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On a Qualitative Comparison of the Proposed Randomized Response Technique with Hussain and Shabbir (2007)

F.B. Adebola[1] & A.O. Adepetun[2*]
Department of Mathematical Sciences, Federal University of Technology ,PMB 704,Akure, Ondo State, Nigeria.
Email: akinolaoladiran@yahoo.com

Abstract
Randomized Response Techniques (RRTs) were developed for the purpose of improving response rate by protecting surveyee's privacy and avoiding answer bias.In this paper, we succinctly investigated the trend of the response rate (π) with both the proposed Randomized Response Technique (RRT) and the conventional one. We found that the proposed (RRT) is better than the conventional one at each response rate since it gives smaller variance.

Keywords: Response rate, Randomized Response Techniques (RRTs), error variance, unpleasant character.

1.Introduction

Obtaining information directly about a unpleasant character such as shoplifting, cheating in an examination, tax evasion etc. in a human population survey is a difficult activity. A survey may receive untruthfully answers from the survey respondents when he/she uses direct questioning approach. Because of many reasons, information about incidence of unpleasant characters in the population is necessary. Warner (1965) proposed the initial randomized-response techniques which is an effective survey method to find such estimates while at the same time protecting individual's anonymity. To date, a large number of developments and variants of Warner's Randomized Response Technique (RRT) have been put forward by several researchers. Greenberg et al. (1969), Mangat and Singh (1990), Mangat (1994), Christofides (2003), Kim and Warde (2004), Adebola and Adepetun (2011) are some of the many to be noted. In the next section, we present conventional technique, Proposed Randomized Response Technique and subsequently its efficiency over the conventional one in terms of the response rate and the variances respectively.

2. The Conventional Technique

Hussain and Shabbir (2007) proposed a Randomized Response Technique (RRT) based on the random use of one of the two randomization devices R_1 and R_2. In design, the two randomization devices R_1 and R_2 are the same as that of Warner's (1965) device but with different probabilities of selecting the sensitive question. The idea behind this suggestion is to decrease the suspicion among the respondents by providing them choice to randomly choose the randomization device itself. As a result, respondents may divulge their true status. A simple random sample with replacement (SRSWR) sampling is assumed to select a sample of size n. Let $\propto$ and β be any two positive real numbers chosen such that $q = \frac{\alpha}{\propto+\beta}, (\alpha \neq \beta)$ is the probability of using R_1, where R_1 consists of the two statements of Warner's device but with preset probabilities P_1 and $1 - P_1$ and $1 - q = \frac{\beta}{\propto+\beta}$ is the probability of using R_2 ,where R_2 consists of the two statements of Warner's device also with preset probabilities P_2 and $1 - P_2$ respectively. For the i^{th} respondent, the probability of a "yes" response is given by

$$P(yes) = \emptyset = \frac{\alpha}{\alpha+\beta}[P_1\pi + (1 - P_1)(1 - \pi)] + \frac{\beta}{\alpha+\beta}[P_2\pi + (1 - P_2)(1 - \pi)] \qquad (2.1)$$

To provide the equal privacy protection in both the randomization devices R_1 and R_2, we put $P_1 = 1 - P_2$ into equation (2.1), obtained:

$$\emptyset = \frac{\pi[(\alpha-\beta)(2P_1-1)] + P_1\beta + P_2\alpha}{\alpha+\beta} \tag{2.2}$$

Hence,

$$\pi = \frac{\emptyset(\alpha+\beta) - P_1\beta - P_2\alpha}{(2P_1-1)(\alpha-\beta)}, P_1 \neq {}^1/_2, \alpha \neq \beta \tag{2.3}$$

The unbiased moment estimator of true probability of yes response (response rate) π was given by

$$\hat{\pi} = \frac{\hat{\emptyset}(\alpha+\beta) - P_1\beta - P_2\alpha}{(2P_1-1)(\alpha-\beta)} \tag{2.4}$$

