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## Rotary Brown Stock Pulp Washers through Mathematical Models – A Review

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### Abstract

Increasing the accuracy of a model generally increases cost and decreases flexibility. The goal in creating a model is usually to obtain a “Sufficiently accurate” and flexible model at a low cost. Pulp and Paper making process involves many complex unit operations and process. The process of pulping and paper making is intensive with respect to raw materials, energy, water and chemicals. If the plants are heat operated carefully it can also become pollution intensive. The key operation influencing the economy of the plant as well as controlling the pollution load is the washing process. Almost all the mills in the country use rotary vacuum washers for separating the black liquor from the pulp and for ensuring clean pulp to be available for downstream processing. The parameters which influence the operation of a brown stock washer are many and the phenomenon involved in the washing operation are complex. The present paper makes an attempt to look at the various parameters influencing the washing and look at various mathematical models used to predict the washer performance.

**Keywords:** accuracy, flexibility, low cost, pulp washing, black liquor.

### 1. Introduction

From a business perspective, it is clear that an improved ability to simulate, predict, or understand certain real-world systems through mathematical modeling provides a distinct competitive advantage. Furthermore, just as in pure science, as computing power becomes cheaper, modeling becomes an increasingly cost-effective alternative to direct experimentation. A washer yields weak black liquor and washer pulp. The black liquor solids which remain in the washed pulp are sources of biological oxygen demand (BOD) chemical oxygen demand (COD) absorbable organic halogens (AOX) etc. in effluent streams. Also poorly washed pulp increases the amount of dioxins and chlorinated furans during bleaching (Hise et al. [1990]).

According to Stromberg [1991] one kg of COD consumes about 0.4 to 0.8 kg of active chlorine. Cullinan [1991] has shown that soda loss (as Na<sub>2</sub>SO<sub>4</sub>) in a linear function of COD. According to Mccubbin et al. [1995] the average Canadian mill discharges nearly 2.0 kg AOX/ton of finished product. Estimated cost of removing the AOX was \$84/kg of AOX. AOX values of Indian mills using elemental chlorine for bleaching are much higher.

The present study is mainly intended to show the effect of different parameters on the brown stock washing. Different investigators have contradictory views regarding the effect of pH temperature, concentration of solids, consistency, and amount of wash water and velocity etc. on washing efficiency. Mathematical models connected with brown stock washing and some efficiency parameters are also given.

### 2. Moral Motive of Pulp Washing

In a rotary vacuum filter pulp is diluted in the vat with weak wash liquor and pulp consistency is lowered to an extent of around 1. Due to the phenomenon of diffusion the black liquor solids present within the fibers are removed. The phenomenon of adsorption desorption and dispersion also affects the washing operation.

Diffusion of black liquor solids out of fibers depends upon time allowed. Volume of shower water and its application, vat consistency rpm, channeling and foaming etc. Hakamaki et al. [1985] have shown that introduction of 5 by volume of dispersed air decreases brown stock washer capacity by approximately 30.

### 3. Technique and Equipments

A mathematical algorithm to solve some coupled equations is developed and implemented into a computer program using MATLAB. Rotary vacuum washer, digester washer, wash press, belt washer atmospheric diffusers and febrifuge washer etc. are used to wash the pulp. Stromberg [1994] has presented an overview of different washers regarding consistency, temperature range, retention time, displacement ration efficiency and COD. Rotary vacuum washer consisting of a battery manner is still very common in majority of industries in the world.

### 4. Models For Cake Washing Zone

The general equation used by different investigators to analyze the flow of liquor through the packed bed can be written as (Kukreja et al.[1995])

$$D_L \frac{\partial^2 c}{\partial z^2} = \frac{w \partial c}{\partial z} + \frac{\partial c}{\partial t} + \frac{\alpha \partial n}{\partial t} \quad (1)$$

Various forms of equation (1) can be obtained by putting different values for the ration a. For each equation two cases might arise by considering or neglecting the effect of longitudinal dispersion coefficient ( $D_L$ )

$$n = k^* c \quad (\text{Linear}) \quad (2)$$

$$\frac{\partial n}{\partial t} = k(c-n) \quad (\text{Finite rate}) \quad (3)$$

$$\frac{\partial n}{\partial t} = k_1 c - k_2 n \quad (\text{Finite rate}) \quad (4)$$

$$n = \frac{ABc}{1+Bc} \quad (\text{Langmuir}) \quad (5)$$

Equation (1) has been solved for the above adsorption desorption isotherms with different initial and boundary conditions, by Lapidus et al. [1952] Kuo (1960) Grahs et al., [1975] Perron et al. [1977] , Kukreja et al. [1995] and other investigators.

