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Heat Transfer in Viscous Free Convective Fluctuating MHD Flow Through Porous Media Past a Vertical Porous Plate with Variable Temperature

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Abstract

In this paper, free convective Magnetohydrodynamics (MHD) flow of a viscous incompressible and electrically conducting fluid past a hot vertical porous plate embedded in a porous medium has been studied. The temperature of the plate varies both in space and time. The main objective of this paper is to study the effect of porosity of the medium coupled with the variation of plate temperature with regards to space and time. The effect of pertinent parameters characterizing the flow has been presented through the graph. The most interesting finding is that presence of porous media has no significant contribution to the flow characteristics. Further, heating and cooling of the plate due to convective current is compensated by the viscous dissipation.

Keywords: MHD flow/Span wise Co-sinusoidal/Free Convection /Heat transfer, porous medium.

1. Introduction

Many Industrial applications use Magnetohyrodynamics (MHD) effects to resolve the complex problems very often occurred in industries. The available hydrodynamics solutions include the effects of magnetic field which is possible as because the most of the industrial fluids are electrically conducting. For example, liquid metal MHD takes its root in hydrodynamics of incompressible media which gains importance in the metallurgical industry, Nuclear reactor, sodium cooling system ,every storage and electrical power generation[1974-76].Free convective flows are of great interest in a number of industrial applications such as fiber and granular insulation, geothermal system etc. Buoyancy is also of importance in an environment where difference between land and air temperature can give rise to complicated flow patterns. The unsteady free convection flow past an infinite porous plate and semi-infinite plate were studied by Nanda and Sharma [1962] .In their first paper they assumed the suction velocity at the plate varying in time as $t^{-1/2}$,where as in the second paper the plate temperature was assumed to oscillate in time about a constant non-zero mean. Free convective flow past a vertical plate has been studied extensively by Ostrach [1953] and many others. The free convective heat transfer on vertical semi-infinite plate was investigated by Berezovsky [1977]. Martynenko et al. [1984] investigated the laminar free convection from a vertical plate.

The basic equations of incompressible MHD flow are non-linear. But there are many interesting cases where the equations became linear in terms of the unknown quantities and may be solved easily. Linear MHD problems are accessible to exact solutions and adopt the approximations that the density and transport properties be constant. No fluid is incompressible but all may be treated as such whenever the pressure changes are small in comparison with the bulk modulus. Mention may be made to the works in [1975, 72]. Ferdows *et al.* [2004] analysed free convection flow with variable suction in presence of thermal radiation. Alam *et al*. [2006] studied Dufour and Soret effect with variable suction on unsteady MHD free convection flow along a porous plate. Majumdar et al. [2007] gave an exact solution for MHD flow past an impulsively started infinite vertical plate in the presence of thermal radiation. Muthucumaraswamy et al. [2009] studied unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion. Recently, Dash et al[2009] have studied free convective MHD flow of a visco-elastic fluid past an infinite vertical porous plate in a rotating frame of reference in the presence of chemical reaction. In the present study we have set the flow through porous media with uniform porous matrix with suction and blowing at the plate surface besides the free convective MHD effects and fluctuating surface temperature.

From the established result it is clear that the suction prevents the imposed non-torsional oscillations spreading away from the oscillating surface (disk) by viscous diffusion for all values of frequency of oscillations. On the contrary the blowing promotes the spreading of the oscillations far away from the disk and hence the boundary layer tends to be infinitely thick when the disk is forced to oscillate with resonant frequency. In other words, in case of blowing and resonance the oscillatory boundary layer flows are no longer possible.

Therefore, in the present study it aims at finding a meaningful solution for a non-linear coupled equation to bring out the effects of suction/blowing with varying span- wise co-sinusoidal time dependent temperature in the presence of uniform porous matrix in a free convective Magnetohydrodynamic flow past a vertical porous plate.

2. Formulation of the problem

An unsteady flow of a viscous incompressible electrically conducting fluid through a porous medium past an insulated, infinite, hot, porous plate lying vertically on the $x^* - z^*$ plane is considered. The x^*-axis is oriented in the direction of the buoyancy force and y^*-axis is taken perpendicular to the plane of the plate. A uniform magnetic field of strength B_0 is applied along the y^*-axis. Let (u^*, v^*, w^*) be the component of velocity in the direction (x^*, y^*, z^*) respectively. The plate being considered infinite in x^* direction, hence all the physical quantities are independent of x^*. Thus, following Acharya and Padhi [1983], ω^* is independent of z^* and the equation of continuity gives $v^* = -V$ (constant) throughout.

We assume the span wise co-sinusoidal temperature of the form

$$T = T^* + \varepsilon(T_0^* - T_\infty^*)\cos(\pi z^* / l - \omega^* t^*),$$

(2.1)

The mean temperature T^* of the plate is supplemented by the secondary temperature $\varepsilon(T_0^* - T_\infty^*)\cos(\pi z^* / l - \omega^* t^*)$ varying with space and time. Under the usual Boussineques approximation the free convective flow through porous media is governed by the following equations:

$$u_t^* + v^* u_y^* = \upsilon(u_{yy}^* + u_{zz}^*) + g\beta(T^* - T_\infty^*) - \sigma B_0^2 u^* / \rho - \upsilon u^* / K_P{}^*, \tag{2.2}$$

$$T_t^* + v^* T_y{}^* = \frac{K}{\rho C_P}(T_{yy}{}^* + T_{zz}{}^*) + \frac{\mu}{\rho C_P}((u_y{}^*)^2 + (u_z{}^*)^2) \tag{2.3}$$

The boundary conditions are given by

$$y^* = 0 \quad : \quad u^* = 0, v^* = V(Const.), T^* = T_0{}^* + \varepsilon(T_0{}^* - T_\infty{}^*)\cos(\pi z^* / l - \omega^* t^*) \tag{2.4}$$

$$y^* \to \infty \quad : u^* \to 0, T^* \to T_\infty{}^*.$$

Introducing the non dimensional quantities defined in the nomenclature, we get,

$$\frac{\omega}{\mathrm{Re}} u_t - u_y = \frac{1}{\mathrm{Re}}(u_{yy} + u_{zz}) + Gr\theta - \frac{1}{\mathrm{Re}}(M^2 + \frac{1}{K_P})u, \tag{2.5}$$

$$\frac{\omega}{\mathrm{Re}} \theta_t - \theta_y = \frac{1}{\mathrm{Pr}\,\mathrm{Re}}(\theta_{yy} + \theta_{zz}) + \mathrm{Re}\, Ec(u_y{}^2 + u_z{}^2), \tag{2.6}$$

with corresponding boundary conditions:

$$y=0: u=0, \theta=1+\varepsilon\cos(\pi z-t), \quad y\to\infty: u\to 0, \theta\to 0. \tag{2.7}$$