Where $\hat{\emptyset} = \frac{y}{n}$ and y is the number of respondents reporting a "yes" answer when $P_1 = 1 - P_2$. The variance of the estimator was given then by

$$V(\hat{\pi})_{conv} = \frac{\pi(1-\pi)}{n} + \frac{(P_2\alpha + P_1\beta)(P_1\alpha + P_2\beta)}{n(2P_1-1)^2(\alpha-\beta)^2(\alpha+\beta)^2} \tag{2.5}$$

3.The Proposed Technique

It was quite obvious that despite the successful attempts by several authors in developing an efficient Randomized Response Techniques (RRTs), the developed techniques only considered a two-option of "yes" and "no" response. As a result of which we propose a new Randomized Response Technique (RRT) that will be based on the random use of one of the three randomization devices, R_1, R_2 and R_3. In design, the three randomization devices R_1, R_2 and R_3 are similar to that of Warner's device but with different probabilities of selection. In addition to α and β proposed earlier by Hussain and Shabbir, we introduce δ, a positive real number such that $q = \frac{\alpha}{\alpha+\beta+\delta}, \alpha \neq \beta \neq \delta$ is the probability of using R_1, where R_1 consists of the two statements of Warner's device and the new introduce device also with preset probabilities P_1, P_2 and P_3 respectively. By adopting Hussain and Shabbir's probability of a "yes" response for the i^{th} respondent, the probability of a "yes" response when the third option "undecided" is included is given by

$$Q(yes) = \varphi = \frac{\alpha}{\alpha+\beta+\delta}[P_1\pi + (1-P_1)(1-\pi)] + \frac{\beta}{\alpha+\beta+\delta}[P_2\pi + (1-P_2)(1-\pi)] + \frac{\delta}{\alpha+\beta+\delta}[P_3\pi + (1-P_3)(1-\pi)] \tag{3.1}$$

In order to provide the equal privacy protection in the three randomization devices R_1, R_2, and R_3, we put $P_1 = 1 - P_2 - P_3$ into equation (3.1), obtained:

$$\pi = \frac{\varphi(\alpha+\beta+\delta) - [(\alpha+\beta+\delta) - P_1\alpha - P_2\beta - P_3\delta]}{2P_1\alpha + 2P_2\beta + 2P_3\delta - \alpha - \beta - \delta} \tag{3.2}$$

Hence, the unbiased sample estimate of π is given as

$$\hat{\pi} = \frac{\hat{\varphi}(\alpha+\beta+\delta) - [(\alpha+\beta+\delta) - P_1\alpha - P_2\beta - P_3\delta]}{2P_1\alpha + 2P_2\beta + 2P_3\delta - \alpha - \beta - \delta} \tag{3.3}$$

Where $\hat{\varphi} = \frac{x}{n}$ and x is the number of respondents reporting a "yes" answer when $P_1 = 1 - P_2 - P_3$. The variance of the estimator is given then by

$$V(\hat{\pi}) = \frac{\pi(1-\pi)}{n} + \frac{(P_1\alpha + P_2\beta + P_3\delta)(P_3\alpha + P_2\beta + P_1\delta)}{n[2P_1(\alpha-\delta) + 2P_2(\beta-\delta) - (\alpha+\beta-\delta)]^2(\alpha+\beta+\delta)^2} \tag{3.4}$$

4.Proposed Technique versus Conventional Technique

We show that the new RRT is better than the existing ones by comparing it with the conventional one at varying response rate keeping the sample size constant. Hence, the proposed tripartite Randomized Response Technique (RRT) will be better than the conventional Randomized Response Technique (RRT) if we have

$$V(\hat{\pi})_{conv} - V(\hat{\pi})_{prop} > 0 \tag{4.1}$$

Or if

$$\frac{\pi(1-\pi)}{n} + \frac{(P_2\alpha+P_1\beta)(P_1\alpha+P_2\beta)}{n(2P_1-1)^2(\alpha-\beta)^2(\alpha+\beta)^2} - \frac{\pi(1-\pi)}{n} - \frac{(P_1\alpha+P_2\beta+P_3\delta)(P_3\alpha+P_2\beta+P_1\delta)}{n[2P_1(\alpha-\delta)+2P_2(\beta-\delta)-(\alpha+\beta-\delta)]^2(\alpha+\beta+\delta)^2} > 0 \tag{4.2}$$