By using the microscopic analysis black liquor concentration of different stream can be found, which can be used to measure the performance of brown stock washing system as a whole.

### 5. Performance of Different Parameters

The parameters used to describe the performance of pulp washers can be divided into three categories

- Wash liquor usage parameters
- Solute removal parameters
- Efficiency parameters

#### 5.1 Wash liquor Usage parameters

During pulp washing operation, the amount of wash water added, is sent to the evaporator for further treatment. If more chemicals are used in this section, it will have perilous effect on environment whereas excess use of water will increase the load on evaporator. Therefore there needs to be a balance in the amount of wash water added and the impurities to be removed. Dilution factor, wash liquor ratio, weight

liquor ratio, filter entrainment and thickening factor are some commonly used parameters. For the sake of brevity dilution factor is explained here, remaining parameters are discussed in Arora et al. (2008).

The difference between wash liquor entering and wash liquor in the washed pulp is known as dilution factor or excess wash water.

$$DF = L_c - L_d \quad (6)$$

Dilution factor represents the net amount of water that is added during washing.  $DF = 0$ , implies that the black liquor in the pulp pad has been replaced by an equal amount of wash water. Negative DF means that the amount of wash water used is less than the amount of liquor leaving with washed pulp.

## 5.2 Solute Removal parameters

These parameters describe the amount of solids removed during a washing stage or washing operation and can be used to predict the amount of bleach chemical consumption. The values of these parameters increase when the wash liquor usage parameters are increased. Displacement ratio is the most prominent among such parameters and is discussed below.

Displacement ratio of any stage is defined as the ratio of actual reduction of dissolved solids to the maximum possible reduction of dissolved solids.

$$DR = (x_i - x_d) / (x_i - x_s) \quad (7)$$

DR value always lies between 0 to 1. When  $DR = 0$  it means that the actual reduction of dissolved solids is equal to the maximum possible reduction of dissolved solids, but this is an ideal stipulation and can not be met in the industry.

## 5.3 Efficiency parameters

Efficiency parameters constitute an important part of the study of pulp washing operation. Soda loss (salt cake loss) is a widely accepted method to check the efficiency of a washer. Percent efficiency is also used for this purpose by some industries. Norden and modified Norden efficiency factors are used to find out the number of stages to achieve the desired efficiency. Equivalent displacement ratio can be used to compare the efficiency of two different types of washers.

Percentage of black liquor solids removed during the washing operation is known as efficiency of the system. Kukreja et al (1995) have proposed an expression in terms of concentration and consistency by assuming that density of all streams leaving and entering the washer is same.

$$\%E = \left[ 1 - \frac{(c_d - c_s)(100 - c_{y,d})}{(c_i - c_s)(100 - c_{y,i})} \right] 100 \quad (8)$$

The efficiency factor of a washing system without side stream can be defined as the number of mixing stages in series with complete mixing of underflow and overflow required achieving the same departing underflow and overflowing as those of the washing system, when the entering flows of the mixing stage system are the same as those of the washing system. Mathematically Norden's efficiency factor (NEF) for a single stage can be written as,

$$E = \frac{\log \left[ \frac{L_i (x_i - x_f)}{L_d (x_d - x_f)} \right]}{\log (L_f / L_d)} \quad (9)$$

According to Oxby et al. [1986] NEF is highly sensitive to flow rate measurement errors. They have proposed an expression in terms of concentration and consistency measurements as follows,

$$E = \frac{\log \left[ \frac{L_i (x_i - x_f)}{L_d (x_d - x_f)} \right]}{\log \left( \frac{L_i (x_i - x_f) L_d (x_f - x_s)}{L_d (x_f - x_s) L_i (x_i - x_f)} \right)} \quad (10)$$

Phillips et al (1977) have developed Modified Norden's efficiency factor (MNEF) for a stage, as the number of ideal counter current mixing stage equivalent to a washing system operating at standard discharge consistency of 10 % or 12 % and at the same dilution factor.

$$E_{st} = \frac{\log \left[ \frac{L_1 (X_1 - X_2)}{L_2 (X_2 - X_3)} \right]}{\log [1 + (DF/L_{st})]} \quad (11)$$

Where,  $L_{st} = (100 - C_{yst}) / C_{yst}$

NEF and MNEF have the advantage of assigning an efficiency number to equipment. NEF and MNEF of the entire system can be found by adding the value of each individual stage.

