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Non-Linear Buckling Analysis of Non-Prismatic Steel Columns Subjected to Axial Compression Loads

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Abstract

This study investigates the effect of non-uniform cross-section on the behavior of steel columns subjected to axial compression load. A nonlinear finite element model using ANSYS 12.0 has been adopted to investigate the behavior of square and circular steel columns. The steel is assumed to behave as an elastic-plastic material with strain hardening in compression. The type of elements have been used to model the steel; SOLID45. The axial-load displacement curves and the deformation shapes were predicted. A parametric study on columns with square and circular section has been done. All the analyzed columns have the same value of cross-section volume for the column, yielding strength, and boundary conditions with different length of column. The results show that the behavior of non-prismatic column is always by the tapering ratio and the slenderness ratio on the elastic buckling. As the taper ratio increases, the elastic buckling load increases, in the main while the maximum ultimate load occurs in the (prismatic column) comparing with the non-prismatic column.

Keywords: Non-uniform cross-section, Steel columns, nonlinear finite element, Elastic buckling load.

1. Introduction

Structures are generally designed on the basis of economy and safety. For common structures, economy often requires the use of standard members because these members are relatively inexpensive and easy to obtain. For many structures, however, using tapered members may both increase structural efficiency and be economical. For small projects, this may not translate into overall economy, but more complex, unique, or large structures may (and often do) take full advantage of the structural efficiency tapered members offer by reducing the amount of material required while strategically stiffening certain parts of a member, thereby increasing the overall performance of the structure. The use of steel members with non-uniform cross- sections either as columns or as distressed parts of a structure with or without bending moments is very common in steel constructions. There is a wide variety of structures such as buildings frames, bridge members, masts or cranes, etc, which are designed with members of non-uniform cross-sections in order to minimize the required material.

A first approach and study the above-mentioned problems of columns with variable cross-sections were made by **[Dinnik, 1929 and 1932]**. The main results of these studies were translated in English by **[Maletsand 1925 and 1932]** the same problem was studied by **[Ostwald, 1910]**, by **[Ono, 1914]** and by **[Morley, 1914 and 1917].** On the history of early studies on these topics, one can refer to **[Timoshenko, 1953]. [Bleich, 1924]** studied compression members the cross sectional moment of inertia of which was varying by a half sine curve. On the other hand, the significance of the initial imperfections was noted very early and studied mainly experimentally by **[Mrston, 1897],** by **[Jensen, 1908]** and by **[Lilly, 1911],** the studies of which were gathered by **[Salmon, 1925]**. In **[Gere and Carter, 1962]** provided equations and design curves for calculating the critical buckling loads of columns with many different cross sections and four different fixity conditions: pinned-pinned, fixed-free, fixed-pinned, and fixed-fixed. This greatly enhanced the information that had previously been available for non-prismatic column design, particularly for tapered wide flange sections.

Numerical solution methods for non-prismatic columns are presented by **[Ram and Rao, 1951], [Ku, 1979] and [Chen et al., 1989].** Tapered column buckling under stepped axial loads was researched by analyzed using numerical integration and a discredited column made up of prismatic sections. Tapered box columns under biaxial loading were analyzed by **[Liew et al, 1989]** using moment curvature-thrust relationships and Horne's stability criteria.

In **[Liew et al, 1989]**, presented the formulation to solving the governing equation of the problem through a numerical method where the eigen shapes of the member are employed. Non-uniform steel members with or without initial geometrical or loading imperfections, that are loaded by axial forces applied concentrically or eccentrically and by concentrated moments applied at the ends or at intermediate points, are studied. More specifically, steel members with varying cross-sections, tapered or stepped or members consisting by two different tapered parts are considered.

In the present study, the influence of the taper ratio and cross-sectional geometry on the stability of non-uniform steel members that are subjected to axial loads is investigated. The problem is studied by focusing on dimensions for square section and round section with cross-sections that may vary along the length, are usually met in steel structures. The methodology is based on the formulation of the mathematical expression for the

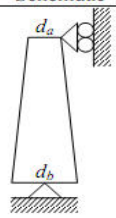
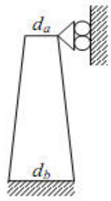
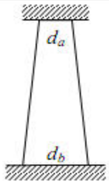
Euler buckling load of an ideal column follows from consideration of equilibrium, the mechanics of bending, geometry of the column, and material properties within the initial linear range. The effect of taper ratio on the buckling strength of such beams is not dealt with in detail in the bibliography. Employment of a detailed finite element analysis using the commercial finite element code **ANASYS (12.0)** serves herewith for verification purposes only and both analytical and numerical results correlate with reasonable accuracy.

2. Numerical Analysis

2.1 Buckling load for a non- prismatic column

In [Gere and Carter, 1962] published a paper in the ASCE Journal of the Structural Division that provided equations and design curves for calculating the critical buckling loads of columns with many different cross sections and four different fixity conditions: pinned-pinned, fixed-free, fixed-pinned, and fixed-fixed. This greatly enhanced the information that had previously been available for non-prismatic column design, particularly for tapered wide flange sections. Gere and Carter used methods developed by Timoshenko (solution of the differential equation of the deflection curve) and Newmark (method of successive approximations of the deflection curve), along with an innovative shape factor to account for the cross-sectional variation along the column, to ultimately provide a relatively concise set of equations to calculate the critical buckling load. Many of the situations that were analyzed could be solved in closed form, but required a difficult solution process based on Bessel functions with many related variables. However, for columns with a linear taper, circular cross section, and pinned-pinned, fixed-pinned, or fixed-fixed end conditions, the resulting equation is simply the product of the square of the ratio of end diameters and the critical buckling load [P_{cr_o}] of a prismatic column for the given end conditions Table (1).

Table (1), Exact buckling load for linearly tapering solid columns of circular or square cross section **[Gere and Carter, 1962].**

End Conditions	Schematic	Buckling Load
Pinned-Pinned		$P_{cr} = \left(\frac{\pi^2 EI_a}{L^2}\right)\left(\frac{d_b}{d_a}\right)^2$
Fixed-Pinned		$P_{cr} = \left(\frac{\pi^2 EI_a}{(0.6997L)^2}\right)\left(\frac{d_b}{d_a}\right)^2$
Fixed-Fixed		$P_{cr} = \left(\frac{4\pi^2 EI_a}{L^2}\right)\left(\frac{d_b}{d_a}\right)^2$

The research on tapered columns has yielded a great number of ways to characterize the taper of a given member. For linearly tapering members of various cross sections, a shape factor is generally devised that is based on the variation of the moment of inertia along the length of the member. If more complex taper scenarios are required, then the shape factor is often based on the variation of some cross-sectional dimension along the length of the member. **[Gere and Carter, 1962]** Provide the most general expression for the variation of the moment of inertia for a linearly tapering column:

$$I(x) = I_a\left[1+\left(\frac{d_a}{d_b}-1\right)\frac{x}{L}\right]^n \tag{1}$$

where $I(x)$ is the moment of inertia at any distance (x) from the small end of the column (end a as defined in Fig. 1), (I_a) is the moment of inertia at the small end of the column, (da) and (db) are the general dimensions at the small and large ends of the column, respectively, and n is the shape factor given by the equation:

$$n = \frac{\log \frac{I_b}{I_a}}{\log \frac{d_b}{d_a}} \tag{2}$$

This allows for the shape factor to be determined for any given cross section. However, for most simple shapes, n may be evaluated by inspection. For example, knowing that $I = \frac{\pi d^4}{64}$ for a rounded cross section, one can deduce that for linear variation of the diameter, the moment of inertia will vary by the power of 4.

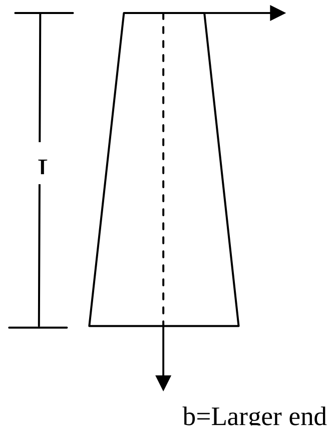


Figure 1 Notation used to characterize tapered column geometry

3. The Numerical Modeling and Application

3.1 The Finite Element Modeling

The commercial finite element software **ANSYS 12.0** was adopted for the numerical simulation. Some previous researchers were used shell element to model the steel **[Xiong and Zha,2007] , [Guo et al,2007], [Kwon et al, 2007]**, and **[Ellobody,2007]**,while **[Mohi-Aldeen, 2008]** and**[Zinkaah, 2010]** were used solid element. Thus, the type of element has been used to model the steel section; is **SOLID45** for the 3-D modeling of RHS column. The element is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions as shown in Fig.(2). The element has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities **[ANSYS, 2004].**

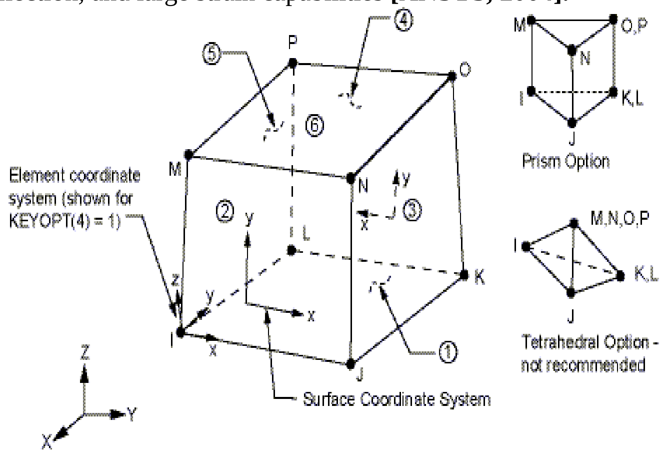


Figure 2 Solid45-3D Solid **[ANSYS, 2004].**

Material properties specified in **ANSYS 12.0** included a Young`s modulus of steel Es of 200000MPa and Poisson`s ratio νs of 0.3 **[Guo et al,2007].** The steel is assumed to behave as an elastic-plastic material with strain hardening in compression. The idealized stress-strain curve used in the numerical analysis is shown in Fig. 3 **[Xiong and Zha, 2007], [Mohi Aldeen, 2008], and [Zinkaah, 2010].** The Fy in Fig. (3) Represent the ultimate stress steel.

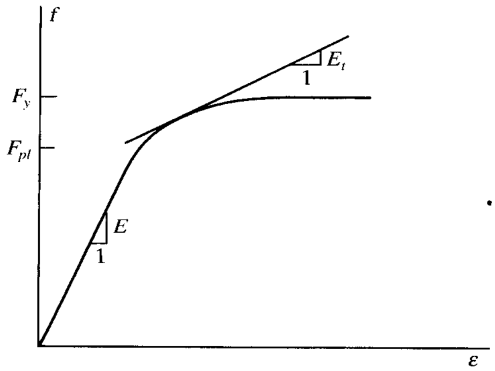


Figure 3 Stress-strain curve**[Xiong and Zha,2007], [Mohi Aldeen, 2008], and [Zinkaah, 2010]**

3.2 Application

In spite of the software **ANSYS 12.0** dose not required to proving the validity and accuracy in analysis of steel columns, two specimens modeled then the axial load–axial displacement curve are compared with the numerical results, to verify the accuracy of the analysis by **ANSYS 12.0**. The specimens represent a square and rounded solid steel columns. The finite element modeling and the boundary conditions of the analyzing column are showed in Fig. 4, plowed for linear buckling analysis. Such a detailed FE model of a dimensions with taper ratio varying a/b=0.1-1 build-up steel cross section with L and No. of element is equal to L/10 .It is found that the analytical results presented herein correlate well with the corresponding FEA results with a maximum error less than 0.2%.

