \qquad \beta = \frac{\gamma}{c}, \qquad \gamma = \sqrt{\frac{c_{44}}{\rho}},$$

$$\Omega_1 = \gamma\Omega, \qquad h = \frac{a}{b}, \qquad q = \frac{2\pi}{l}, \qquad m = \frac{2\pi b}{l}.$$

The calculations of the roots of the frequency equation (27), represent a major task and require a rather extensive effort for numerical computation. Calculations have been carried out for the case of Titanium dioxide (Rutile TiO_2), which belongs to the tetragonal system (crystal symmetry for it is 4/mmm). It has 6 elastic constants [19].

$$c_{11} = 26.6(10^{11})\ dyne/cm^2 \qquad c_{33} = 46.99(10^{11})\ dyne/cm^2$$

$$c_{12} = 17.33(10^{11})\ dyne/cm^2 \qquad c_{44} = 12.39(10^{11})\ dyne/cm^2$$

$$c_{13} = 13.62(10^{11})\ dyne/cm^2 \qquad c_{66} = 17.33(10^{11})\ dyne/cm^2$$

Also, the density is $\rho = 4.26\ gm/cm^3$, the velocity of light is $c = 3(10^{10})\ cm/\sec$ and the permeability is $\mu_o = 1\ Gauss/Oersted \quad c = 3(10^{10})\ cm/\sec$.

7. Discussion and Conclusion

The dimensionless frequency spectrum Ω for the longitudinal vibrations, as a function of the ratio thickness $h = (a/b)$, for the value of non-dimensional wave number $m = 1$, is calculated and given in form of graphs. The values of the effective primary magnetic field H_o are chosen as ($H_o = 10^5, 10^6, 10^7$ Oersted). The frequency equation is solved numerically, and for this purpose a matrix determinant computation routine is used for different Ω and h along with a root finding method to refine steps close to its roots. For each pair (Ω and h), Eqs. (27) and (30) are solved by using "interval halving" iteration technique [34]. The results in these cases are presented in the Figures (1-9) to illustrate the effects of the primary magnetic field on the longitudinal vibrations of an orthotropic circular cylinder.

It is clear from Figure 1. The first mode of dimensionless frequency Ω decreases as the ratio thickness h increases for ($H_o = 10^5, 10^6$ Oersted). However, it increases monotonically as function of h for the value of primary magnetic field increases ($H_o = 10^7$ Oersted). The same behavior is observed for the case of ($H_o = 10^5, 10^6, 10^7$ Oersted) and it is shown in Figures 2 and 3 for the second and third modes of dimensionless frequency Ω. Also, in this case the effects of the primary magnetic field when ($H_o = 10^5, 10^6$ Oersted) are very small and the curves are almost identical. In Figure 4 a comparison between the first three modes of the frequency Ω versus different values of h for $H_o = 10^7$ is illustrated. Furthermore, our numerical calculations show that all the mode of the frequency Ω is not sensitive to the primary magnetic field H_o less than 10^5 Oersted. So, for the values of H_o less than 10^5, it can be neglected as their relative variations become less than 10^3.

It is clarified that when $m = 0$, the frequency equation (27) degenerates into two independent equations: (i) One of them is for uncoupled radial vibrations (which contains the radial displacement u only). (ii) The second shows axial shear vibrations (which contains the axial displacement w only). The first, second and third modes of the dimensionless frequency Ω as function of the h of radial vibrations for various values of $H_o = (1, 5, 10)10^6$ are presented in Figures 5, 6 and 7 respectively. Furthermore, in the same case, a comparison between the first three modes of the frequency Ω as a function of h when $H_o = 5\times10^6$ is shown in Figure 8. It is visible that in this case all modes are increase when increasing the imposed magnetic field H_o. Figure 9, represents the first three modes of dimensionless frequency Ω of axial shear vibrations against the variation of h when $m=0$. It was found that in this second special case, the frequency Ω of axial shear vibrations is not affected with the values of the primary magnetic field H_o. Finally, some existing results in the literature are considered as the special case of this study, for example Refs. [6, 7, 8, 10, 15, 16].