## 6. Factor Affecting Rate of Solute Removal

Several factors which affects the rate of solute removal from pulp fibers during washing are kappa number, type of pulping (i.e. wood species, pulping methods, temperature, pH, concentration of solute and other cations in the wash liquor and time allowed for leaching.

### 6.1 Kappa Number and Type of Pulp

Kappa number is used to describe the relative hardness, the degree of delignification obtained in a chemical pulping process, the bleach ability or whitening optitude of a pulp. However, it should be noted that there is no general and unambiguous relationship between the Kappa number and the lignin content. Hartler N. and Rydin S. (1975) has studied the bisulphate pulp of spruce. Rosen (1975) has found that the level of sorbed sodium was higher in pine (softwood) pulps than the white oak (hardwood) pulp. Grahs (1976) has used pine sulphate pulps where as Xuan et al.(1978) used pine soda oxygen pulp and pine Kraft pulps for their experiments. Smith et al. (1993) have shown that the leachable lignin and Kappa number are linearly related.

### 6.2 Temperature

According to Loney et al. (2001) studied the effect of high output light-curing on temperature transfer through resin composite and dentin. There exists no discernible relationship between sorbed sodium and temperature in the range of 100-190 °F (Rosen 1975). Similarly Trinh et al. (1987) have found no relationship for Na between 30-50° C. Smith et al. (1993), for Douglas fir pulp, between 20-90°C. Have shown that relationship between rate of leaching of lignin and temperature is approximately linear.

### 6.3 pH

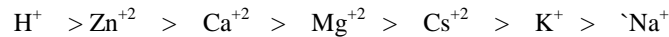
Liquids having different pH value were used as wash liquids. According to Potucek et al. (2002) the distilled water was acidified to pH value ranging from 2.1 to 6.2 with sulfuric acid, while pH value in alkaline region up to 10.9 were adjusted by the addition of sodium hydroxide. Hartler et al. (1975) when the pH value of Kraft pulp is lowered towards neutrality, dissociation of free phenclac groups in the Kraft lignin is reversed resulting in desorption of sodium. Rosen (1975) has found that there is a decrease of about 8lb of Na<sub>2</sub>SO<sub>4</sub>/AD ton (4.03 kg Na<sub>2</sub>SO<sub>4</sub>/OD ton) for each pH unit. This relationship was found to be independent of both Kappa number and pulp type. Trinh et al. (1987) have found that at low pH (3-6) sorbed sodium increases rapidly. Smith et al. (1993) have shown that high pH is required to remove a significant amount of lignin.

### 6.4 Concentration of Solutes

Rosen (1975) , Xuan et al. (1978) and Smith et al. (1993) have observed no effect of sodium ion concentration in the wash liquor on sorbed sodium, where as other investigators like,Hartler et al. (1975), Grahs (1976) and Trinh et al.(1987) have found that at low liquor concentration the sorbed sodium decreased rapidly towards zero . At high liquor concentration the sorbed sodium approaches an asymptotic value. This type of behavior can be well described by Langmuir equation,

$$S = ABc / (1+Bc) \quad (13)$$

Ohlsson et al. (1975) have given a series of cations that are known to have a greater affinity for cellulose than sodium,



## 6.5 Time of Leaching

Grahs (1976) has reported that rates of adsorption and the times for mass transport for lignin and sodium are different. Trinh et al. (1987) have shown that within 1 minute majority of sodium ions and lignin present in the liquor inside and outside the fiber walls is removed by the wash liquor. However, even after 2 days leaching of lignin from the continued. Cullinan (1991) has suggested that the difference in the leaching rate of lignin and sodium is due to their molecular mass.

## 7. FACTOR AFFECTING BROWN STOCK WASHING

Some of the key factor which affects the efficiency is cake thickness consistency velocity and amount of wash water used rpp pressure prop etc. specific surface area of fiber also affected the performance of a rotary washer.

### 7.1 Cake Thickness

Cake thickness is found to influence the bed porosity significantly. Gren et al. (1973) have a found that the sustenance yield in cieaa by increasing cake thickness (10 cm) Graphs (1976) observed same effect for the bed linings 0.102-0.166m, for pine and sprees pulps of kappa number 32. Gren et at. (1985) found that washing efficiency increase with bed length (10-90mm). Shower flow rate was 0.0127 cm/s. Trinh et al.(1989)for thickness>25mm and consistency>138 have shown the pressure drop across the pad becomes so high causing poor displacement washing . Han et al. (1988) have known that when mat thickness increases E factor also increase.