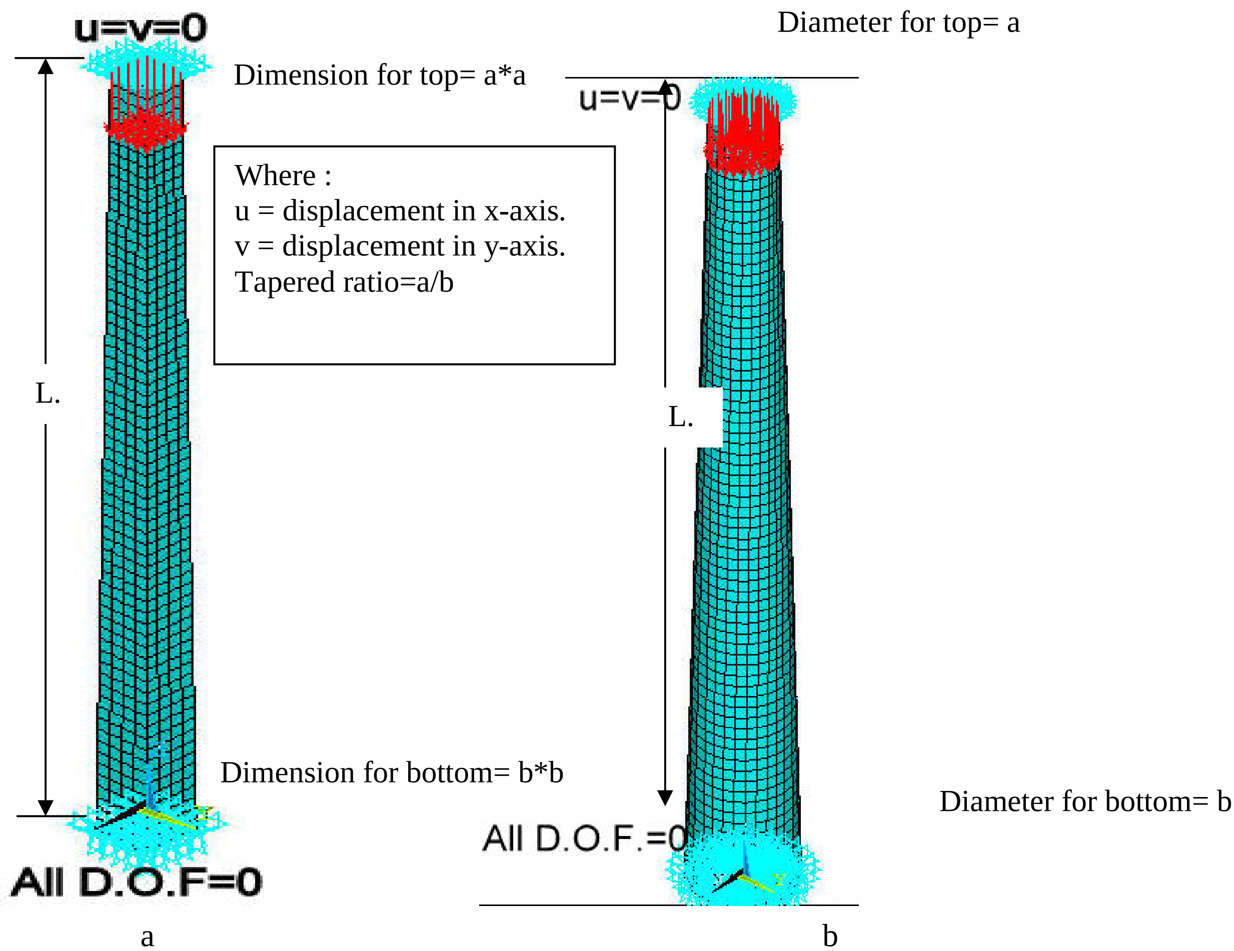


Fig ure 4 Meshing and boundary conditions of columns
a) Square cross section b**)** Rounded cross section

3.2.1 Applied load

The deformation shape, the values of and the load-axial displacement curve for Square cross section column and circular cross section are shown in **Figs. 5** and **6** respectively.

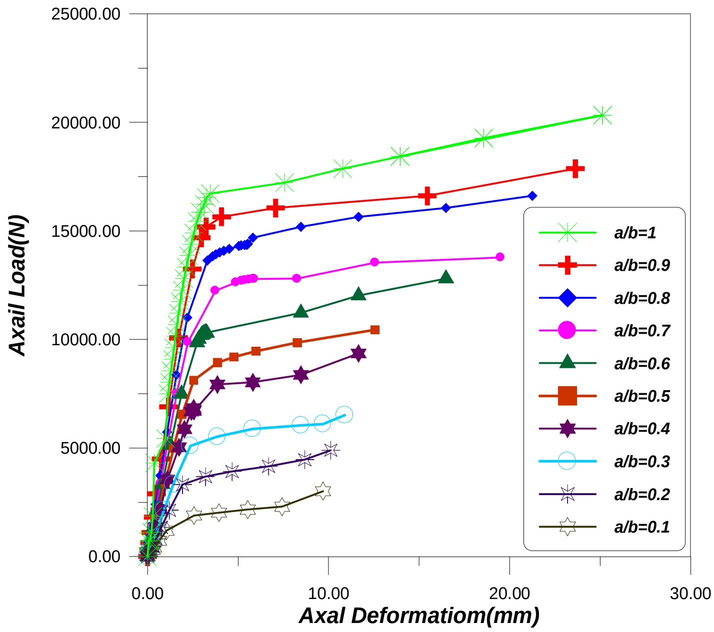


Figure 5 Axial load vs. axial displacement curve of square column

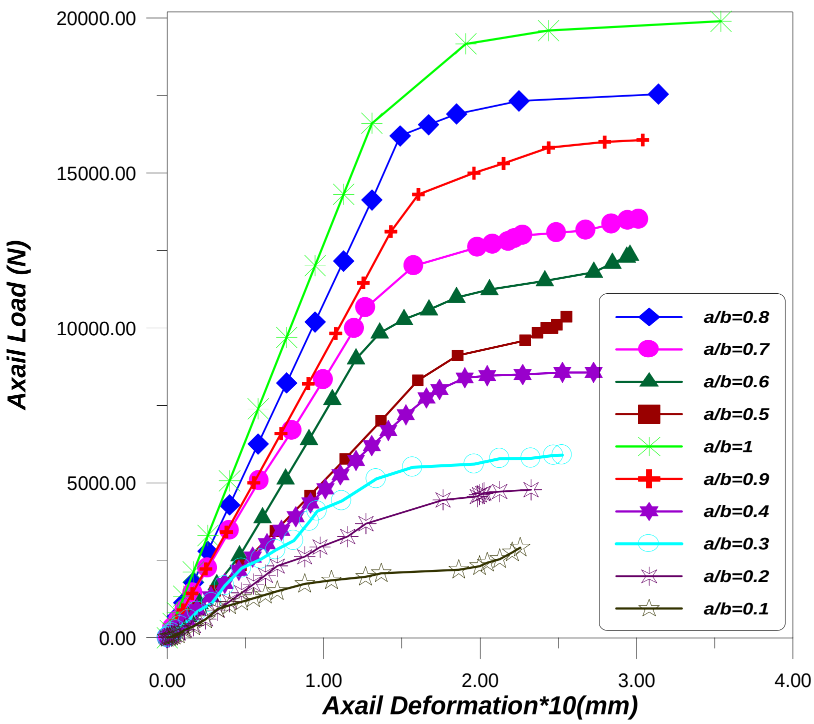


Figure 6 Axial load vs. axial displacement curve of circular column

In **Fig. 7** show the comparing between two columns for the same conditions.

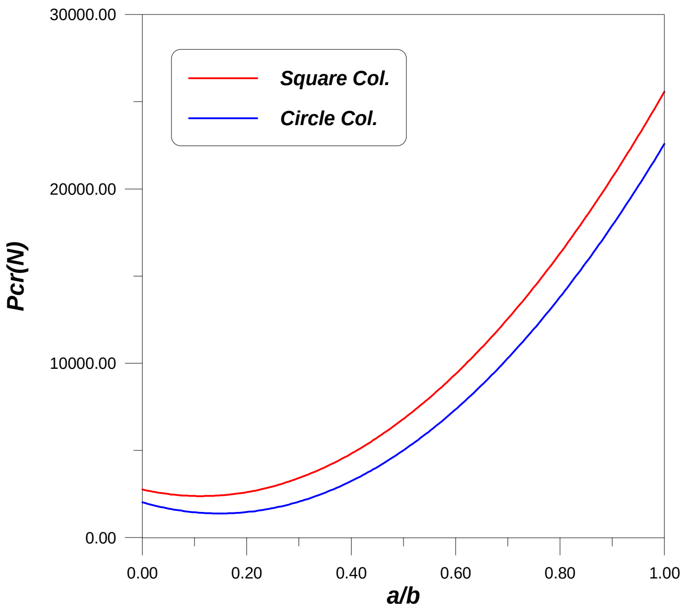


Figure 6 Critical load vs. tapered ratio (a/b) curve of two types of columns.

In **Fig. 7,** one can see the critical load ratio P_{cr}/P_{cr0} versus the taper ratio a/b for various lengths of a tapered column. The loading consists from a concentrated load P applied axially on the smaller end of column. It must be noted at this point that although the differences in the critical loads seem to be small, the corresponding differences in the critical load values may be significant depending on the both the taper ratio a/b. For example, the load P_{cr0} which used as a reference load in Table (1), corresponds to a uniform column with dimension for square cross section (20*20mm) and 22.567mm diameter for rounded cross section, for two columns have the same volume, the results presented (86.023%) convergence between the columns.

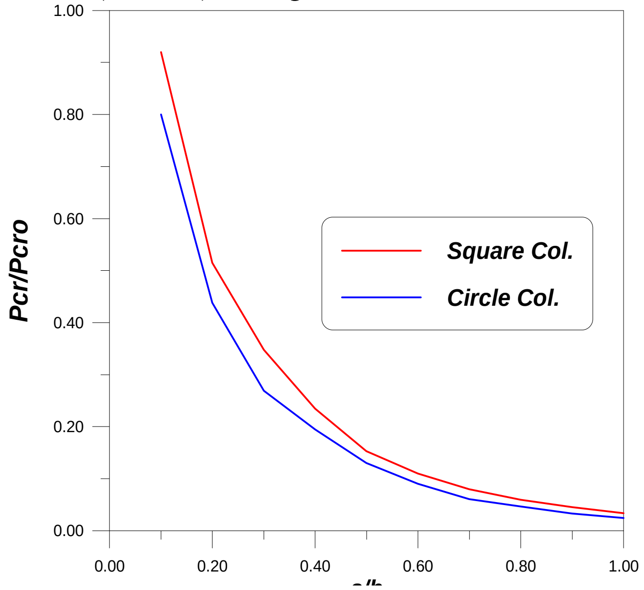


Figure 7 Critical load ratio P_{cr}/P_{cr0} versus taper ratio a/b for the columns

The deformation shape, the values of displacement in x-direction, the values of axial displacement and the load-axial displacement deformed shape for columns square with different cross section for columns are shown in **Figs.8-17** respectively. Form the results, the modes shape of column were changed due to varying of tapering ratio.