Since the amplitude, $\varepsilon(<<1)$, of the plate temperature is very small, we represent the velocity and temperature in the neighborhood of the plate as

$$u(y,z,t)=u_0(y)+\varepsilon u_1(y,z,t)+o(\varepsilon^2), \quad \theta(y,z,t)=\theta_0(y)+\varepsilon\theta_1(y,z,t)+o(\varepsilon^2), \tag{2.8}$$

Comparing the coefficient of like powers of ε after substituting (2.8) in (2.5) and (2.6), we get the following zeroth order equations:

$$u_{0yy}+\mathrm{Re}\,u_{0y}-(M^2+\frac{1}{K_P})u_0=-\mathrm{Re}\,Gr\theta_0, \tag{2.9}$$

$$\theta_{0yy}+\Pr\mathrm{Re}\,\theta_{0y}=-\Pr\mathrm{Re}^2\,Ecu_{0y}{}^2 \tag{2.10}$$

For solving the above coupled equations we use the following perturbation equations with perturbation parameter Ec, the Eckert number,

$$\left.\begin{aligned} u_0&=u_{01}+Ecu_{02}+o(Ec^2)\\ \theta_0&=\theta_{01}+Ec\theta_{02}+o(Ec^2)\end{aligned}\right\} \tag{2.11}$$

Substituting (2.11) into (2.9) and (2.10) we get the following zeroth and first order equations of Ec.

$$u_{01}''+\mathrm{Re}\,u_0'-(M^2+\frac{1}{K_P})u_{01}=-\mathrm{Re}\,Gr\theta_{01}, \tag{2.12}$$

$$\theta_{01}''+\Pr\mathrm{Re}\,\theta_{01}'=0, \tag{2.13}$$

$$u_{02}''+\mathrm{Re}\,u_{02}'-(M^2+\frac{1}{K_P})u_{02}=-\mathrm{Re}\,Gr\theta_{02}, \tag{2.14}$$

$$\theta_{02}''+\Pr\mathrm{Re}\,\theta_{02}'=-\Pr\mathrm{Re}^2\,u_{01}'{}^2, \tag{2.15}$$

The corresponding boundary conditions are:

$$y=0; u_{01}=0, \theta_{01}=1, u_{02}=0, \theta_{02}=0, \quad y\to\infty; u_{01}=0, \theta_{01}=0, u_{02}=0, \theta_{02}=0. \tag{2.16}$$

The solution of the equations (2.12) to (2.15) under the boundary conditions (2.16) are

$$u_{01}(y)=C_3(e^{-\Pr\mathrm{Re}\,y}-e^{-m_1 y}), \tag{2.17}$$

$$\theta_{01}(y) = e^{-\Pr \mathrm{Re}\, y}, \tag{2.18}$$

$$u_{02}(y) = C_4 e^{-m_1 y} + A_7 e^{-\Pr \mathrm{Re}\, y} + A_8 e^{-2\Pr \mathrm{Re}\, y} + A_9 e^{-2m_1 y} + A_{10} e^{-(\Pr \mathrm{Re} + m_1)y}, \tag{2.19}$$

$$\theta_{02}(y) = A_3 e^{-\Pr \mathrm{Re}\, y} + A_4 e^{-2\Pr \mathrm{Re}\, y} + A_5 e^{-2m_1 y} + A_6 e^{-(\Pr \mathrm{Re} + m_1)y}, \tag{2.20}$$

The terms of the coefficient of ε give the following first order equations:

$$\omega u_{1t} - \mathrm{Re}\, u_{1y} = (u_{1yy} + u_{1zz}) + Gr \,\mathrm{Re}\, \theta_1 - (M^2 + \frac{1}{K_P}) u_1, \tag{2.21}$$

$$\omega \theta_{1t} - \mathrm{Re}\, \theta_{1y} = \frac{1}{\Pr}(\theta_{1yy} + \theta_{1zz}) + \mathrm{Re}^2 u_{0y} u_{1y}. \tag{2.22}$$

In order to solve (2.21) and (2.22),it is convenient to adopt complex notations for velocity and temperature profile as,

$$u_1(y,z,t) = \phi(y) e^{i(\pi z - t)}. \tag{2.23}$$

$$\theta_1(y,z,t) = \phi(y) e^{i(\pi z - t)}.$$

The solutions obtained in terms of complex notations, the real part of which have physical significance.

Now, substituting (2.23) into (2.21) and (2.22) we get the following coupled equations:

$$\phi''(y) + \mathrm{Re}\, \phi'(y) + [\omega i - \pi^2 - (M^2 + \frac{1}{K_P})]\phi = -\mathrm{Re}\, Gr \psi(y), \tag{2.24}$$

$$\psi''(y) + \Pr \mathrm{Re}\, \psi'(y) + (\omega \Pr i - \pi^2)\psi = -2\,\mathrm{Re} \Pr u_{oy} \phi', \tag{2.25}$$

Again, to uncouple above equations we assume the following perturbed forms:

$$\begin{aligned} \phi &= \phi_0 + Ec\phi_1 + o(Ec^2), \\ \psi &= \psi_0 + Ec\psi_1 + o(Ec^2), \end{aligned} \tag{2.26}$$

Substituting (2.26) into (2.24) and (2.25) and equating the coefficient of like powers of Ec we get the subsequent equations:

$$\phi_0'' + \mathrm{Re}\, \phi_0' + [\omega i - \pi^2 - (M^2 + \frac{1}{K_P})]\phi_0 = -\mathrm{Re}\, Gr \psi_0, \tag{2.27}$$

$$\psi_0'' + \Pr \mathrm{Re}\, \psi_0' + (\omega \Pr i - \pi^2)\psi_0 = 0, \tag{2.28}$$

$$\phi_1'' + \mathrm{Re}\, \phi_1' + [\omega i - \pi^2 - (M^2 + \frac{1}{K})]\phi_1 = -\mathrm{Re}\, Gr \psi_1, \tag{2.29}$$

$$\psi_1'' + \Pr \mathrm{Re}\,\psi_1' + (\omega \Pr i - \pi^2)\psi_1 = -2\,\mathrm{Re}^2 \Pr u_{01}'\phi_0', \tag{2.30}$$