Or if

$$\frac{(P_2\alpha+P_1\beta)(P_1\alpha+P_2\beta)}{n(2P_1-1)^2(\alpha-\beta)^2(\alpha+\beta)^2} - \frac{(P_1\alpha+P_2\beta+P_3\delta)(P_3\alpha+P_2\beta+P_1\delta)}{n[2P_1(\alpha-\delta)+2P_2(\beta-\delta)-(\alpha+\beta-\delta)]^2(\alpha+\beta+\delta)^2} > 0 \tag{4.3}$$

The condition given in (4.3) is true, for P_1, P_2, P_3 and π ranging from 0.1 to 0.9 if α and β, α and δ, β and δ differ from each other by at least 9 where α, β, and δ are any suitable positive real numbers.

Table 4.1: Comparison between conventional RRT and proposed RRT when $P_1 = 0.6, P_2 = 0.3, P_3 = 0.1$, $\alpha = 20, \beta = 11, \delta = 2$, n=50, for varying of response rate (π)

π	P_1	P_2	P_3	α	β	Δ	Conventional Variance	Proposed Variance
0.1	0.6	0.3	0.1	20	11	2	0.00304	0.00226
0.2	0.6	0.3	0.1	20	11	2	0.00444	0.00366
0.3	0.6	0.3	0.1	20	11	2	0.00544	0.00466
0.4	0.6	0.3	0.1	20	11	2	0.00604	0.00526
0.5	0.6	0.3	0.1	20	11	2	0.00624	0.00546

Table 4.2: Comparison between conventional RRT and proposed RRT when $P_1 = 0.4, P_2 = 0.4, P_3 = 0.2$, α 20, $\beta = 11, \delta = 2$, n=50, for varying of response rate (π)

π	P_1	P_2	P_3	α	β	δ	Conventional Variance	Proposed Variance
0.1	0.4	0.4	0.2	20	11	2	0.00279	0.00184
0.2	0.4	0.4	0.2	20	11	2	0.00419	0.00324
0.3	0.4	0.4	0.2	20	11	2	0.00519	0.00424
0.4	0.4	0.4	0.2	20	11	2	0.00579	0.00484
0.5	0.4	0.4	0.2	20	11	2	0.00599	0.00504

Table 4.3: Comparison between conventional RRT and proposed RRT when $P_1 = 0.2, P_2 = 0.5, P_3 = 0.3, \alpha = 20, \beta = 11, \delta = 2$, *n=50, for varying of response rate (π)*

π	P_1	P_2	P_3	α	β	δ	Conventional Variance	Proposed Variance
0.1	0.2	0.5	0.3	20	11	2	0.00188	0.00181
0.2	0.2	0.5	0.3	20	11	2	0.00328	0.00321
0.3	0.2	0.5	0.3	20	11	2	0.00428	0.00421
0.4	0.2	0.5	0.3	20	11	2	0.00488	0.00481
0.5	0.2	0.5	0.3	20	11	2	0.00508	0.00501

Table 4.4: Comparison between conventional RRT and proposed RRT when $P_1 = 0.15, P_2 = 0.6, P_3 = 0.25, \alpha = 20, \beta = 11, \delta = 2$, *n=50, for varying of response rate (π)*