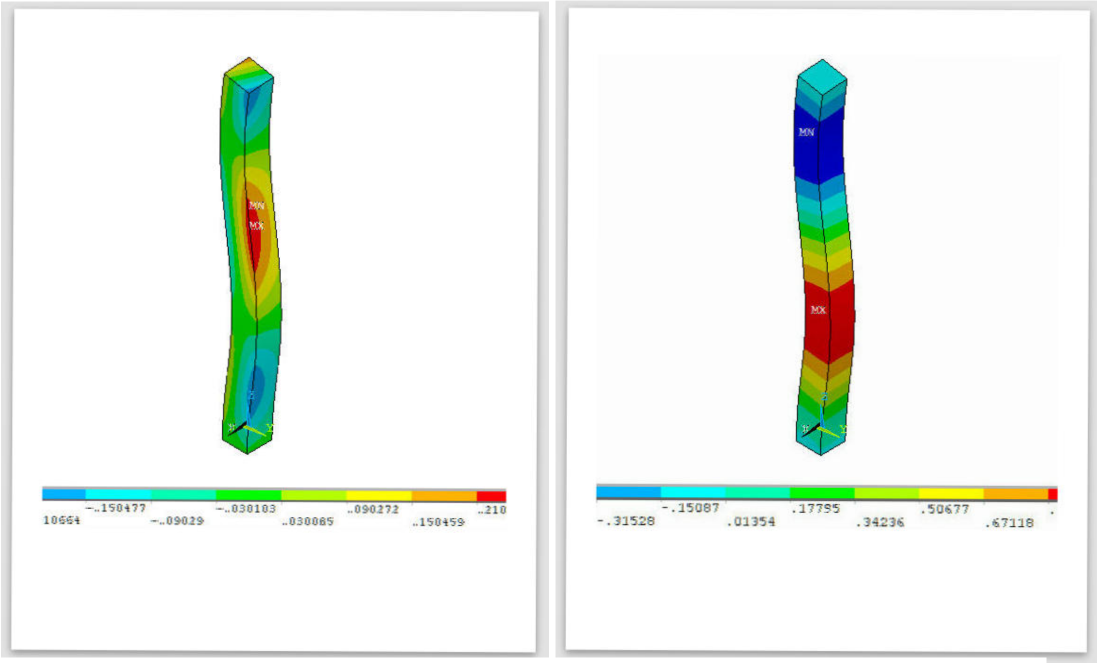


Figure 8 Deformation shape, axial and lateral deformation for tapered ratio a/b=1

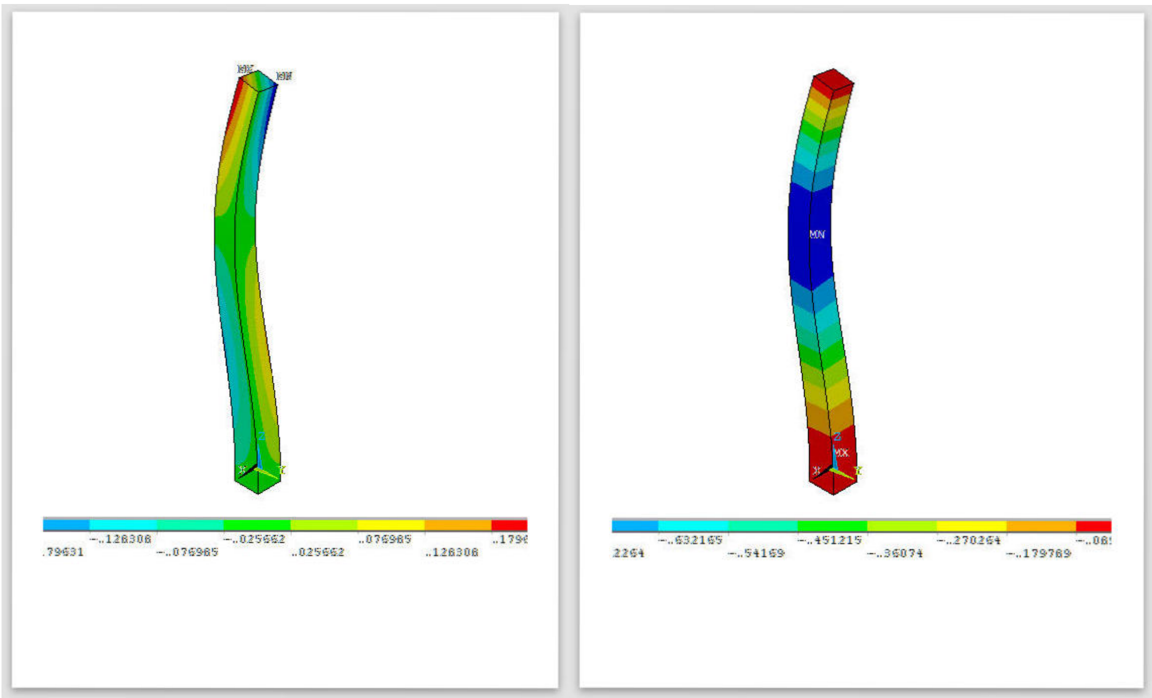


Figure 9 Deformation shape, axial and lateral deformation for tapered ratio a/b=0.9

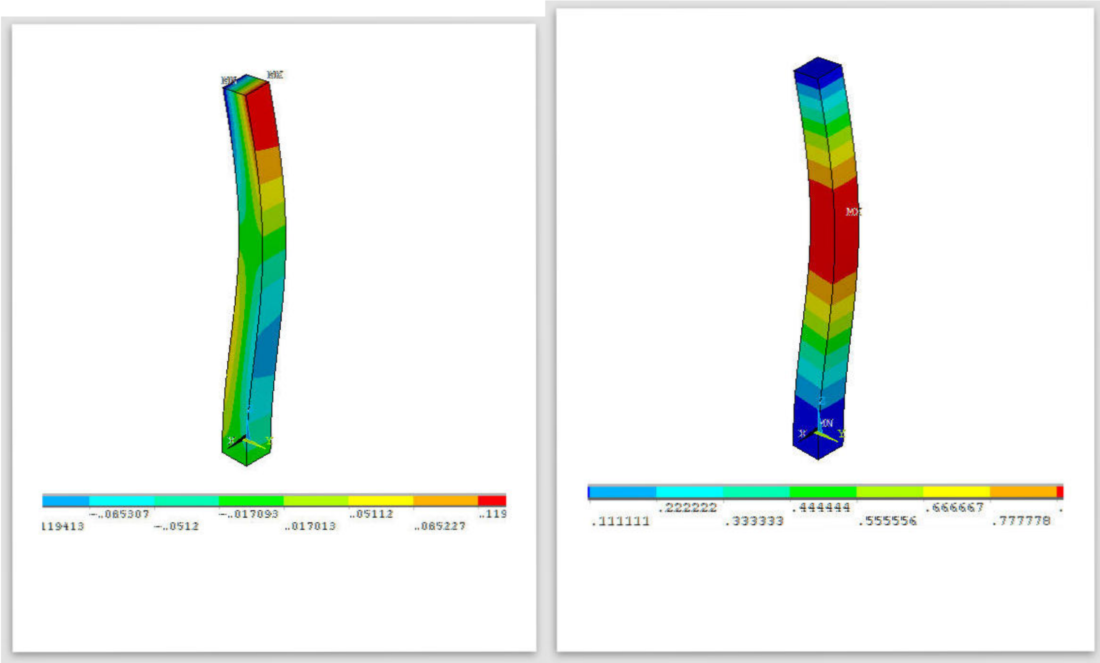


Figure 10 Deformation shape, axial and lateral deformation for tapered ratio a/b=0.8

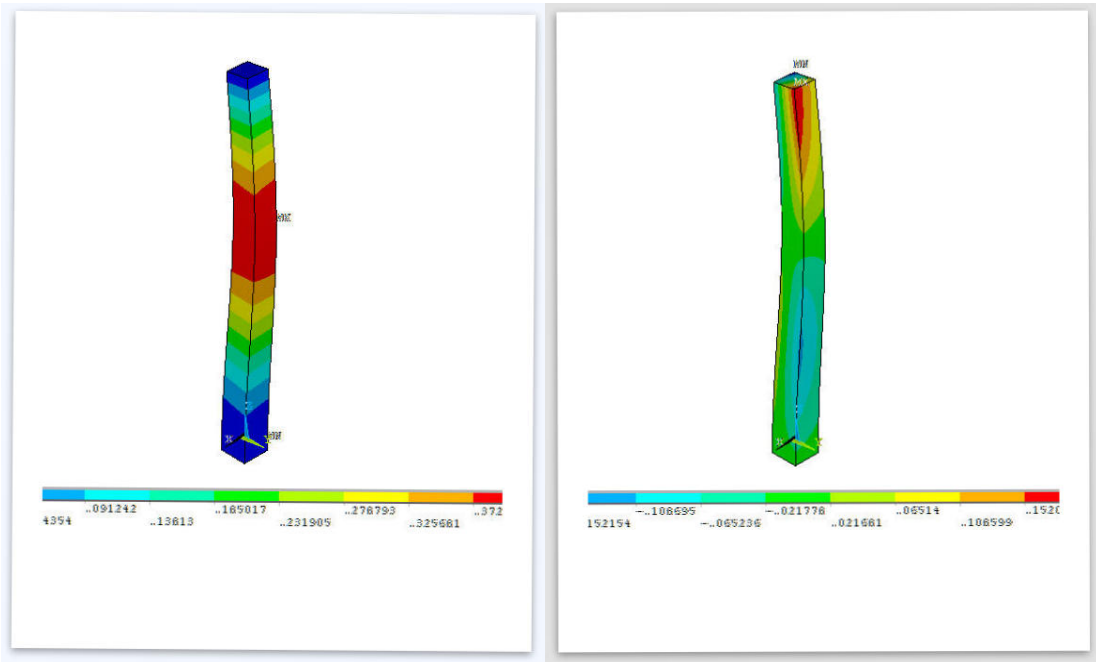


Figure11 Deformation shape, axial and lateral deformation for tapered ratio a/b=0.7