Where the prime denotes the differentiation with respect to y.
The corresponding boundary conditions are:

$$\begin{aligned} &y = 0; \phi_0 = 0, \phi_1 = 0, \psi_0 = 1, \psi_1 = 0, \\ &y \to \infty; \phi_0 = 0, \phi_1 = 0, \psi_0 = 0, \psi_1 = 0, \end{aligned} \tag{2.31}$$

The solutions of equations (2.27) to (2.30) under the boundary conditions (2.31) are:

$$\phi_0(y) = A_{11}(e^{-m_2 y} - e^{-m_3 y}), \tag{2.32}$$

$$\psi_0(y) = e^{-m_2 y}, \tag{2.33}$$

$$\psi_1(y) = A_{12}e^{-(\Pr \mathrm{Re}+m_2)y} + A_{13}e^{-(\Pr \mathrm{Re}+m_3)y} + A_{14}e^{-(m_1+m_2)y} + A_{15}e^{-(m_1+m_2)y} + A_{16}e^{-m_4 y}, \tag{2.34}$$

$$\phi_1(y) = A_{17}e^{-(\Pr \mathrm{Re}+m_2)y} + A_{18}e^{-(\Pr \mathrm{Re}+m_3)y} + A_{19}e^{-(m_1+m_2)y} + A_{20}e^{-(m_1+m_3)y} + A_{21}e^{-m_4 y} + A_{22}e^{-m_3 y}. \tag{2.35}$$

The important flow characteristics of the problem are the plate shear stress and the rate of heat transfer at the plate. The expressions for shear stress (τ) and Nusselt number (Nu) are given by

$$\tau = \tau^* l / \mu V = \mathrm{Re}\,al\frac{du}{dy} \quad \text{at } y = 0$$

$$\begin{aligned} &= C_3(-\Pr \mathrm{Re} + m_1) - Ec[C_4 m_1 + A_7 \Pr \mathrm{Re} + 2A_8 \Pr \mathrm{Re} + 2m_1 A_9 + A_{10}(\Pr \mathrm{Re} + m_1)] \\ &+\varepsilon|F|Cos(\pi z - t + \alpha) \end{aligned} \tag{2.36}$$

Where

$$F = F_r + iF_i$$

$$= A_{11}(m_3 - m_2) - Ec[(m_3 A_{22} + m_4 A_{21} + A_{17}(\Pr \mathrm{Re} + m_2) + A_{18}(\Pr \mathrm{Re} + m_3) + A_{19}(m_1 + m_2) + A_{20}(m_3 + m_1)]$$

The amplitude and the phase angle are given by

$$|F| = \sqrt{F_r^2 + F_i^2}\ , \quad \alpha = \tan^{-1}\frac{F_i}{F_r}$$

$$Nu = -\frac{q^* l}{k(T_0^* - T_\infty^*)} = \text{Real } \frac{d\theta}{dy} \quad \text{at } y = 0$$

$$\begin{aligned} &= -\Pr \mathrm{Re} - Ec[\Pr \mathrm{Re}(A_3 + 2A_4) + 2m_1 A_5 + A_6(\Pr \mathrm{Re} + m_1 + \beta)] \\ &-\varepsilon|G|Cos(\pi z - t + \beta) \end{aligned} \tag{2.37}$$

Where $G = G_r + iG_i$

$$= m_2 + Ec[(m_{14}A_{16} + A_{12}(\Pr\mathrm{Re} + m_2) + A_{13}(\Pr\mathrm{Re} + m_3) + A_{14}(m_1 + m_2) + A_{14}(m_1 + m_3)]$$

The amplitude and the phase angle are given by

$$|G| = \sqrt{G_r^2 + G_i^2}\,, \quad \beta = \tan^{-1}\frac{G_i}{G_r}.$$

3. Result and Discussions

This section analyses the velocity, temperature, and amplitude, phase angle of shear stress and rate of heat transfer. The present discussion brings the following cases as particular case.

- $M = 0$, represents the case of non-conducting fluid without magnetic field.
- $K_P \to \infty$, represents without porous medium.

The most important part of the discussion is due to the presence of the sinusoidal variation of surface temperature with space and time and the forcing forces such as Lorentz force, resistive force, due to porosity of the medium, thermal buoyancy and cross flow due to permeable surface.

From equation (2.5) the following results follows.

In the absence of cross flow, $V = 0 \Rightarrow \mathrm{Re}\left(=\frac{Vl}{\upsilon}\right) = 0$, the u component of velocity remains unaffected by convective acceleration and thermal buoyancy force and the equation (2.5) reduces to

$$u_t = \frac{1}{\omega}\left[\left(u_{yy} + u_{zz}\right) - \left(M^2 + \frac{1}{K_P}\right)u\right]$$

More over, the viscosity contributes significantly with a combined retarding effect caused by magnetic force and resistance due to porous medium with an inverse multiplicity of the frequency of the temperature function. Further, $M = 0, K_P \to \infty$, reduces the problem to a simple unsteady motion given by $u_t = \frac{1}{\omega}\left(u_{yy} + u_{zz}\right)$.

Thus, in the absence of cross flow, that is, in case of a non-permeable surface, frequency of the fluctuating temperature parameter $\omega = \left(\frac{\omega^* l^2}{\upsilon}\right)$ acts as a scaling factor.In case of impermeable surface, that is, in absence of cross flow, equation (6) reduces to $\theta_t = \frac{1}{\omega\Pr}\left(\theta_{yy} + \theta_{zz}\right)$.

This shows that the unsteady temperature gradient is reduced by high prandtle number fluid as well as increasing frequency of the fluctuating temperature and viscous dissipation fails to effect the temperature distribution.