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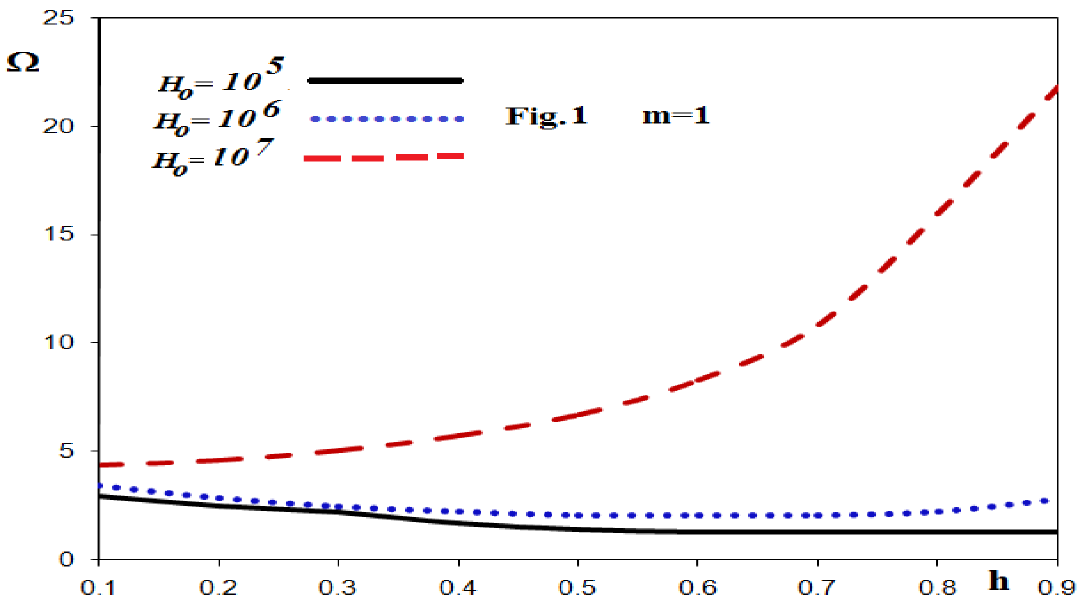

Figure 1. The first mode of dimensionless frequency Ω for longitudinal vibrations versus different values of h=a/b for different values of H_o, when *m=1.*

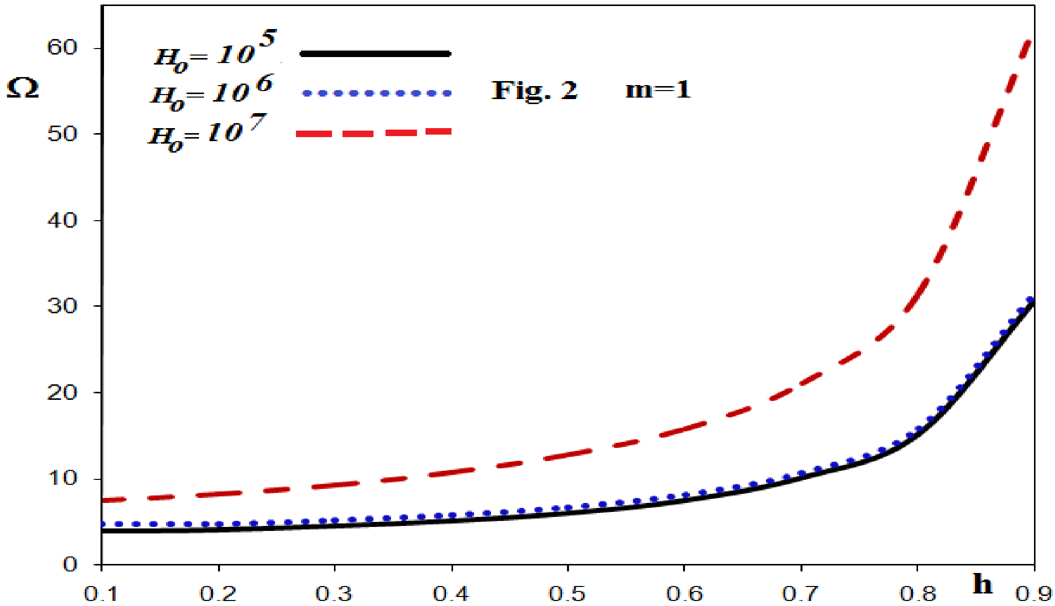

Figure 2. The second mode of dimensionless frequency Ω for longitudinal vibrations versus different values of h=a/b for different values of H_o, when *m=1.*