π	P_1	P_2	P_3	α	β	δ	Conventional Variance	Proposed Variance
0.1	0.15	0.6	0.25	20	11	2	0.00187	0.00181
0.2	0.15	0.6	0.25	20	11	2	0.00327	0.00321
0.3	0.15	0.6	0.25	20	11	2	0.00427	0.00421
0.4	0.15	0.6	0.25	20	11	2	0.00487	0.00481
0.5	0.15	0.6	0.25	20	11	2	0.00507	0.00501

Figure 4.1: Graph showing comparison between conventional RRT and proposed RRT when P_1=0.6,P_2=0.3,P_3=0.1, α=20,β=11,δ=2, n=50, for varying of response rate (π)

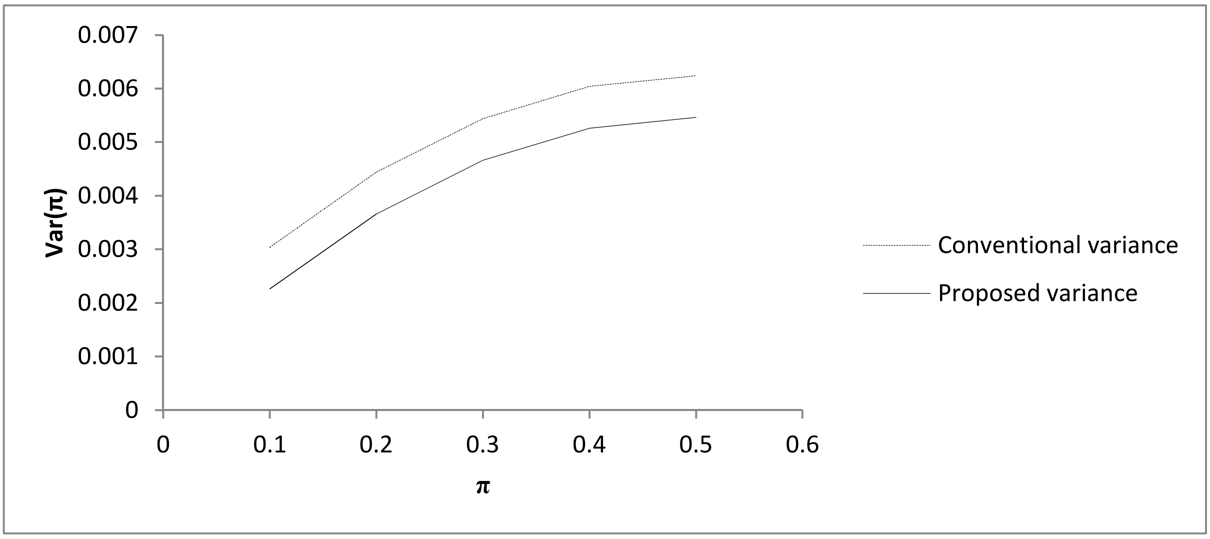


Figure 4.2: Graph showing comparison between conventional RRT and proposed RRT when $P_1 = 0.4, P_2 = 0.4, P_3 = 0.2$, $\alpha = 20, \beta = 11, \delta = 2$, n=50, for varying of response rate (π)