From fig.1 it is observed that maximum velocity occurs in case of cooling of the plate $(Gr > 0)$ without magnetic field (curve IX) and reverse effect is observed exclusively due to heating of the plate $(Gr < 0)$ with a back flow (curve X). Thermal buoyancy effect has a significant contribution over the flow field.

From curves (VIII & IX), it is seen that the Lorentz force has a retarding effect which is in conformity with the earlier reported result [14].

In the present study, velocity decrease is 7 times and there by steady state is reached within a few layers of the flow domain. Further, it is seen that flow reversal occurs only in case of heating of the plate (curve X& XX) both in porous and non porous medium.

Moreover, it is interesting to note that increase in Pr leads to significant decrease of the velocity (curve I (air) and IV (water)) throughout the flow field but an increase in cross flow Reynolds number, increases the velocity in the vicinity of the plate. Afterwards, it decreases rapidly. From curves, I & VIII it is observed that an increase in viscous dissipation increases the velocity at all points. Thus, the energy loss sets a cooling current vis-à-vis accelerating the velocity to reach the steady state.

These observations are clearly indicated by the additive and subtractive terms in the equation discussed earlier.

Fig.2 exhibits asymptotically decreasing behavior of the temperature distribution. For higher Pr fluid (Pr =7.0), the temperature reduces drastically (curves I&IV).

It is interesting to remark that the cooling and heating of the plate (curves I & X) makes no difference in temperature distribution which is compensated due to increase in dissipative energy loss.

Fig.3 shows the variation of amplitude of shear stress in both the cases i.e. presence and absence of porous medium. Shear stress is almost linear for all values of frequency of fluctuation. For high Reynolds number as well as greater buoyancy force shearing stress increases (curves II, XII, VII & XVII).

From fig.4 a steady increasing behavior is marked with an increasing frequency parameter but when ω exceed 15.0 (approx.), slight decrease is marked. There is always a phase lead for all the parameters and for all values of ω. A decrease in phase angle is marked due to increase in Reynolds number and magnetic parameter but the reverse effect is observed in case of Prandtl number .Phase angle remains unaffected due to buoyancy effect, dissipative loss and presence of porous medium.

Fig.5 and fig.6 exhibits the variation of amplitude $|G|$ and phase angle $\tan\beta$ in case of heat transfer. Further, $|G|$ exhibits three layer characters due to high, medium and low value of Pr.

4. Conclusion

- Heating of the plate leads to back flow.
- Lorentz force has retarding effects.
- Presence of porous medium has no significant contribution.
- Viscous dissipation generates a cooling current which accelerates the velocity.
- There are three layer variation of amplitude of heat transfer with a phase lead.
- A phase lead is marked in case of shear stress and a phase lag for heat transfer which remains unaffected by magnetic parameter, Grashoff number and Eckert number

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Nomenclature:

(x^*, y^*, z^*)	Cartesian Co-ordinate system	V	Suction velocity
l	Wave length	ε	Amplitude of the span wise co-sinusoidal temperature
υ	Kinematics coefficient of viscosity	μ	Coefficient of viscosity
$\left.\begin{matrix} y = y^*/l \\ z = z^*/l \end{matrix}\right\}$	Dimensionless space variable	θ	Dimensionless temperature $(T^* - T_\infty^*)/(T_0^* - T_\infty^*)$
β	Coefficient of volumetric expansion	B_0	Uniform magnetic field strength
g	Acceleration due to gravity	β	Coefficient of volume approximation
T_0^*	Temperature of the plate	T_∞^*	ambient temperature

ρ	Density	C_P	Specific heat at constant pressure
K	Thermal conductivity	ω	Frequency of temperature fluctuation- $\omega^* l^2 / \upsilon$
Re	Reynolds number- Vl/υ	Pr	Prandtl number- $\mu C_P / K$
Gr	Grashoff number- $\upsilon g\beta(T_0^* - T_\infty^*)/V^3$	Ec	Eckert number- $V^2 / C_P(T_0^* - T_\infty^*)$
M	Hartmann number- $B_0 l \sqrt{\sigma/\mu}$	τ	Dimensionless skin friction
Nu	Nusselt number	K_P	Permeability of the medium

Appendix

$$m_1 = \frac{\mathrm{Re} + \sqrt{\mathrm{Re}^2 + 4(M^2 + \frac{1}{K_P})}}{2}, \quad m_2 = \frac{\mathrm{Pr}\,\mathrm{Re} + \sqrt{\mathrm{Pr}^2\,\mathrm{Re}^2 - 4(\mathrm{Pr}\, i\omega - \pi^2)}}{2}$$

$$m_3 = \frac{\mathrm{Re} + \sqrt{\mathrm{Re}^2 - 4(i\omega - \pi^2 - (M^2 + \frac{1}{K_P}))}}{2}, \quad m_4 = \frac{\mathrm{Pr}\,\mathrm{Re} + \sqrt{\mathrm{Pr}^2\,\mathrm{Re}^2 - 4(\mathrm{Pr}\, i\omega - \pi^2)}}{2}$$

$$C_3 = \frac{-\mathrm{Re}\, Gr}{\mathrm{Pr}^2\,\mathrm{Re}^2 - \mathrm{Pr}\,\mathrm{Re}^2 - (M^2 + \frac{1}{K_P})}, \quad A_3 = -\sum_{i=4}^{6} A_i, \quad A_4 = -\frac{\mathrm{Pr}\,\mathrm{Re}^2\, C_3^{\,2}}{2},$$

$$A_5 = -\frac{\mathrm{Pr}\,\mathrm{Re}^2\, C_3^{\,2} m_1^{\,2}}{4m_1^{\,2} - 2m_1^2 Pe\,\mathrm{Re}}, \quad A_6 = \frac{2\,\mathrm{Pr}^2\,\mathrm{Re}^3\, C_3^{\,2} m_1}{(\mathrm{Pr}\,\mathrm{Re} + m_1)^2 - 2m_1^2\,\mathrm{Pr}\,\mathrm{Re}}, \quad A_7 = \frac{-\mathrm{Re}\, Gr A_3}{\mathrm{Pr}^2\,\mathrm{Re}^2 - \mathrm{Pr}\,\mathrm{Re}^2 - (M^2 + \frac{1}{K_P})}$$