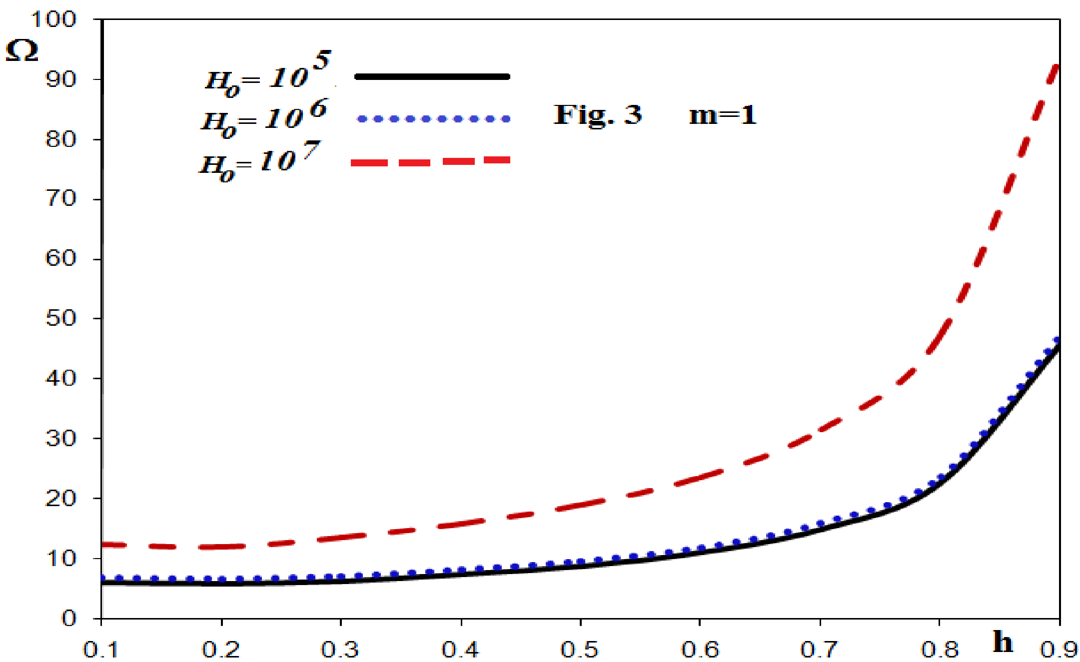

Figure 3. The second mode of dimensionless frequency Ω for longitudinal vibrations versus different values of h=a/b for different values of H_o, when *m=1.*