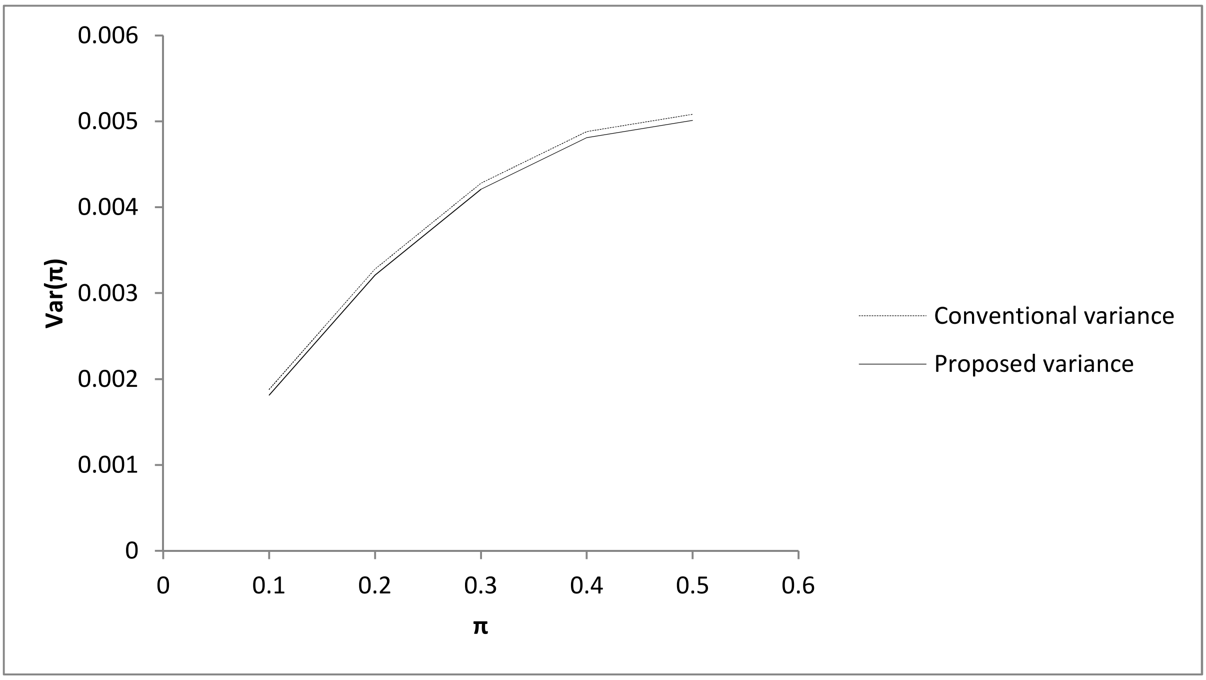


Figure 4.3: Graph showing comparison between conventional RRT and proposed RRT when $P_1 = 0.2, P_2 = 0.5, P_3 = 0.3$, $\alpha = 20, \beta = 11, \delta = 2$, n=50, for varying of response rate (π)

