

H-Function and a problem related to a String

S. N. Singh

Head, Department of Mathematics Jamtara College
 Jamtara (Jharkhand)
 email : singhsn@live.in

Raj Mehta

Dept . of Mathematics,
 Guru Ramdas Khalsa Institute of Science & Technology,
 Barela. Distt. Jabalpur (M.P.) 483001, INDIA
 email : raj.2512@rediffmail.com, raj.251264@gmail.com

Abstract

The aim of this paper is to obtain the solution of a problem related to a String with the help of H-function of one variable.

Keywords: Motion, Velocity, Transverse displacement

1. Introduction:

The H-function of one variable [3, p.10] is defined as:

$$H_{p, q}^{m, n} [x] \left(\begin{matrix} (a_j, \alpha_j)_{1, p} \\ (b_j, \beta_j)_{1, q} \end{matrix} \right) = (1/2\pi i) \int_L \theta(s) x^s ds \quad (1.1)$$

where $i = \sqrt{-1}$,

$$\theta(s) = \frac{\prod_{j=1}^m \Gamma(b_j - \beta_j s) \prod_{j=1}^n \Gamma(1 - a_j + \alpha_j s)}{q \prod_{j=1}^q \Gamma(1 - b_j + \beta_j s) p \prod_{j=1}^p \Gamma(a_j - \alpha_j s)}$$

where

$$\sum_{j=1}^n \alpha_j - \sum_{j=n+1}^p \alpha_j + \sum_{j=1}^m \beta_j - \sum_{j=m+1}^q \beta_j \equiv M > 0,$$

and $|\arg x| < \frac{1}{2} M\pi$.

In this paper, we shall make application of following modified form of the integral [2, p.372]:

$$\int_0^\pi (\sin x)^{\omega-1} \sin nx \, dx = \frac{\pi \sin \frac{1}{2} n\pi \Gamma(\omega)}{2^{\omega-1} \Gamma\{\frac{1}{2}(\omega+n+1)\} \Gamma\{\frac{1}{2}(\omega-n+1)\}} \quad (1.2)$$

$\text{Re}(\omega) > 0$.

2. Integral:

The integral to be established here is

$$\int_0^\pi (\sin x)^{\omega-1} \sin nx \, H_{p, q}^{m, l} [z (\sin x)^\lambda] \left(\begin{matrix} (a_j, \alpha_j)_{1, p} \\ (b_j, \beta_j)_{1, q} \end{matrix} \right) dx$$

$$= 2^{1-\omega} \pi \sin \frac{1}{2} n\pi H_{p+1, q+2}^{m, l+1} \left[z 2^{-\lambda} \left(\begin{matrix} (1-\omega, \lambda), (a_j, \alpha_j)_{1, p} \\ (b_j, \beta_j)_{1, q}, (1/2 - \omega/2 \pm n/2, \lambda/2) \end{matrix} \right) \right], \quad (2.1)$$

where

$$\sum_{j=1}^l \alpha_j - \sum_{j=l+1}^p \alpha_j + \sum_{j=1}^m \beta_j - \sum_{j=m+1}^q \beta_j \equiv M > 0,$$

$|\arg z| < \frac{1}{2} M\pi$, $\lambda \geq 0$ and $\text{Re}(\omega) > 0$.

Proof:

Replace the H- function by its equivalent contour integral as given in (1.1), change the order of integration, evaluate the inner integral with the help of (1.2) and finally interpret it with (1.1), to get (2.1).

3. Problem related to String:

In this section, we consider a string, stretched between the point 0 and π on the x-axis and initially at rest, is released from the position $y = f(x)$. Air resistance opposes its motion, which is proportional to the velocity at each point. Let the unit of time be chosen so that the equation of motion becomes

$$y_{tt}(x, t) = y_{xx}(x, t) - 2\beta y_t(x, t), \quad (3.1)$$

where β is a positive constant. Assuming that $0 < \beta < 1$, solution of (3.1) is given by [1, p.119]:

$$y(x, t) = \exp(-\beta t) \sum_{n=1}^{\infty} b_n [\cos \alpha_n t + (\beta/\alpha_n) \sin \alpha_n t] \sin nx, \quad (3.2)$$

where

$$\alpha_n = \sqrt{(n^2 - \beta^2)}$$

$$b_n = (2/\pi) \int_0^{\pi} f(x) \sin nx \, dx, \quad n = 1, 2, \dots \quad (3.3)$$

for the transverse displacement.

Now choose

$$f(x) = (\sin x)^{\omega-1} H_{p, q}^{m, l} [z (\sin x)^{\lambda} |_{(b_j, \beta_j)_{1, q}}^{(a_j, \alpha_j)_{1, p}}] \quad (3.4)$$

4. Solution of the Problem:

Combining (3.4) and (3.3) and making the use of the integral (2.1), we derive

$$b_n = 2^{2-\omega} \sin \frac{1}{2} n\pi H_{p+1, q+2}^{m, l+1} [z 2^{-\lambda} |_{(b_j, \beta_j)_{1, q}, (1/2 - \omega/2 \pm n/2, \lambda/2)}^{(1-\omega, \lambda), (a_j, \alpha_j)_{1, p}}], \quad (4.1)$$

Putting the value of b_n from (4.1) in (3.2), we get the following required solution of the problem:

$$y(x, t) = 2^{2-\omega} \exp(-\beta t) \sum_{n=1}^{\infty} \left\{ \sin \frac{1}{2} n\pi [\cos \alpha_n t + (\beta/\alpha_n) \sin \alpha_n t] \sin nx, \times \right. \\ \left. \times H_{p+1, q+2}^{m, l+1} [z 2^{-\lambda} |_{(b_j, \beta_j)_{1, q}, (1/2 - \omega/2 \pm n/2, \lambda/2)}^{(1-\omega, \lambda), (a_j, \alpha_j)_{1, p}}] \right\} \quad (4.2)$$

Provided the condition stated with (2.1) are satisfied.

References

1. Churchill, R.V.: Fourier Series and Boundary Value Problems, McGraw-Hill, New York (1988).
2. Gradshteyn, I. S. and Ryzhik, I. M.: Tables of Integrals, Series and Products, Academic Press, Inc. New York, 1980.
3. Srivastava, H. M., Gupta, K. C. and Goyal, S. P.: The H-function of one and two variables with applications, South Assian Publishers, New Delhi, 1982.

This academic article was published by The International Institute for Science, Technology and Education (IISTE). The IISTE is a pioneer in the Open Access Publishing service based in the U.S. and Europe. The aim of the institute is Accelerating Global Knowledge Sharing.

More information about the publisher can be found in the IISTE's homepage:

<http://www.iiste.org>

The IISTE is currently hosting more than 30 peer-reviewed academic journals and collaborating with academic institutions around the world. **Prospective authors of IISTE journals can find the submission instruction on the following page:**

<http://www.iiste.org/Journals/>

The IISTE editorial team promises to review and publish all the qualified submissions in a fast manner. All the journals articles are available online to the readers all over the world without financial, legal, or technical barriers other than those inseparable from gaining access to the internet itself. Printed version of the journals is also available upon request of readers and authors.

IISTE Knowledge Sharing Partners

EBSCO, Index Copernicus, Ulrich's Periodicals Directory, JournalTOCS, PKP Open Archives Harvester, Bielefeld Academic Search Engine, Elektronische Zeitschriftenbibliothek EZB, Open J-Gate, OCLC WorldCat, Universe Digital Library, NewJour, Google Scholar