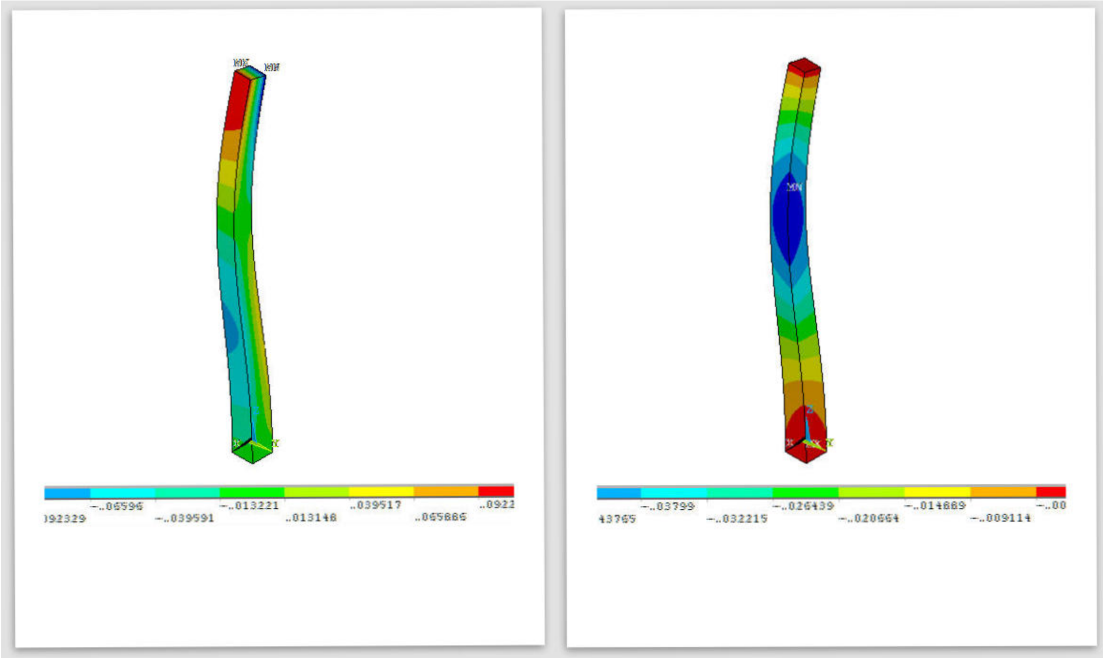


Figure 12 Deformation shape, axial and lateral deformation for tapered ratio a/b=0.6

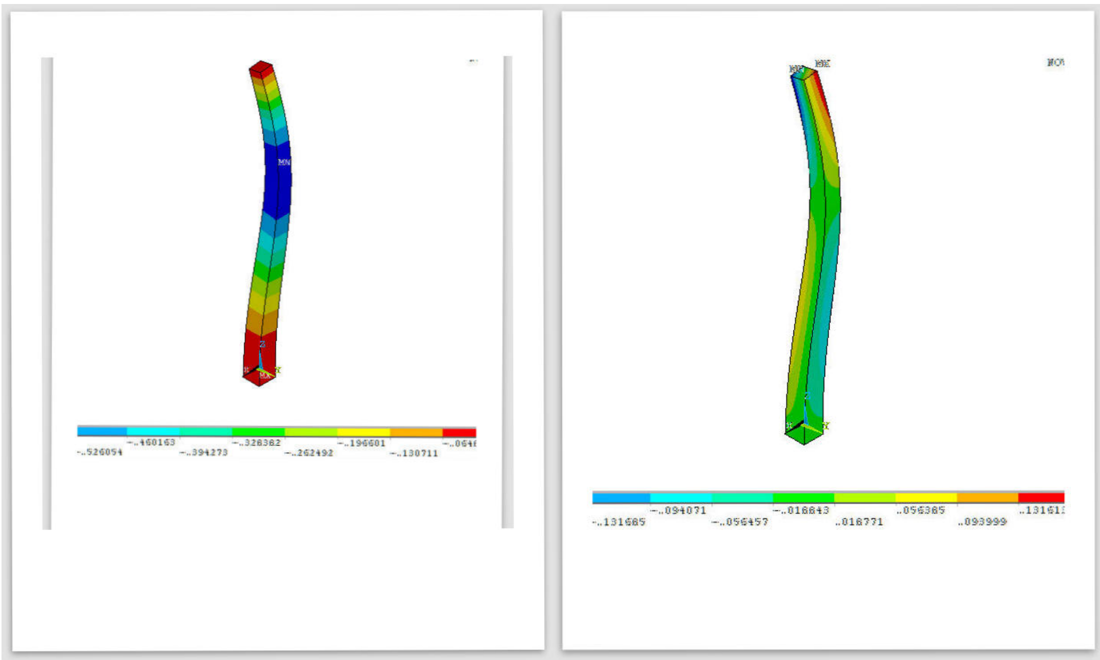


Figure 13 Deformation shape, axial and lateral deformation for tapered ratio a/b=0.5

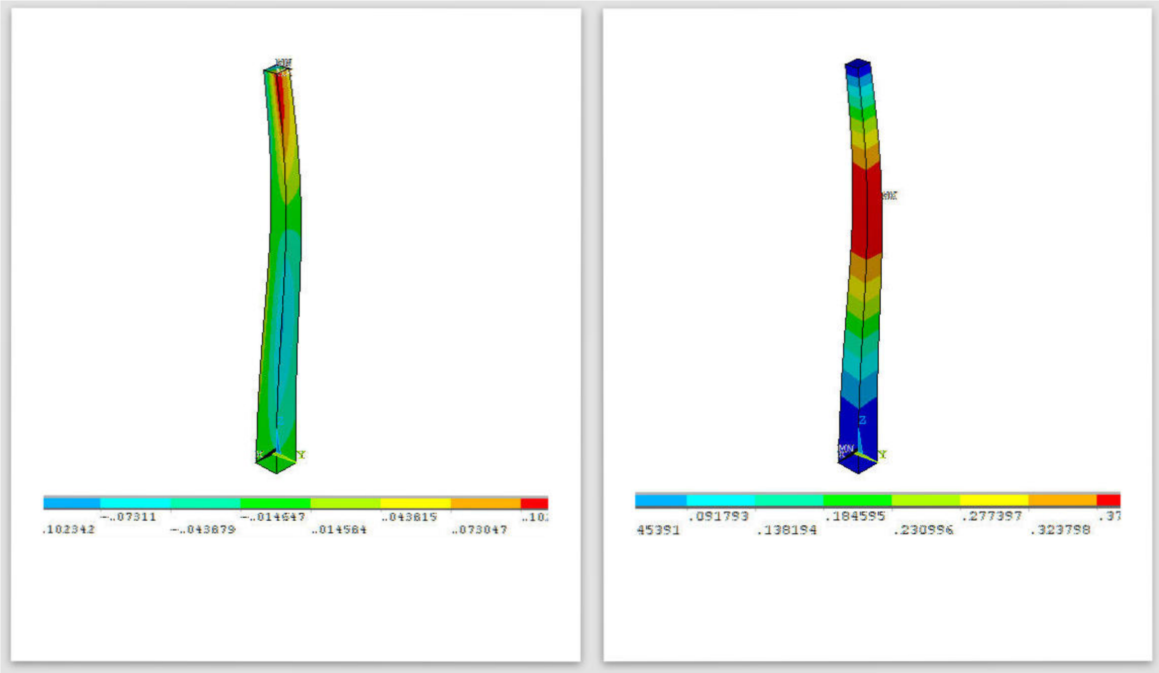


Figure 14 Deformation shape, axial and lateral deformation for tapered ratio a/b=0.4

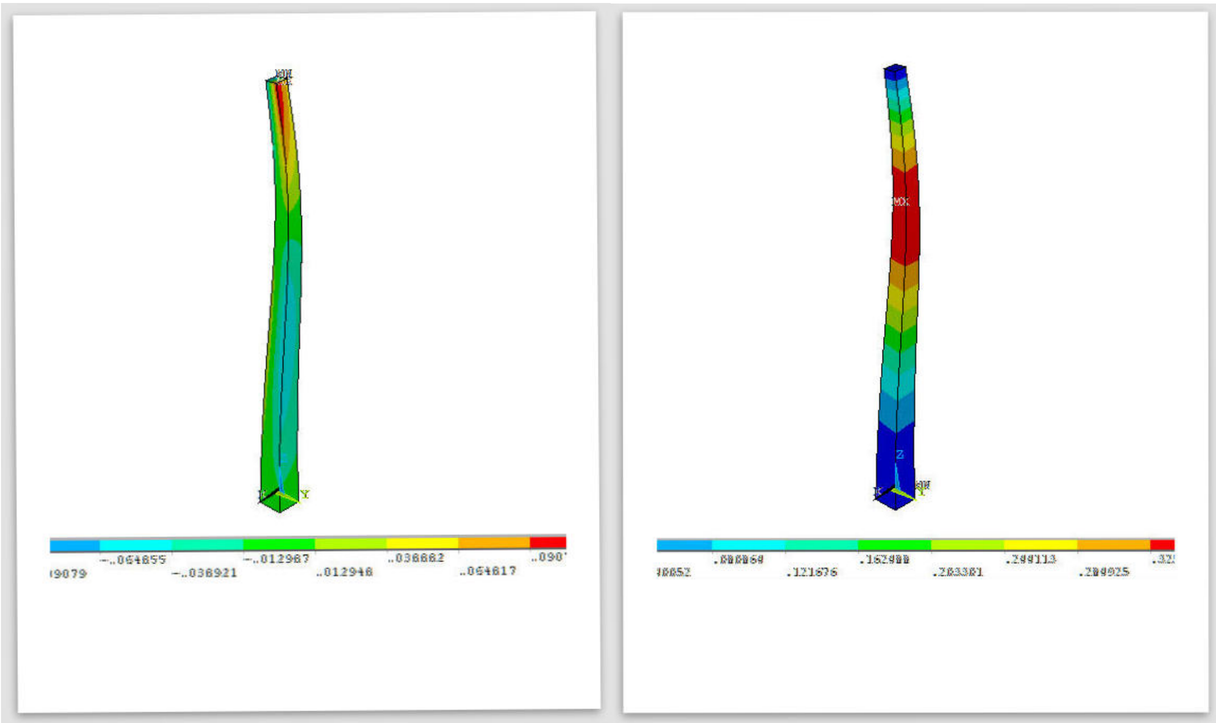


Figure 15 Deformation shape, axial and lateral deformation for tapered ratio a/b=0.3