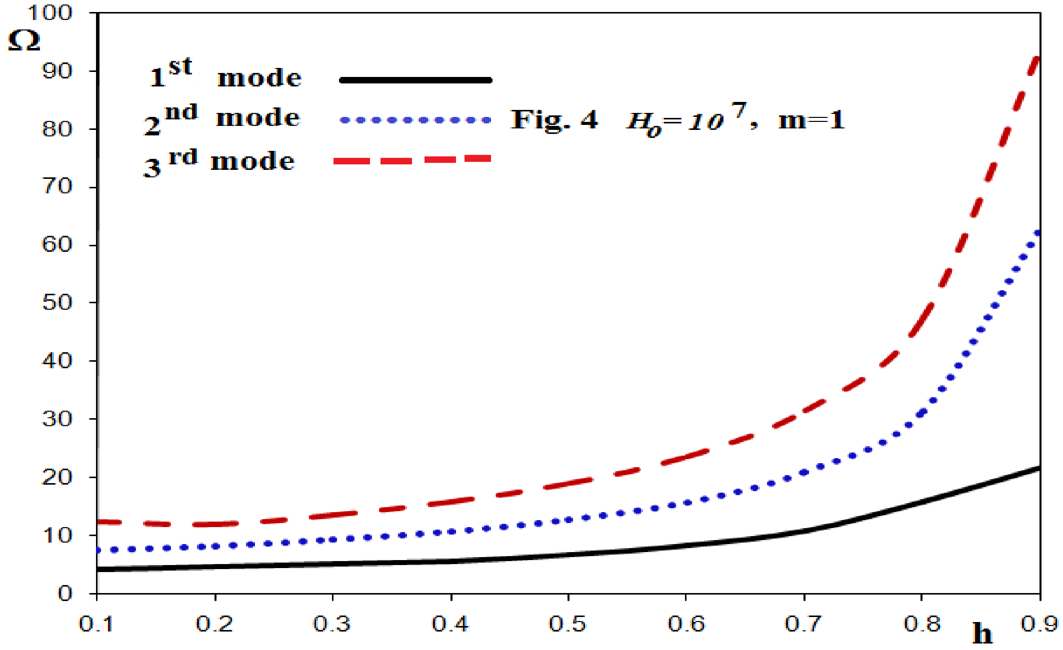

Figure 4. The first three modes of dimensionless frequency Ω of longitudinal vibrations versus different values of h=a/b for $H_o = 10^7$, when *m=1.*

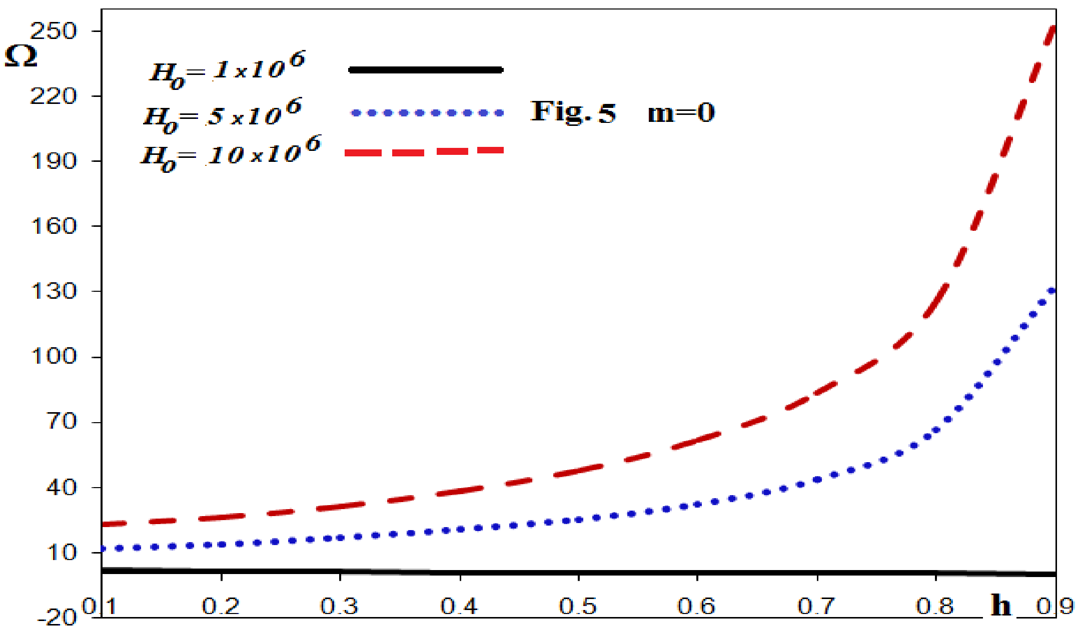

Figure 5. The first mode of dimensionless frequency Ω of radial vibrations versus different values of h=a/b for different values of H_o, when *m=0.*