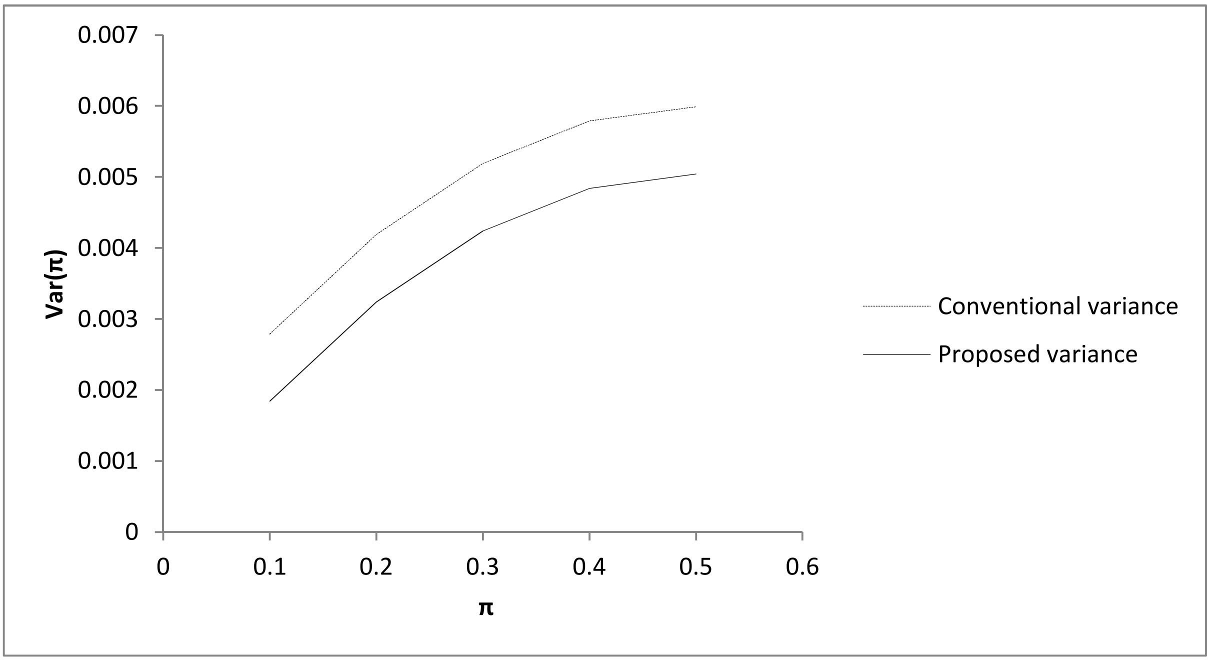
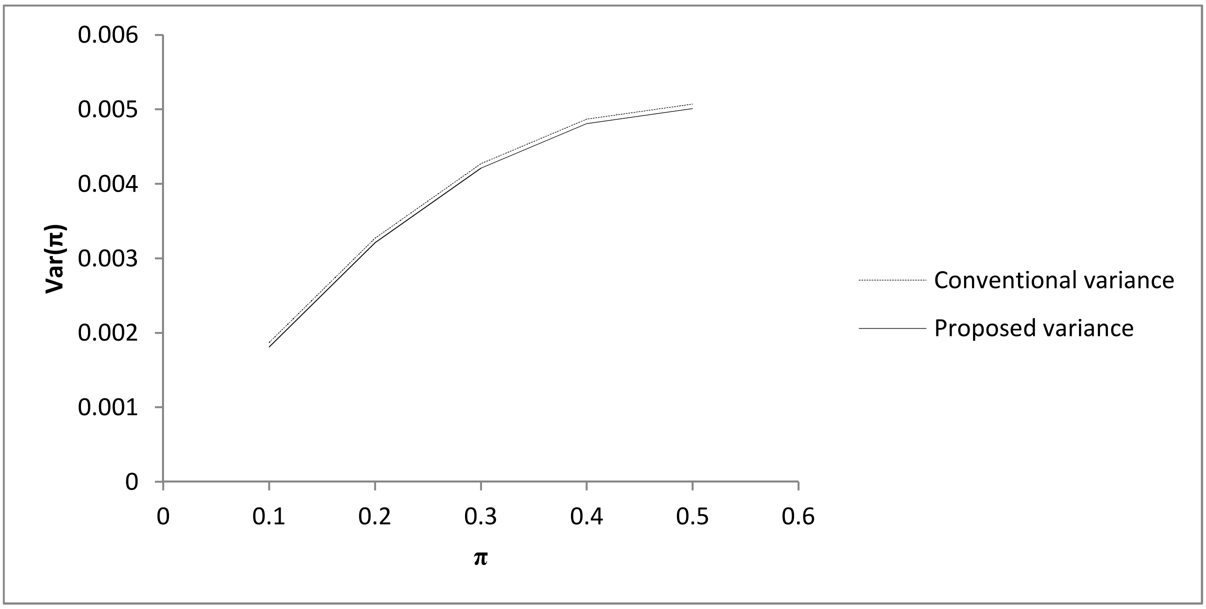


Figure 4.4: Graph showing comparison between conventional RRT and proposed RRT when $P_1 = 0.15, P_2 = 0.6, P_3 = 0.25$, $\alpha = 20, \beta = 11, \delta = 2$, n=50, for varying of response rate (π)



5.Conclusion

In this study, we reviewed the conventional Randomized Response Technique (RRT).

The efficiency of our proposed Randomized Response Technique over that of the conventional one was also verified when both techniques were compared using the response rate approach. It was obvious in the results on Tables and Figures 4.1,4.2,4.3 and 4.4 above that the proposed technique is indeed better than the conventional one.

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