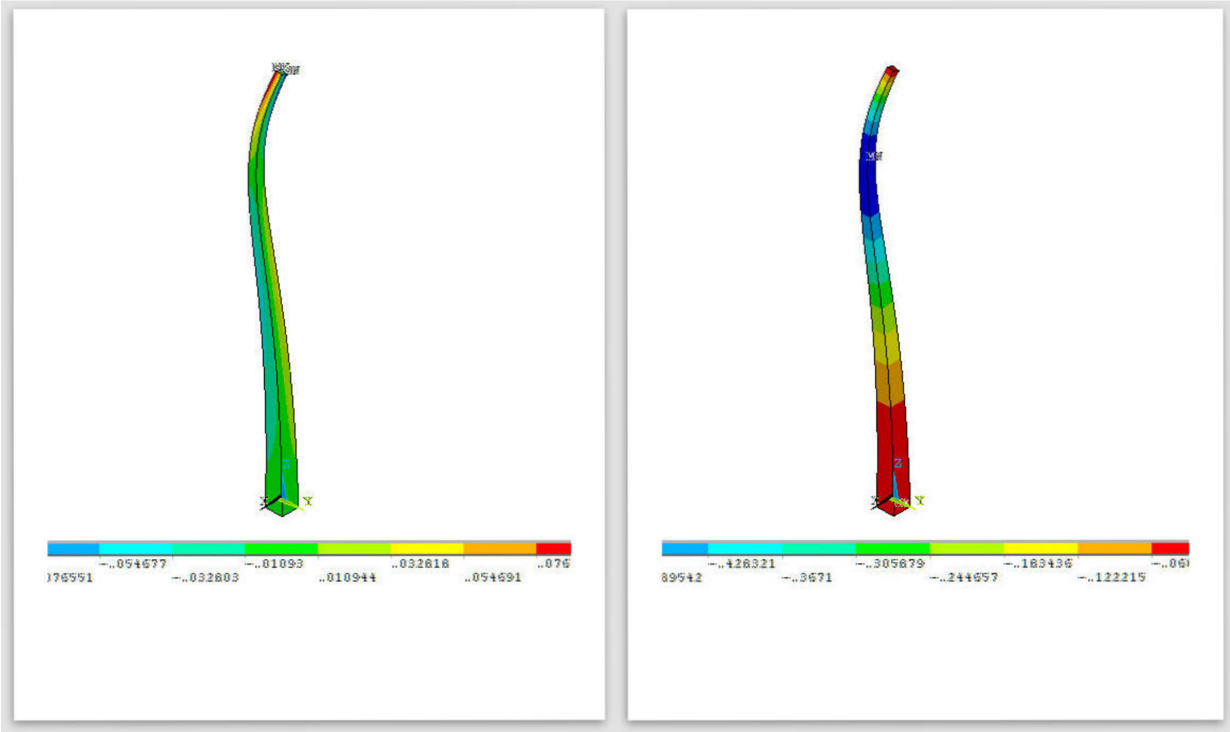


Figure 16 Deformation shape, axial and lateral deformation for tapered ratio a/b=0.2

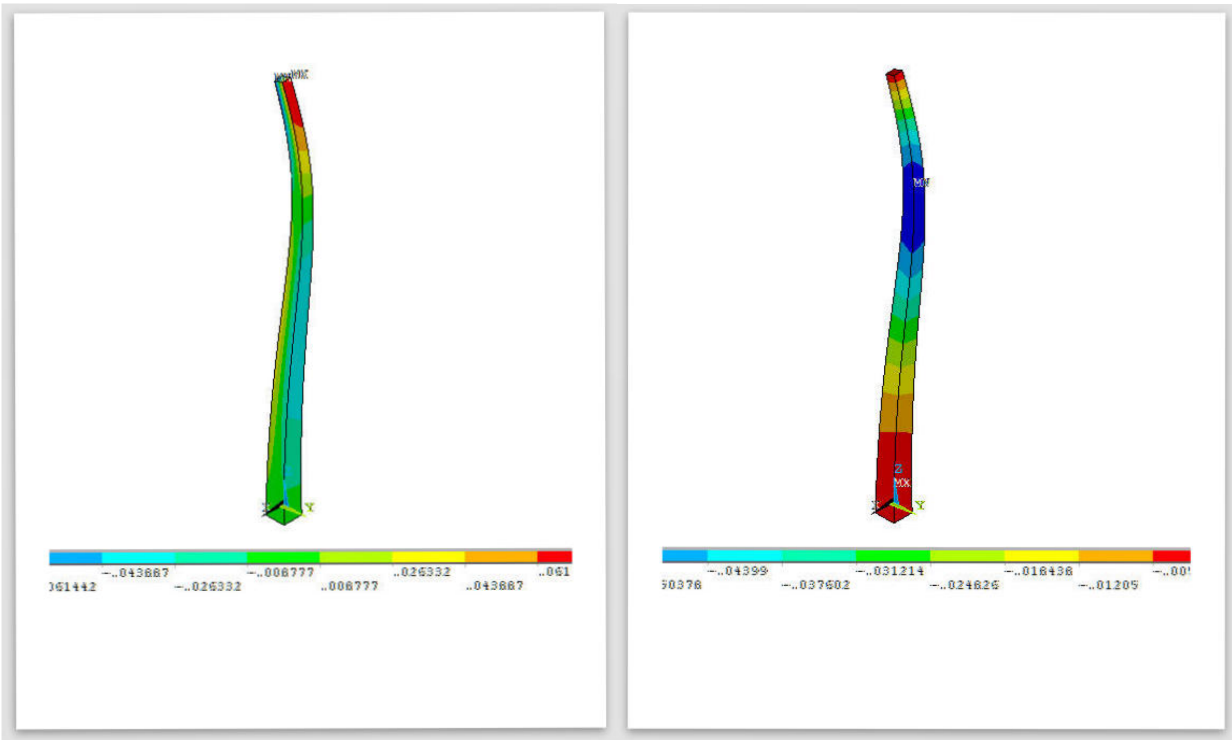


Figure17 Deformation shape, axial and lateral deformation for tapered ratio **a/b=0.1**

The deformation shape, the values of displacement in x-direction, the values of axial displacement and the load-axial displacement deformed shape for columns circular with different cross section for columns are shown in **Figs.18-27** respectively. Form the results, the modes shape of column were changed due to varying of tapering ratio.

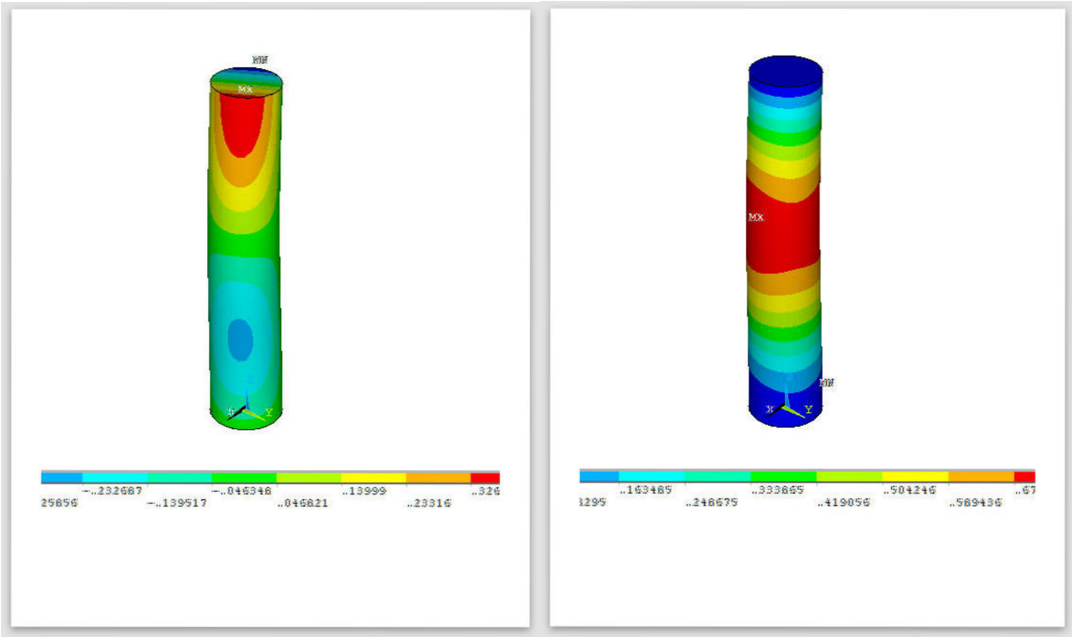


Figure 18 Deformation shape, axial and lateral deformation for tapered ratio a/b=1

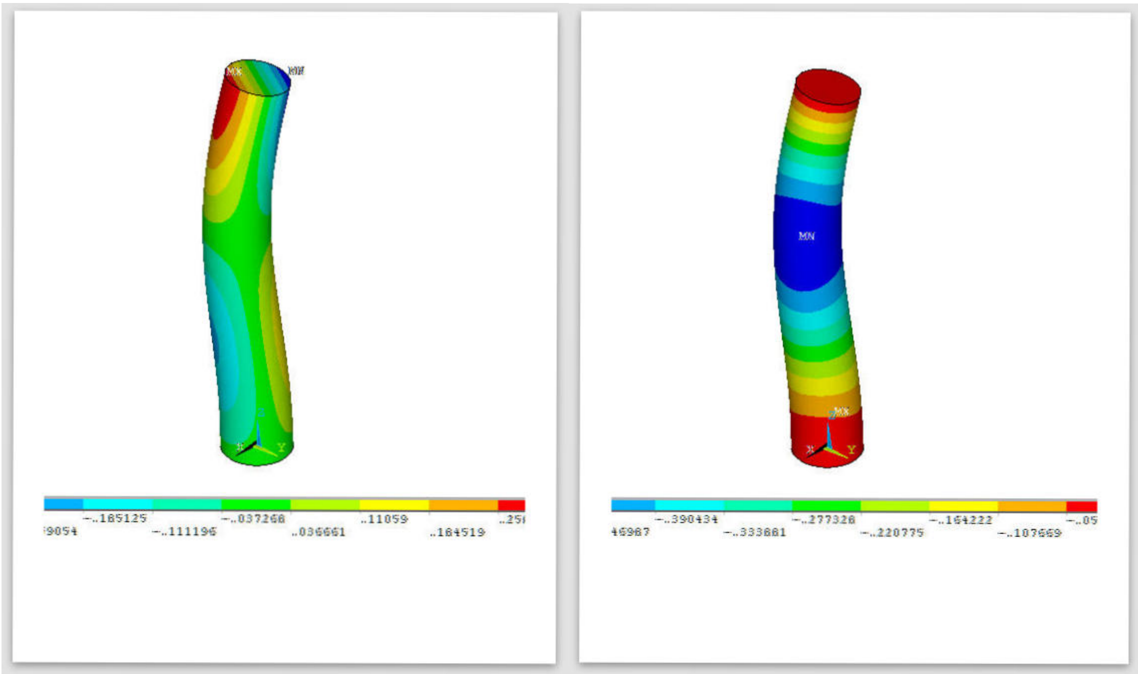


Figure 19 Deformation shape, axial and lateral deformation for tapered ratio a/b=0.9