$$A_8 = \frac{-\mathrm{Re}\, Gr A_4}{4\,\mathrm{Pr}^2\,\mathrm{Re}^2 - 2\,\mathrm{Pr}\,\mathrm{Re}^2 - (M^2 + \frac{1}{K_P})}, \quad A_9 = \frac{-\mathrm{Re}\, Gr A_5}{4m_1^2\,\mathrm{Re}^2 - 2m_1\,\mathrm{Re} - (M^2 + \frac{1}{K_P})}$$

$$A_{10} = \frac{-\mathrm{Re}\, Gr A_6}{(\mathrm{Pr}\,\mathrm{Re} + m_1)^2 - \mathrm{Re}(\mathrm{Pr}\,\mathrm{Re} + m_1) - (M^2 + \frac{1}{K_P})}, \quad C_4 = -\sum_{i=7}^{10} A_i,$$

$$A_{11} = \frac{-\mathrm{Re}\,Gr}{m_2^2 - \mathrm{Re}\,m_2 + \left[i\omega - \pi^2 - M^2 + \frac{1}{K_P})\right]}, A_{12} = -\frac{2\,\mathrm{Re}^3\,Ec\,\mathrm{Pr}^2\,m_2 C_3 A_{11}}{(\mathrm{Pr}\,\mathrm{Re} + m_2)^2 - \mathrm{Pr}\,\mathrm{Re}(\mathrm{Pr}\,\mathrm{Re} + m_2) + (i\omega\,\mathrm{Pr} - \pi^2)}$$

$$A_{13} = \frac{2\,\mathrm{Re}^3\,Ec\,\mathrm{Pr}^2\,m_3 C_3 A_{11}}{(\mathrm{Pr}\,\mathrm{Re} + m_3)^2 - \mathrm{Pr}\,\mathrm{Re}(\mathrm{Pr}\,\mathrm{Re} + m_3) + (i\omega\,\mathrm{Pr} - \pi^2)}, A_{14} = \frac{2\,\mathrm{Re}^2\,Ec\,\mathrm{Pr}\,m_1 m_2 A_{11}}{(m_1 + m_2)^2 + \mathrm{Pr}\,\mathrm{Re}(m_1 + m_2) + (i\omega\,\mathrm{Pr} - \pi^2)}$$

$$A_{15} = -\frac{2\,\mathrm{Re}^2\,Ec\,\mathrm{Pr}\,m_1 m_3 A_{11}}{(m_1 + m_3)^2 + \mathrm{Pr}\,\mathrm{Re}(m_1 + m_3) + (i\omega\,\mathrm{Pr} - \pi^2)}, A_{16} = -\sum_{i=12}^{15} A_i$$

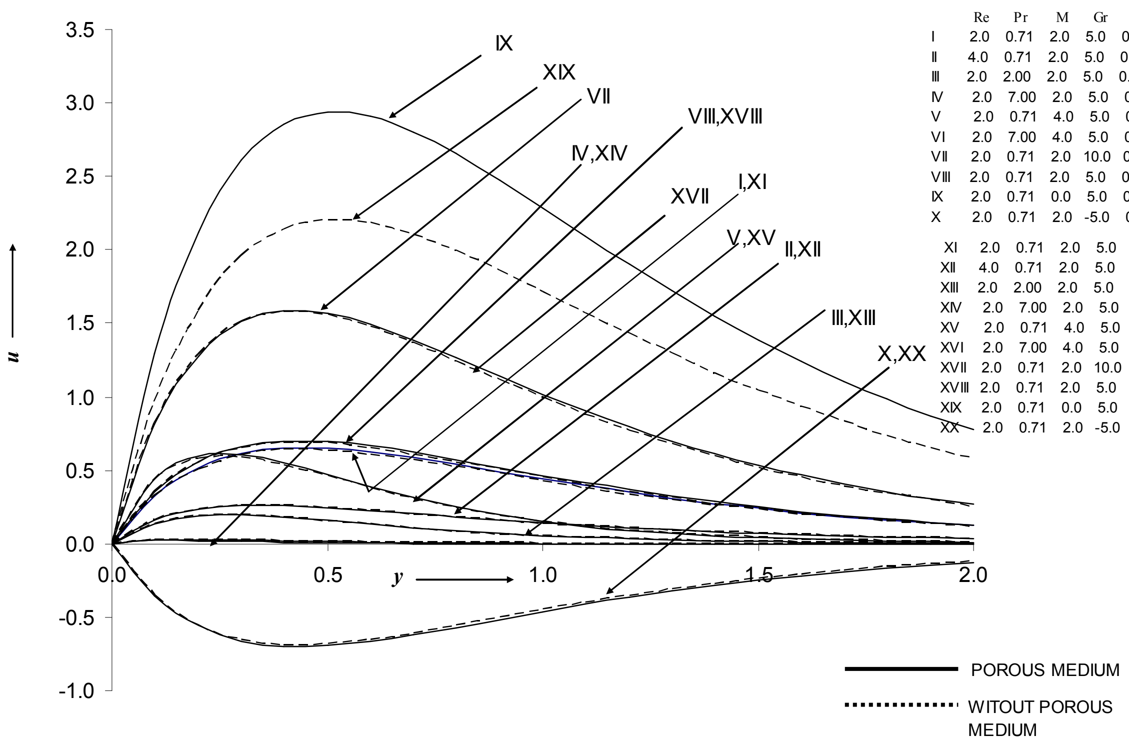
$$A_{17} = -\frac{\mathrm{Re}\,Gr A_{13}}{(\mathrm{Pr}\,\mathrm{Re} + m_3)^2 - \mathrm{Re}(\mathrm{Pr}\,\mathrm{Re} + m_3) + \left[i\omega - \pi^2 - (M^2 + \frac{1}{K_P})\right]}$$

$$A_{18} = -\frac{\mathrm{Re}\,Gr A_{12}}{(\mathrm{Pr}\,\mathrm{Re} + m_2)^2 - \mathrm{Re}(\mathrm{Pr}\,\mathrm{Re} + m_2) + \left[i\omega - \pi^2 - (M^2 + \frac{1}{K_P})\right]}$$

$$A_{19} = -\frac{\mathrm{Re}\,Gr A_{14}}{(m_1 + m_2)^2 - \mathrm{Re}(m_1 + m_2) + \left[i\omega - \pi^2 - (M^2 + \frac{1}{K_P})\right]}$$