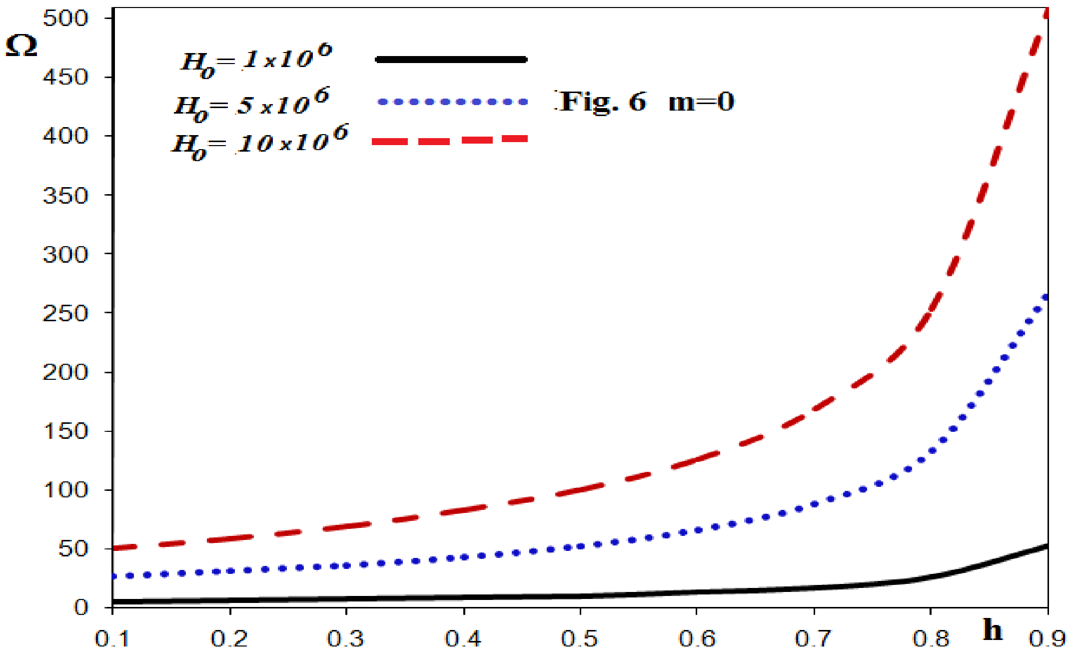

Figure 6. The second mode of dimensionless frequency Ω of radial vibrations

versus different values of h=a/b for different values of H_o, when *m=0.*

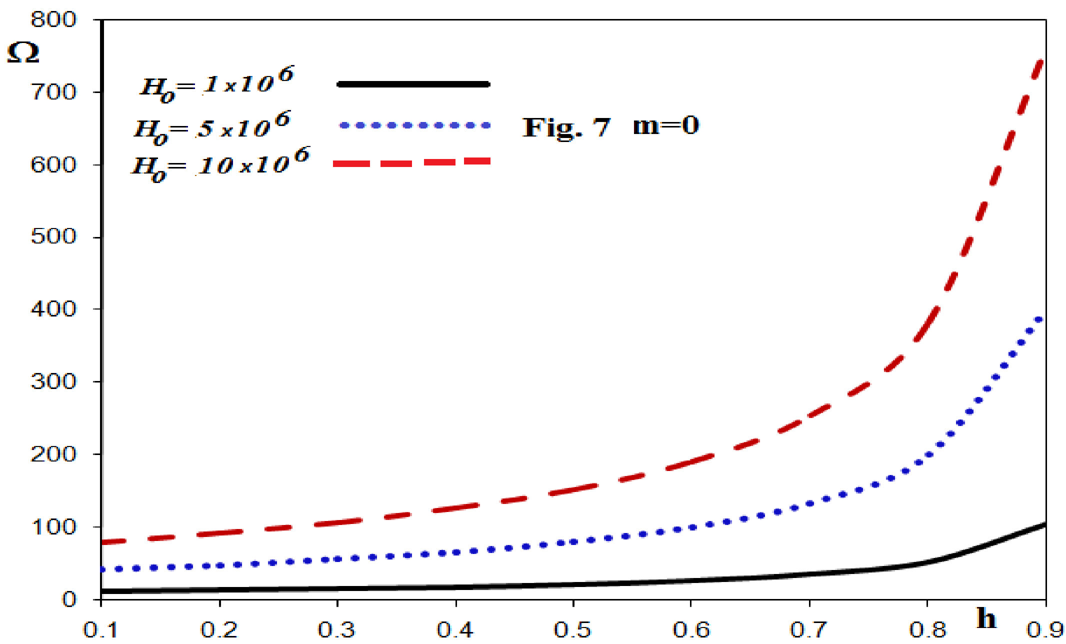

Figure 7. The third mode of dimensionless frequency Ω of radial vibrations

versus different values of h=a/b for different values of H_o, when *m=0.*