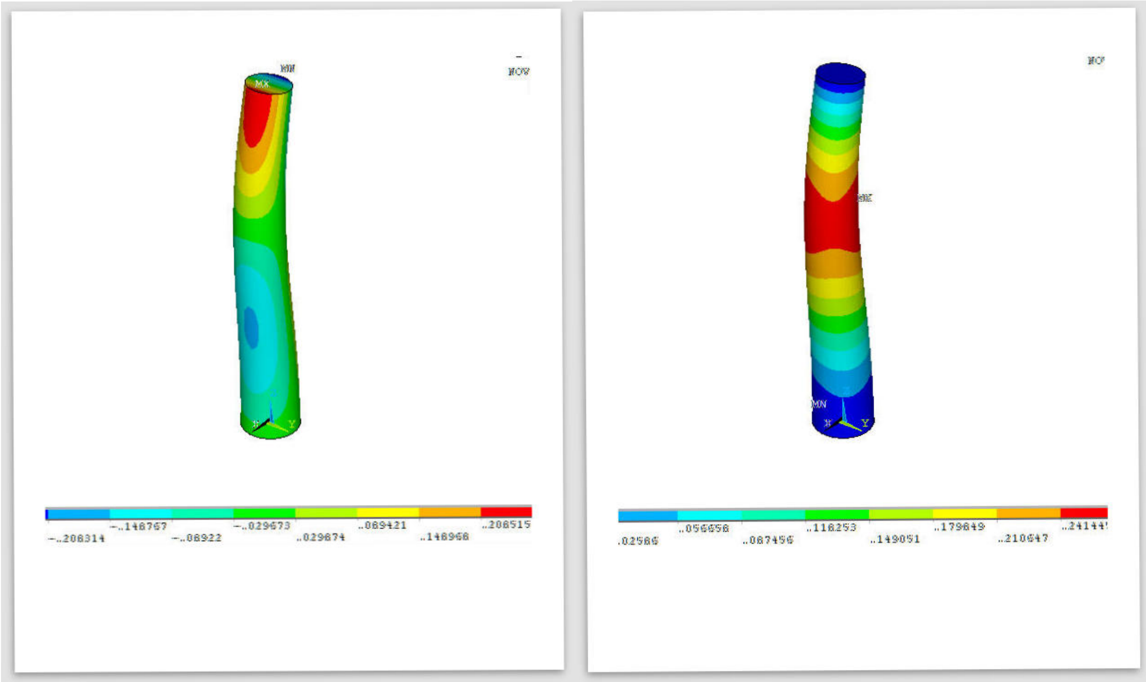


Figure. 20 Deformation shape, axial and lateral deformation for tapered ratio a/b=0.8

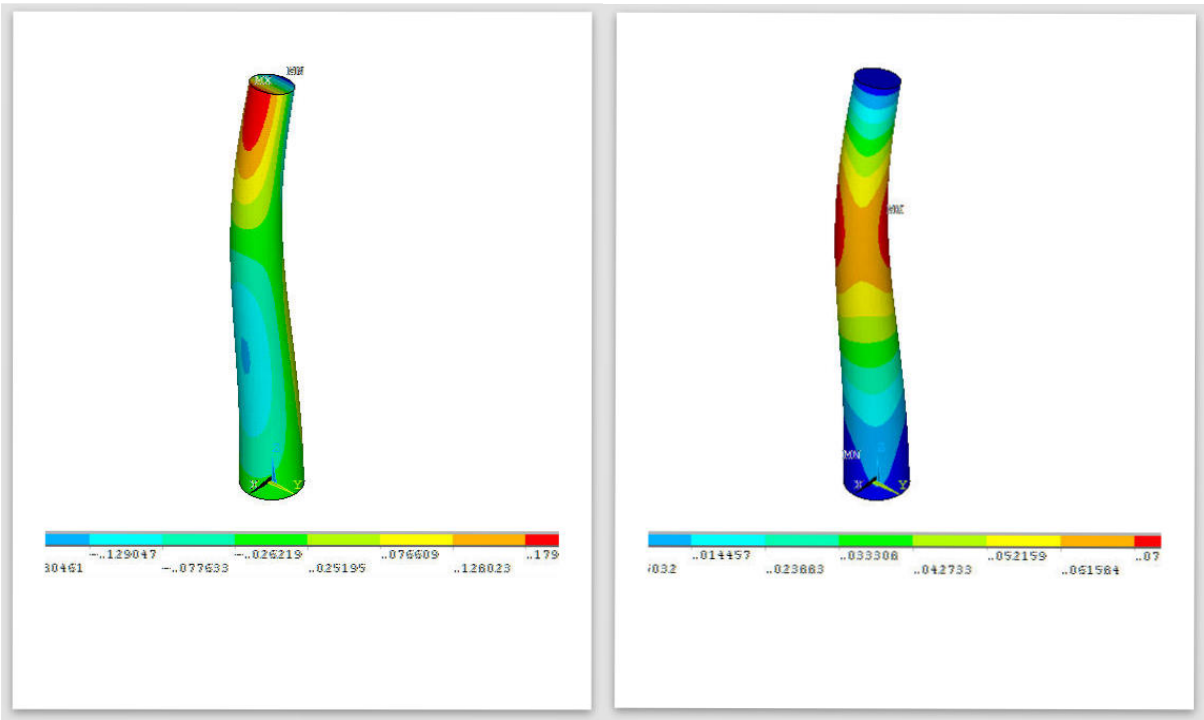


Figure 21 Deformation shape, axial and lateral deformation for tapered ratio a/b=0.7

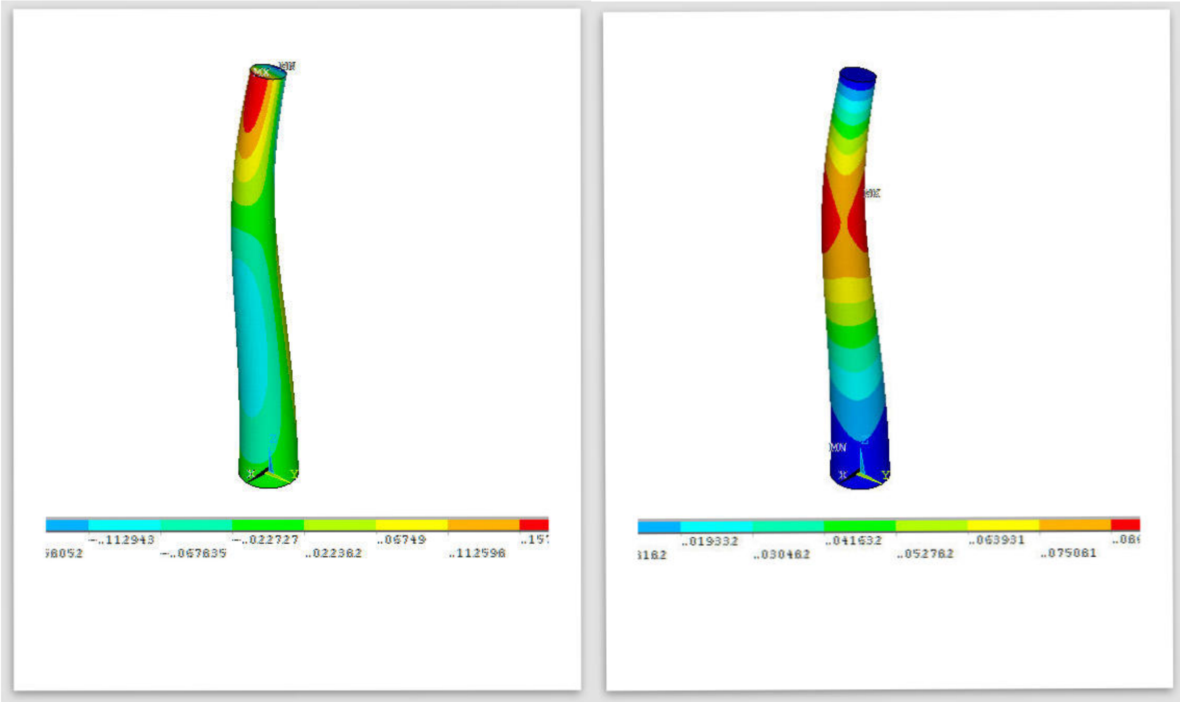


Figure 22 Deformation shape, axial and lateral deformation for tapered ratio a/b=0.6

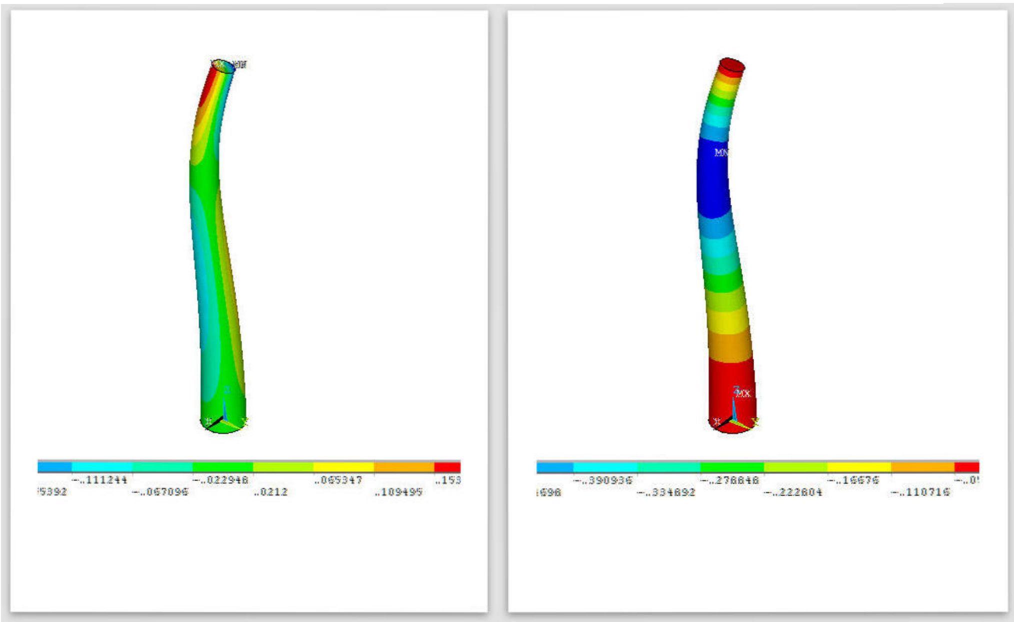


Figure 23 Deformation shape, axial and lateral deformation for tapered ratio a/b=0.5

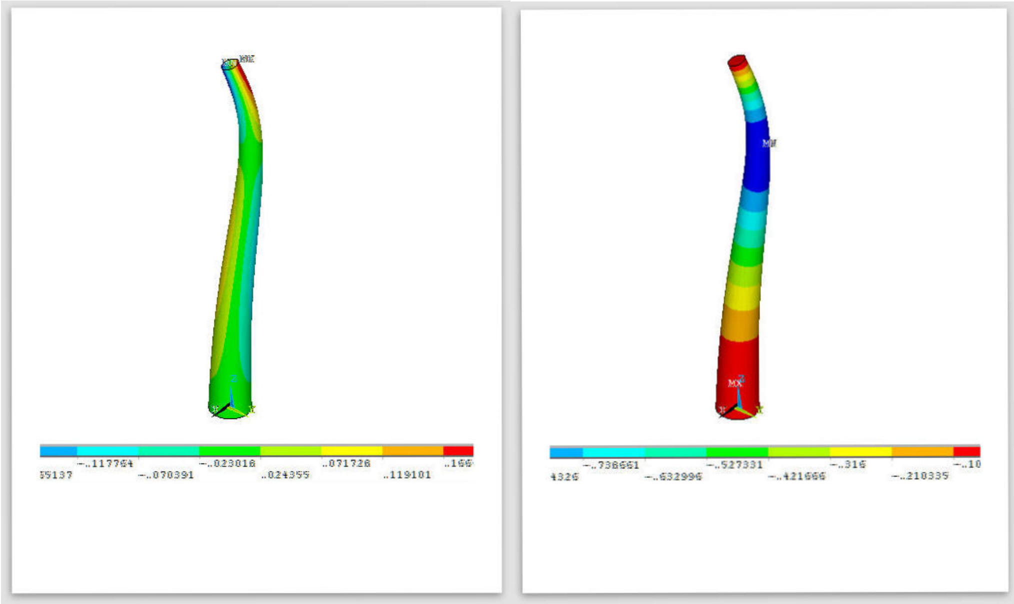


Figure24 Deformation shape, axial and lateral deformation for tapered ratio a/b=0.4