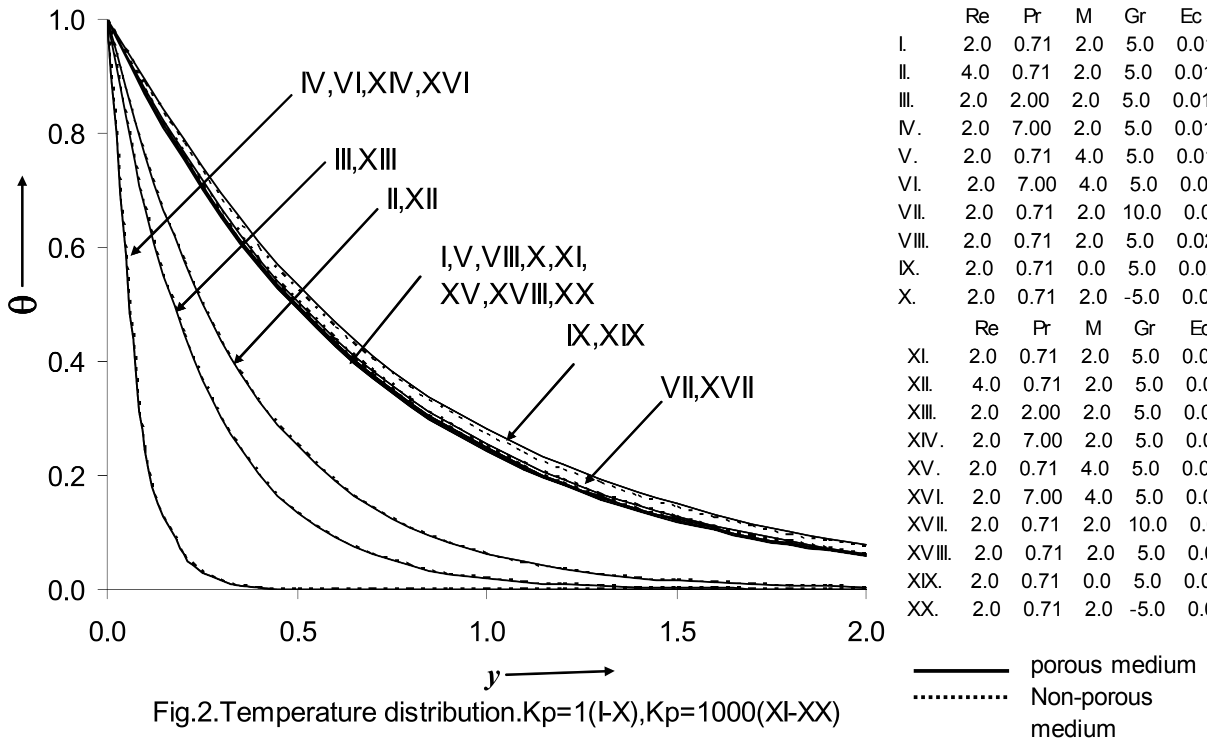
$$A_{20} = -\frac{\mathrm{Re}\,Gr A_{15}}{(m_1 + m_3)^2 - \mathrm{Re}(m_1 + m_3) + \left[i\omega - \pi^2 - (M^2 + \frac{1}{K_P})\right]}$$

$$A_{21} = -\frac{\mathrm{Re}\,Gr A_{16}}{{m_4}^2 - \mathrm{Re}\,m_4 + \left[i\omega - \pi^2 - (M^2 + \frac{1}{K_P})\right]} \quad , A_{22} = -\sum_{i=17}^{21} A_i$$



	Re	Pr	M	Gr	Ec
I	2.0	0.71	2.0	5.0	0.01
II	4.0	0.71	2.0	5.0	0.01
III	2.0	2.00	2.0	5.0	0.01
IV	2.0	7.00	2.0	5.0	0.01
V	2.0	0.71	4.0	5.0	0.01
VI	2.0	7.00	4.0	5.0	0.01
VII	2.0	0.71	2.0	10.0	0.01
VIII	2.0	0.71	2.0	5.0	0.02
IX	2.0	0.71	0.0	5.0	0.02
X	2.0	0.71	2.0	-5.0	0.02
XI	2.0	0.71	2.0	5.0	0.01
XII	4.0	0.71	2.0	5.0	0.01
XIII	2.0	2.00	2.0	5.0	0.01
XIV	2.0	7.00	2.0	5.0	0.01
XV	2.0	0.71	4.0	5.0	0.01
XVI	2.0	7.00	4.0	5.0	0.01
XVII	2.0	0.71	2.0	10.0	0.01
XVIII	2.0	0.71	2.0	5.0	0.02
XIX	2.0	0.71	0.0	5.0	0.02
XX	2.0	0.71	2.0	-5.0	0.02

Fig.1 Velocity disrtibution.Kp=1(I-X),Kp=1000(XI-XX)



	Re	Pr	M	Gr	Ec
I.	2.0	0.71	2.0	5.0	0.01
II.	4.0	0.71	2.0	5.0	0.01
III.	2.0	2.00	2.0	5.0	0.01
IV.	2.0	7.00	2.0	5.0	0.01
V.	2.0	0.71	4.0	5.0	0.01
VI.	2.0	7.00	4.0	5.0	0.01
VII.	2.0	0.71	2.0	10.0	0.01
VIII.	2.0	0.71	2.0	5.0	0.02
IX.	2.0	0.71	0.0	5.0	0.02
X.	2.0	0.71	2.0	-5.0	0.02
XI.	2.0	0.71	2.0	5.0	0.01
XII.	4.0	0.71	2.0	5.0	0.01
XIII.	2.0	2.00	2.0	5.0	0.01
XIV.	2.0	7.00	2.0	5.0	0.01
XV.	2.0	0.71	4.0	5.0	0.01
XVI.	2.0	7.00	4.0	5.0	0.01
XVII.	2.0	0.71	2.0	10.0	0.01
XVIII.	2.0	0.71	2.0	5.0	0.02
XIX.	2.0	0.71	0.0	5.0	0.02
XX.	2.0	0.71	2.0	-5.0	0.02

Fig.2.Temperature distribution.Kp=1(I-X),Kp=1000(XI-XX)