### 7.2 Consistency

Fiber consistency is found to decrease linearly with the cake thickness. Lee (1979) found that increasers in consistency is not beneficial for washing efficiency .Hakamaki et al. (1985) formed pads of different consistence (7-12%) and mat thickness 2 cm and found that NEF increases as consistency is increased. Trinh et al. (1989) have shown that below 13% consistency washing efficiency increasing pad thickness.

### 7.3 Velocity

Gren et al. (1973) selected velocities between  $10^{-4}$  to  $5 \times 10^{-4}$  m/s for bed depths of 4,6,9cm and found slight effect on the sub stance yield Lee (1979) at 40°C increased velocity from 0.0072 to 0.075cm/s and found little effect on the displacement washing efficiency Hakamaki et al. (1985) have shown that NEF decreases by increasing water velocity and have also stated that it is not straight forward to decrease the wash water velocity to obtain high washing g efficiency because drum washer capacity can suffer at low velocity.

Gren et al. (1985) studied the effect of flow rates between 0.4to  $3.2 \times 10^{-4}$  m/s and founding that wash ratio decreases for higher velocities. Trinh et al (1989) have found that washing efficiency increases with superficial velocity at low consistency(3%) and remains unaffected with superficial velocity at high consistency (15%). Pad thickness was 50mm and temperature 40°C.

### 7.4 RPM

Hakamaki et al. (1985) increased the rpm of drum from 1 to 4 and found that the capacity of drum increased by 2.5 times. Kukreja et al. (1995) have shown that by increasing the rpm fiber production rate also increases.

### 7.5 Pressure Drop

Yi-Ning Wang et al. (2003) Products will increase with increasing reaction pressure while decrease with increasing reaction pressure while decrease with increasing cooling temperature. Hakamaaki et al. (1985) performed experiments with a pulp tester and concluded the increase in filtration pressure increase the drum capacity but it is not so remarkable as with rpm and inlet vat consistency. Range of pressure drop was between 0.5-1.5m H<sub>2</sub>O (4900-14700 Pascal). Han et al (1988) have found that mat consistency increases by increasing drum vacuum. Pressure drop was varied between 17.8-43.2 cm hg (23729-57589). Kukreja et al .1995 have also found that pressure drop (17500-25000Pascal) increases the fiber production rate.

### 8. Conclusion

The general transient mathematical model for a rotary vacuum filter used for pulp washing, was developed using phenomenological principles. Sodium loss due to sorption is significant and it is essential to include sorption effects in washing efficiency. Calculation, especially when washing efficiency is very high around 99%. The % error does not exceed 10%. Therefore it is prudent to infer that the model appears to be a realistic one and can be used for the optimization of industrial problem.

Some optimum conditions for a rotary filter are,

- Cake thickness around 5cm,
- Temperature between 40-60° C,
- Rpm around 2,
- Pad consistency less than 13%

The results published by different investigators are inconsistent even for the same parameter. This may be attributable to different pulping techniques, species of wood, equipments used for experiments (Pulp tester, displacement cell, washers) etc.

Hence a uniform analysis is the need of the hour to increase the washing efficiency and to decrease pollution load.

### NOMENCLATURE

A: Maximum amount of sorbed Na, kg Na/kg pulp

B: Rate constant, m<sup>3</sup>/kg.

C: Concentration of Na in Liquor, kg/m<sup>3</sup>.

c: Solute concentration in the liquor Phase, kg/m<sup>3</sup>

*C<sub>i</sub>*: Concentration of solute inside the vat, kg/m<sup>3</sup>

*C<sub>m</sub>*: Mean concentration of solute, kg/m<sup>3</sup>

*C<sub>s</sub>*: Concentration of solute in the wash liquor, kg/m<sup>3</sup>

*C<sub>Y</sub>*: Pulp consistency, %

*C<sub>yd</sub>*: Discharged consistency of pulp, %

*C<sub>yi</sub>*: Inlet vat consistency of pulp, %

*DF*: Dilution factor, kg of liquor/kg of pulp

*D<sub>L</sub>*: Longitudinal dispersion coefficient, m<sup>2</sup>/s

*DR*: Displacement ratio, dimensionless

$K^*$ : Mass transfer coefficients

$k, k_1, K_2$ : Mass transfer coefficients, 1/s

L: Amount of liquor, kg liq/kg pulp

n: Solute conc. On fibers, kg solid/m<sup>3</sup> fiber

S: Amount of sorbed Na

t: Time, s

u: Liquor speed in cake pores, m/s

X: Dissolved solids, %

z: Variable cake thickness, m

Sub scripts

d: Discharged pulp

I : Inside the vat

s: Shower Liquor

st: Standard consistency

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