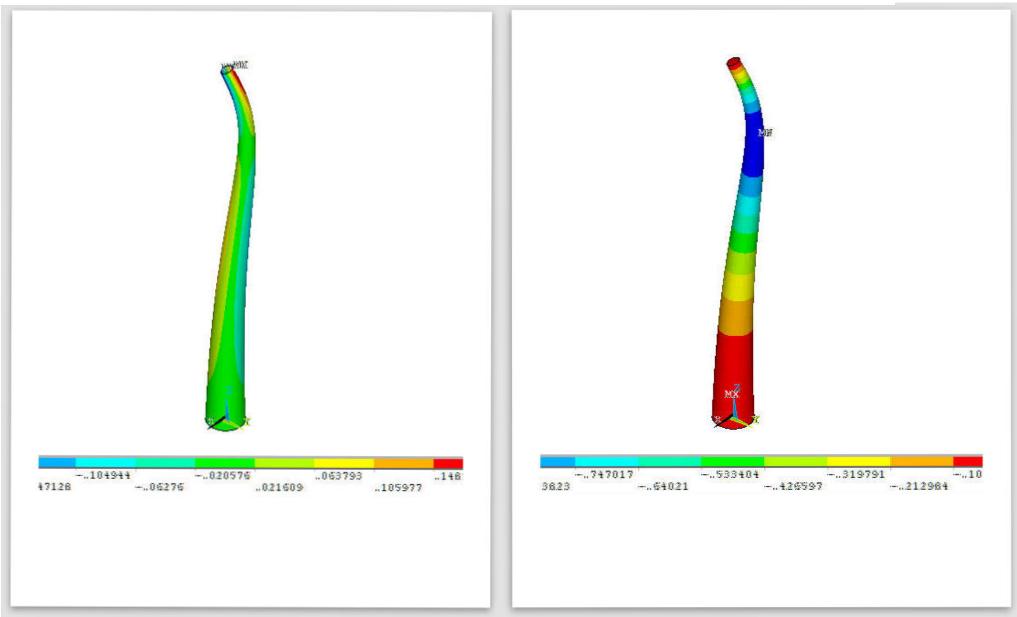


Figure 25 Deformation shape, axial and lateral deformation for tapered ratio a/b=0.3

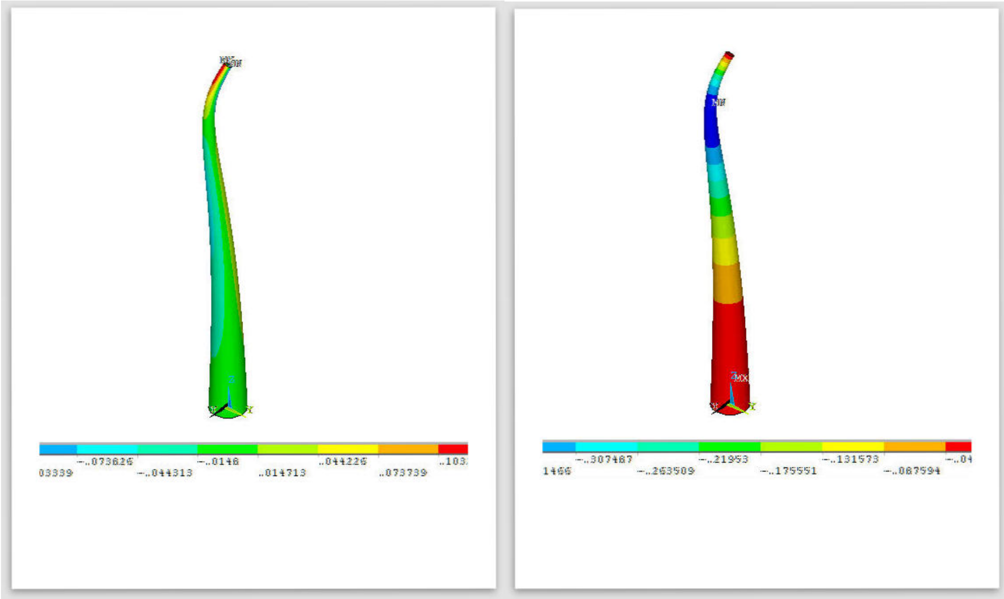


Figure 26 Deformation shape, axial and lateral deformation for tapered ratio a/b=0.2

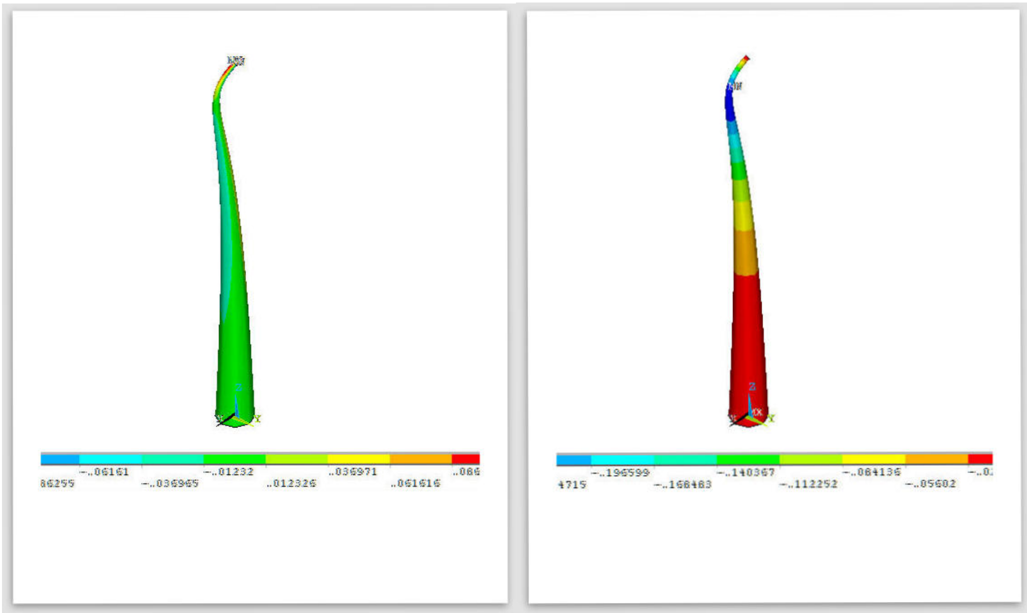


Figure 27 Deformation shape, axial and lateral deformation for tapered ratio a/b=0.1

3.2.2 Mode Shape

In both cases of columns with tapering ratio causes varying of mode shape in the same conditions. In Figs. (28 and 29) show the behaviour of non-prismatic column due to non-uniform cross section a long length of column.

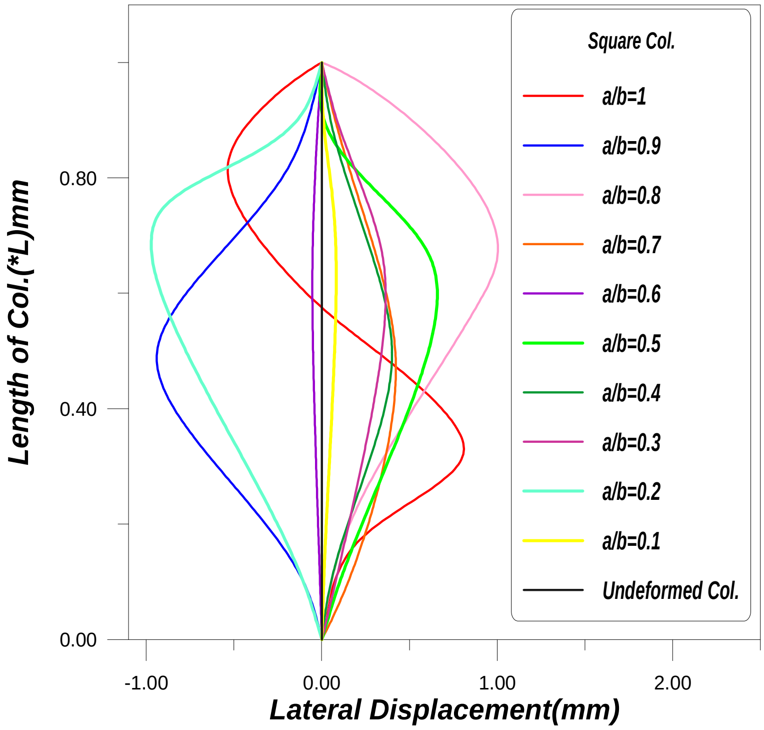


Figure 28 Mode shape for Square Column

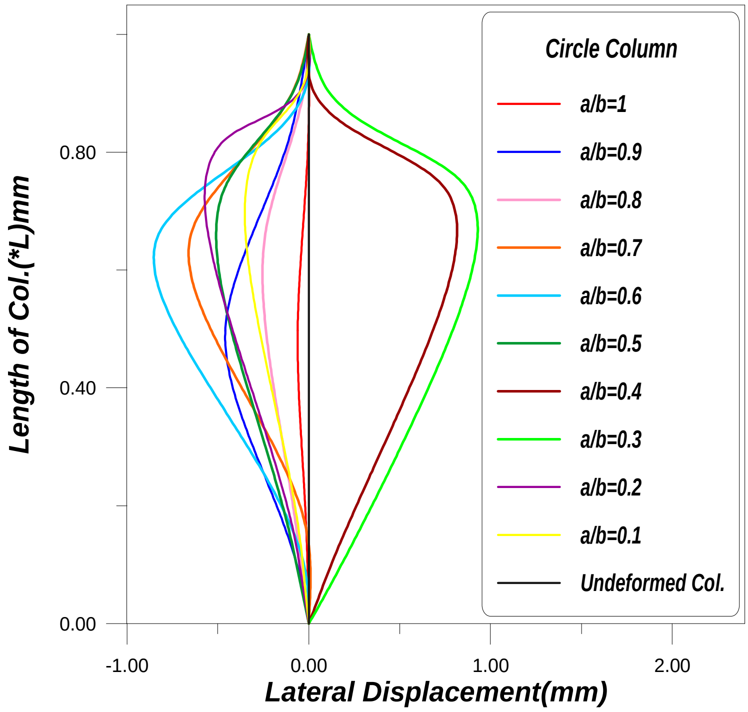


Figure 29 Mode shape for Circle Column

3.2.3 Stresses

The different failure modes are resulted from the difference stress-slenderness characteristice of steel columns.Figs.(30 and 31) shows the results of failure for taperedd ratio. For square column with a/b=(0.1 to 0.4) failed by buckling for arange of slendereness ratios. At a/b=(0.5 to 1), the square column failed by yielding but the buckling failure occure in the circle column at tapered ratio, a/b=(0.7 to 1) in the same arange of slendereness ratios and yielding stresses (Fy=414 MPa.).