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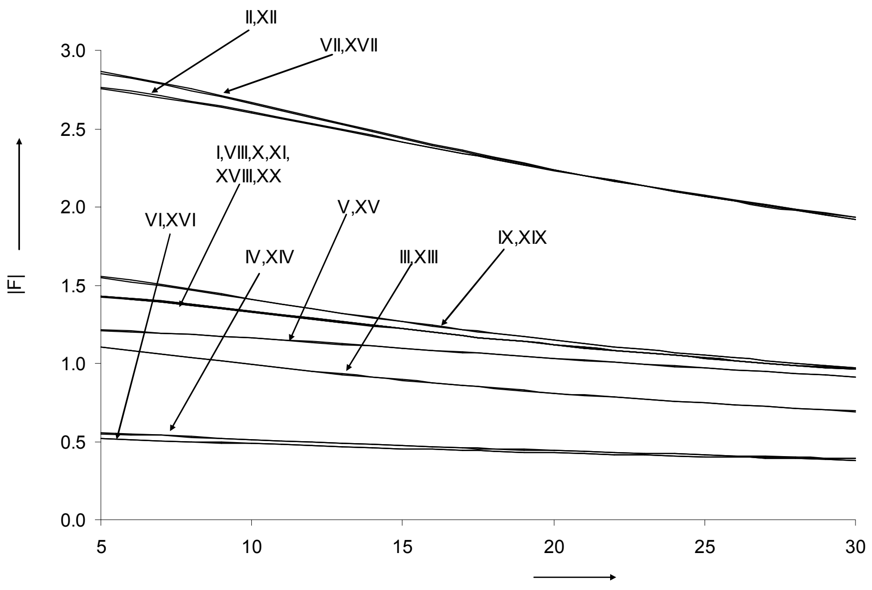
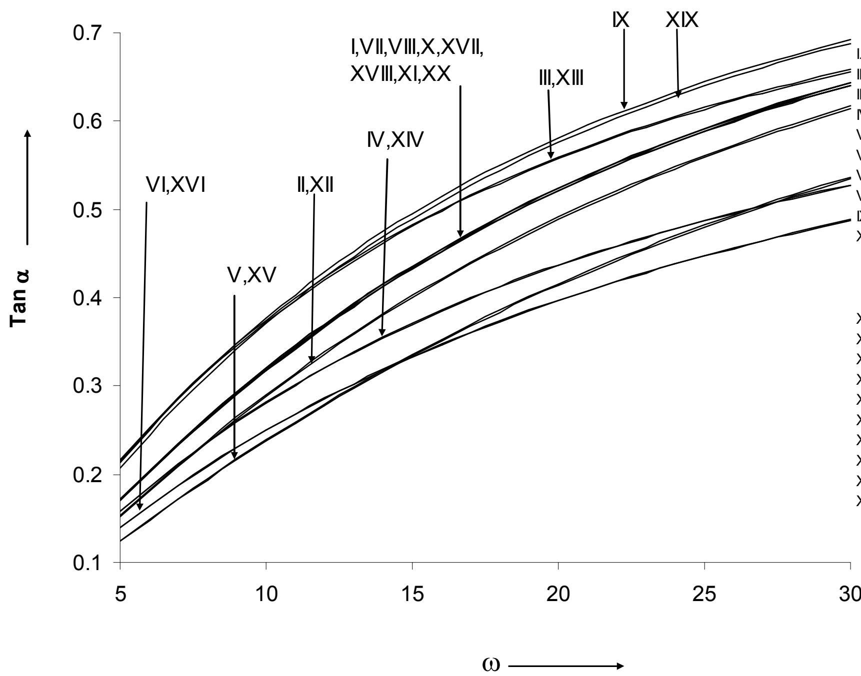


	Re	Pr	M	Gr	Ec
I.	2.0	0.71	2.0	5.0	0.01
II.	4.0	0.71	2.0	5.0	0.01
III.	2.0	2.00	2.0	5.0	0.01
IV.	2.0	7.00	2.0	5.0	0.01
V.	2.0	0.71	4.0	5.0	0.01
VI.	2.0	7.00	4.0	5.0	0.01
VII.	2.0	0.71	2.0	10.0	0.01
VIII.	2.0	0.71	2.0	5.0	0.02
IX.	2.0	0.71	0.0	5.0	0.02
X.	2.0	0.71	2.0	-5.0	0.02

	Re	Pr	M	Gr	Ec
XI.	2.0	0.71	2.0	5.0	0.01
XII.	4.0	0.71	2.0	5.0	0.01
XIII.	2.0	2.00	2.0	5.0	0.01
XIV.	2.0	7.00	2.0	5.0	0.01
XV.	2.0	0.71	4.0	5.0	0.01
XVI.	2.0	7.00	4.0	5.0	0.01
XVII.	2.0	0.71	2.0	10.0	0.01
XVIII.	2.0	0.71	2.0	5.0	0.02
XIX.	2.0	0.71	0.0	5.0	0.02
XX.	2.0	0.71	2.0	-5.0	0.02

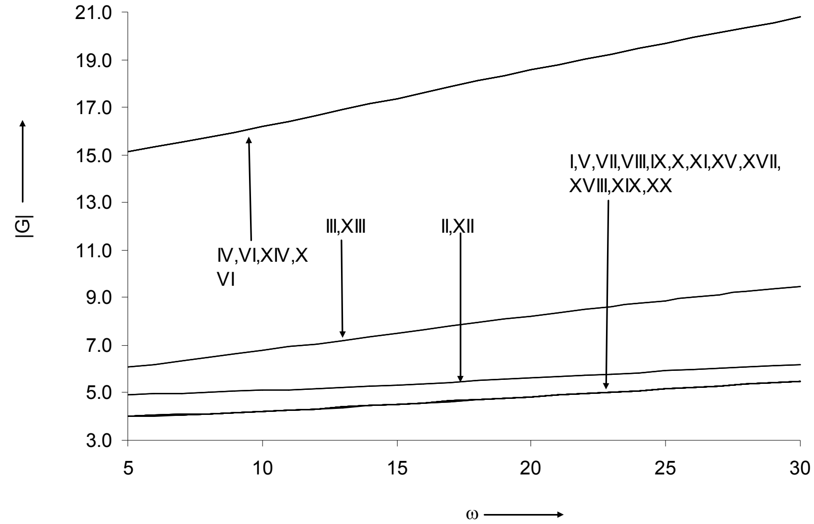
Fig. 3. The amplitude |F| of the shear stress.Kp=1(I-X),Kp=1000(XI-XX)