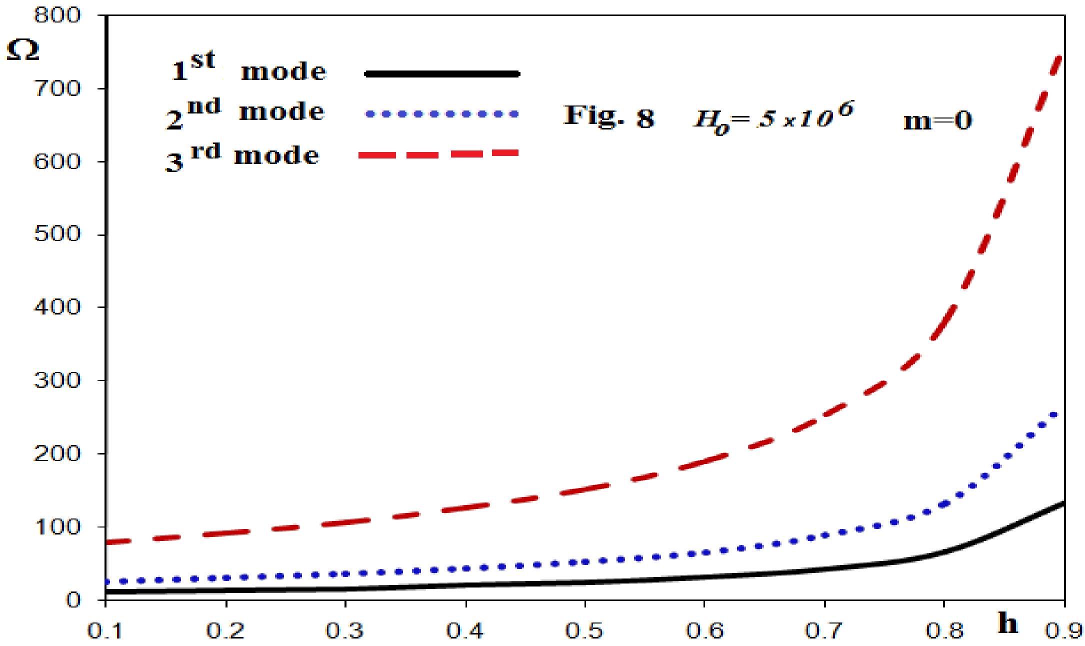

Figure 8. The first three modes of dimensionless frequency Ω of radial vibrations versus different values of h=a/b, for $H_o = 5\times10^6$, when *m=0.*

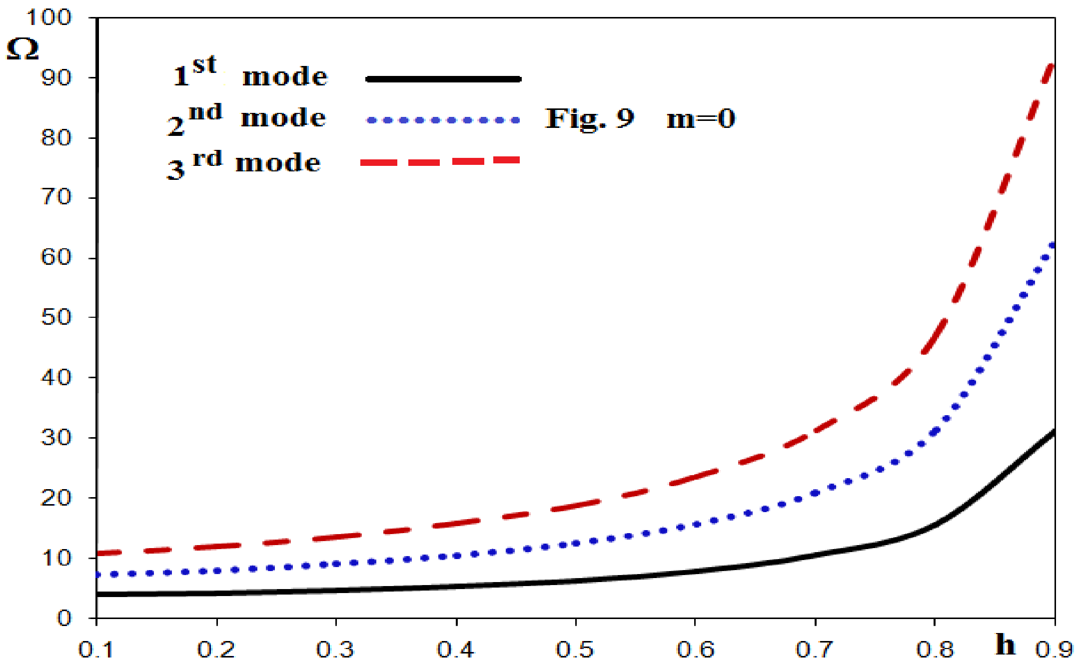

Figure 9. The first three modes of dimensionless frequency Ω of axial shear vibrations versus different values of h=a/b, when *m=0.*