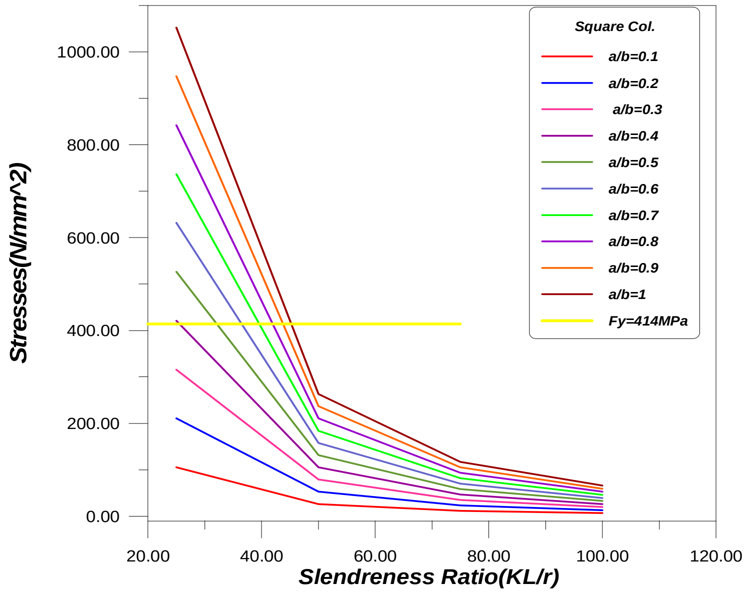


Figure 30 Stresses for varying slenderness ratio and tapered ratio for Square Column

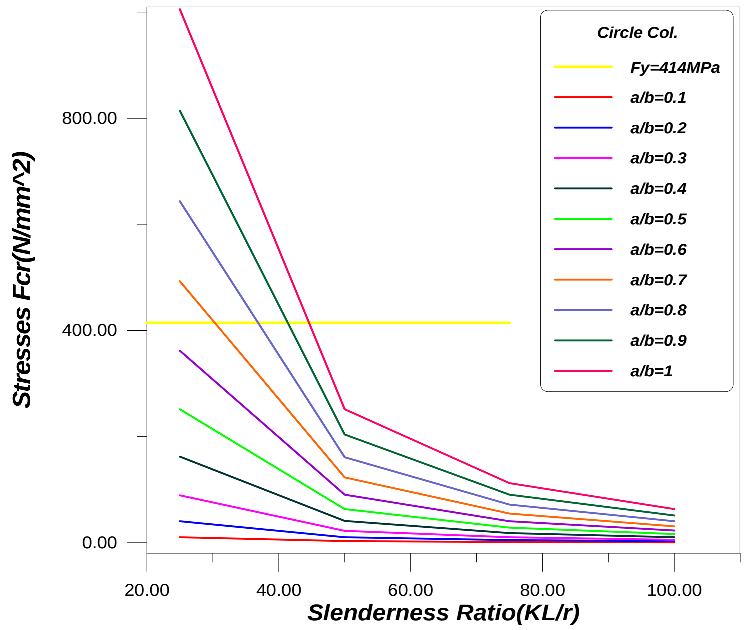


Figure 31 Stresses for varying slenderness ratio and tapered ratio for Circle Column

3.2.3 Comparison of results

In order to demonstrate accuracy, convergency and applicability of the finite element software **ANSYS 12.0**, the buckling problem of columns with non-uniform cross section are analyzed and numerical result **[Gere and Carter, 1962]** are plotted in Fig. 32, shows about 20% convergences.

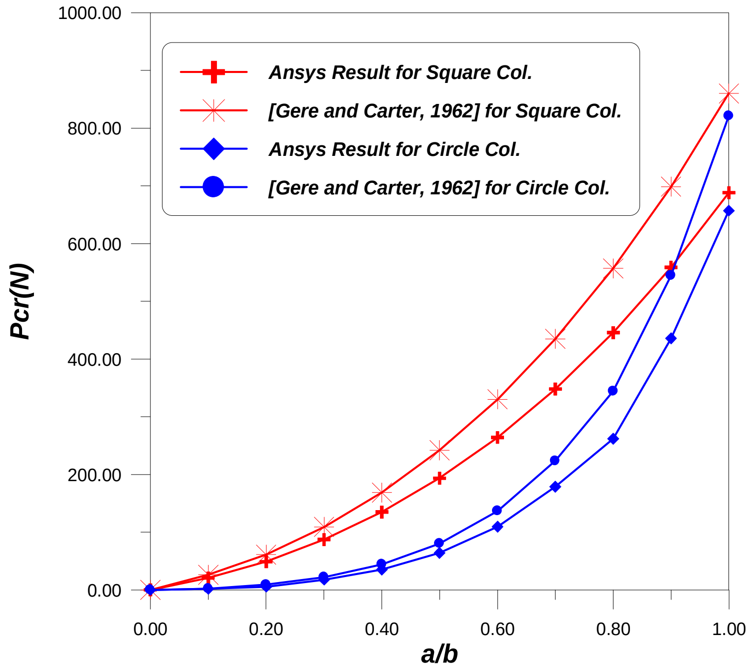


Figure32 Critical load vs. tapered ratio for non-prismatic

3.2.4 Support Conditions

In Figs. 33 and 34, one can see the corrosponding diagram for the same colum , with different cases of supporting condition.

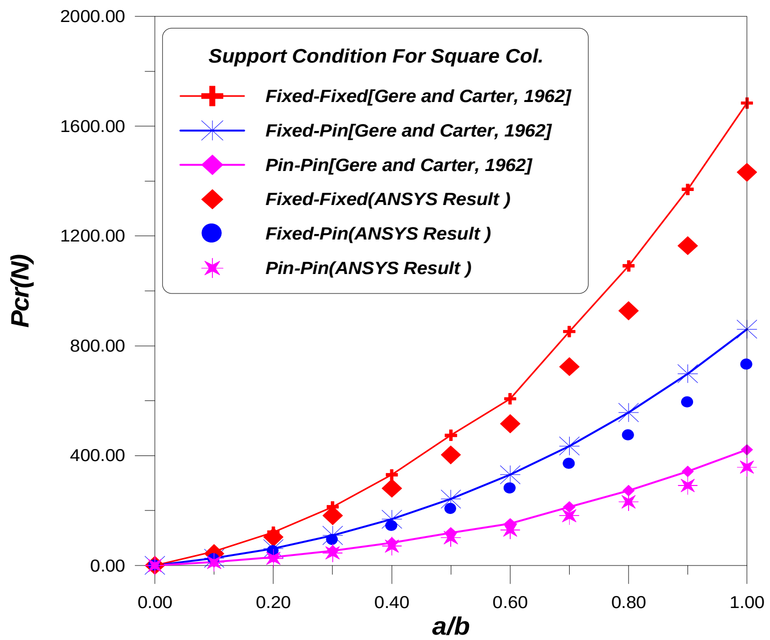


Figure33 Critical load vs. tapered ratio for non-prismatic square column under different cases of

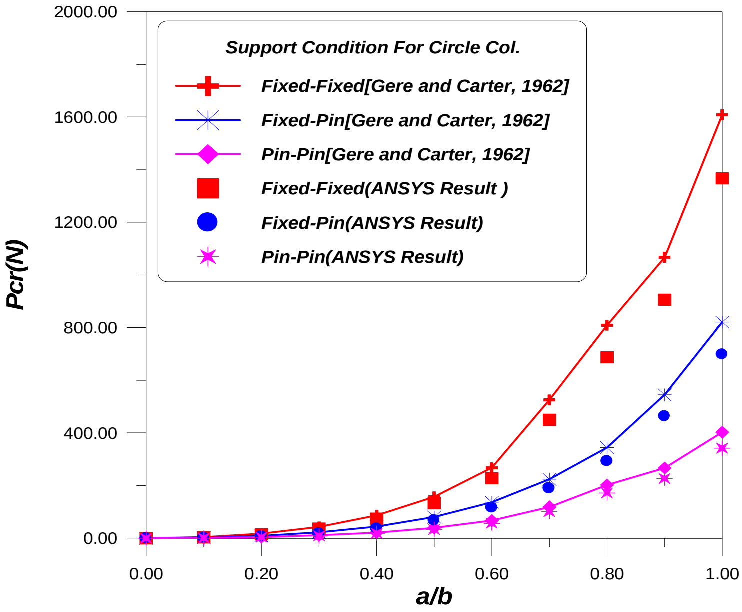


Figure 34 Critical load vs. tapered ratio for non-prismatic circle column under different cases of support

Finally, the effect of taper ratio on number of element through meshing in the finite element software **ANSYS 12.0** was plotted in Fig. 35. At the same time the shape of cross section for steel column was affected on the number of element.

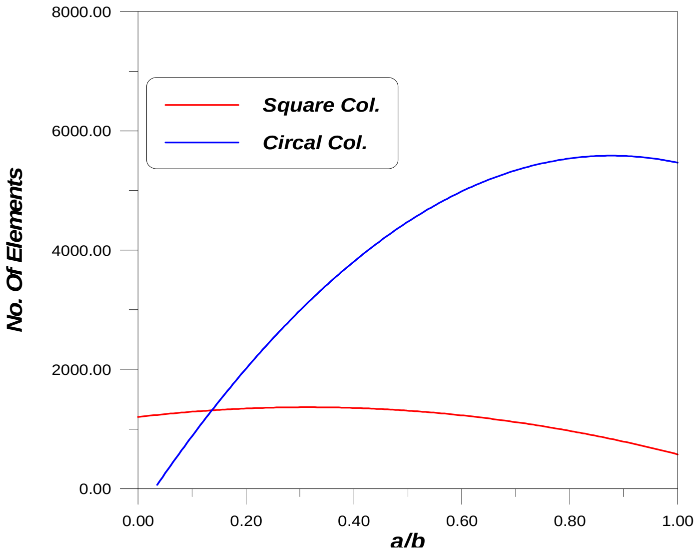


Figure 35 Number of Element vs. Tapered ratio

5. Conclusion

In this study, the elastic buckling behavior of steel tapered column by means of determining the corresponding elastic critical load for elastic buckling. This critical load can be used for the determination of the corresponding member strength. More specifically, the influence of the taper ratio and cross-sectional geometry, the different cross section and the different slenderness ratio of non-uniform steel members that are subjected to axial loads is thoroughly investigated. The most conclusions that can be drawn from this study are:

1. The effect of tapering ratio decreases the critical load.
2. The tapering ratio affected on the location of buckling and mode shape of failure.
3. More different types of failure occure due to increasing of tapering ratio.
4. The tapering ratio affected on the number of element through meshing.
5. The influence of taper ratio is greater for critical load due to different conditions of support.

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