	Re	Pr	M	Gr	Ec
I.	2.0	0.71	2.0	5.0	0.01
II.	4.0	0.71	2.0	5.0	0.01
III.	2.0	2.00	2.0	5.0	0.01
IV.	2.0	7.00	2.0	5.0	0.01
V.	2.0	0.71	4.0	5.0	0.01
VI.	2.0	7.00	4.0	5.0	0.01
VII.	2.0	0.71	2.0	10.0	0.01
VIII.	2.0	0.71	2.0	5.0	0.02
IX.	2.0	0.71	0.0	5.0	0.02
X.	2.0	0.71	2.0	-5.0	0.02

	Re	Pr	M	Gr	Ec
XI.	2.0	0.71	2.0	5.0	0.01
XII.	4.0	0.71	2.0	5.0	0.01
XIII.	2.0	2.00	2.0	5.0	0.01
XIV.	2.0	7.00	2.0	5.0	0.01
XV.	2.0	0.71	4.0	5.0	0.01
XVI.	2.0	7.00	4.0	5.0	0.01
XVII.	2.0	0.71	2.0	10.0	0.01
XVIII.	2.0	0.71	2.0	5.0	0.02
XIX.	2.0	0.71	0.0	5.0	0.02
XX.	2.0	0.71	2.0	-5.0	0.02

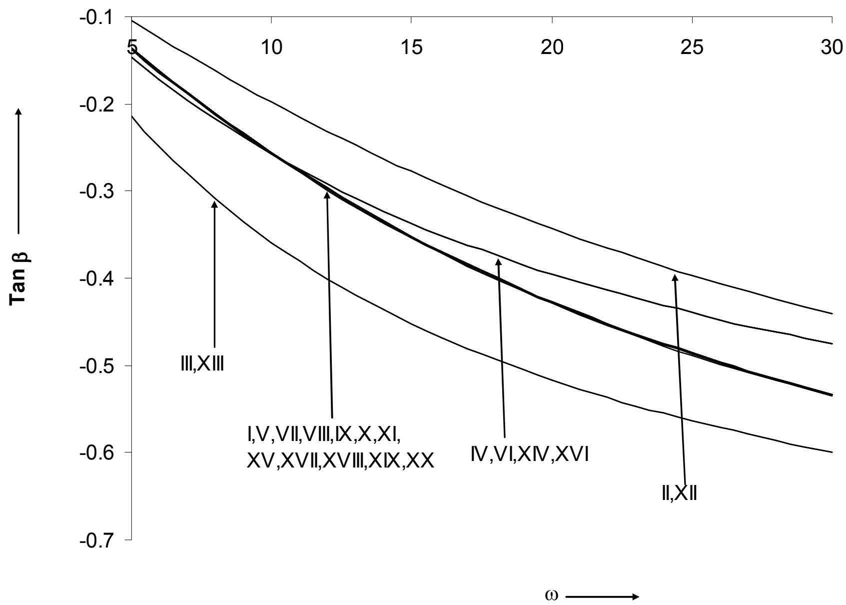
Fig. 4. The phase angle Tanα of shear stress,Kp=1(I-X),Kp=1000(XI-XX)



	Re	Pr	M	Gr	Ec
I.	2.0	0.71	2.0	5.0	0.01
II.	4.0	0.71	2.0	5.0	0.01
III.	2.0	2.00	2.0	5.0	0.01
IV.	2.0	7.00	2.0	5.0	0.01
V.	2.0	0.71	4.0	5.0	0.01
VI.	2.0	7.00	4.0	5.0	0.01
VII.	2.0	0.71	2.0	10.0	0.01
VIII.	2.0	0.71	2.0	5.0	0.02
IX.	2.0	0.71	0.0	5.0	0.02
X.	2.0	0.71	2.0	-5.0	0.02

	Re	Pr	M	Gr	Ec
XI.	2.0	0.71	2.0	5.0	0.01
XII.	4.0	0.71	2.0	5.0	0.01
XIII.	2.0	2.00	2.0	5.0	0.01
XIV.	2.0	7.00	2.0	5.0	0.01
XV.	2.0	0.71	4.0	5.0	0.01
XVI.	2.0	7.00	4.0	5.0	0.01
XVII.	2.0	0.71	2.0	10.0	0.01
XVIII.	2.0	0.71	2.0	5.0	0.02
XIX.	2.0	0.71	0.0	5.0	0.02
XX.	2.0	0.71	2.0	-5.0	0.02

Fig.5. The amplitude |G| of the ate of heat transfer.
Kp=1(I-X),Kp=1000(XI-XX)



	Re	Pr	M	Gr	Ec
I.	2.0	0.71	2.0	5.0	0.01
II.	4.0	0.71	2.0	5.0	0.01
III.	2.0	2.00	2.0	5.0	0.01
IV.	2.0	7.00	2.0	5.0	0.01
V.	2.0	0.71	4.0	5.0	0.01
VI.	2.0	7.00	4.0	5.0	0.01
VII.	2.0	0.71	2.0	10.0	0.01
VIII.	2.0	0.71	2.0	5.0	0.02
IX.	2.0	0.71	0.0	5.0	0.02
X.	2.0	0.71	2.0	-5.0	0.02

	Re	Pr	M	Gr	Ec
XI.	2.0	0.71	2.0	5.0	0.01
XII.	4.0	0.71	2.0	5.0	0.01
XIII.	2.0	2.00	2.0	5.0	0.01
XIV.	2.0	7.00	2.0	5.0	0.01
XV.	2.0	0.71	4.0	5.0	0.01
XVI.	2.0	7.00	4.0	5.0	0.01
XVII.	2.0	0.71	2.0	10.0	0.01
XVIII.	2.0	0.71	2.0	5.0	0.02
XIX.	2.0	0.71	0.0	5.0	0.02
XX.	2.0	0.71	2.0	-5.0	0.02

Fig.6.The phase angle Tanβ of the rate of heat transfer.
Kp=1(I-X),Kp=1000(XI-XX)