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Predictive Model of Moment of Resistance for Rectangular Reinforced Concrete Section under Actions

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Abstract

Generally engineering design requires adequate data, good understanding of methods of analysis and ability to use them effectively and accurately for creative processes. A model is developed for predicting implications of various combinations of sectional dimensions in structural reinforced concrete elements. The method adopted is purely mathematical technique of modeling, concept and application.

The paper thus aimed at increasing design understanding for effectiveness and precision to ensure safety in structural design. A concrete rectangular section of singly reinforced status was used as a case study. The section was analyzed from stress-strain relationship to formulate the governing equation. This was simulated to describe the relationship between the variable parameters and predict the behavior of a rectangular section under loads. The section is specified by breadth b, total depth h and moment of resistance M_R. Thus the adequate sectional dimension can be accurately selected for design.

Keywords: Reinforced Concrete, Stress-Strain Relationship, Rectangular Section and Moment of Resistance

1. Introduction

Issues and communications between structural engineers about a particular element is usually its behaviour under actions of load. Hence, the first of the objectives of the most recent methods of structural design known as 'limit state' emphasized that a particular engineering structure must be safe under worst condition of loadings. This requirement among others must be met, no matter how difficult the task of estimating the loads and variation in the strength of the concrete and steel.

Accepting human limitation upon estimations and calculations of design data, the use of factors of safety is considered to achieve safe and durable structures. However, possession of adequate data, knowledge of materials characteristic and other design information are not exhaustive. Substantive skill in design procedure and understanding of the dimensional combination implications of a particular and walls. They are rigidly connected together to form a monolithic frame. Thus each of the elements/members must be designed to be capable of resisting the subjecting forces. Elements under a set of imposed actions (loads) are also crucial. Analysis of a structural element at the ultimate limit state is normally performed for loadings corresponding to that state.

A reinforced concrete structure consists of bending (horizontal) elements and compressive (vertical) elements. Generally, bending elements are slabs and beams, while compressive elements are columns

Thus the first challenge faced by the designers is the selection of adequate dimension for a particular element under consideration. This paper therefore addressed the issue of reactive behavior of an element under actions at varying cross-sectional dimensions, with particular attention to beam.

2. Methodology

Sectional analysis method was adopted to achieve satisfactory and economic design of a concrete structure. Mosley W. H. and Bungey J. H. (1990) and Mosley W. H. et al (2007), method of analysis was followed to obtain the governing equation and also to define the problem. Singly reinforced rectangular section is as well considered under bending action at balance condition of ultimate limit state.

2.1 The Governing Equation

The behavior of structural concrete is normally represented by a parabolic stress block obtained from stress- strain relationship, but ultimate design stress is given by;

1) For cylinder crushing strength , Mosley W. H. et al (2007):

$$\alpha_r f_{ck}/\gamma_c = 0.85 f_{ck}/1.5$$

$$\text{stress} = 0.567 f_{ck} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

2) For cube crushing strength , Mosley W. H. and Bungey J. H. (1990):

$$\alpha_b f_{cu}/\gamma_c = 0.67 f_{cu}/1.5$$

$$\text{stress} = 0.45 f_{cu} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

where; α_r = factor for bending strength and cylinder crushing strength disparity
α_b = factor for bending strength and cube crushing strength disparity
$f_{ck} = f_{cu}$ = characteristic strength of concrete
γ_c = patial factor of safety.

In this paper, stress from cube crushing strength is adopted.

Considering a singly reinforced beam section under bending, Fig. 01 below.

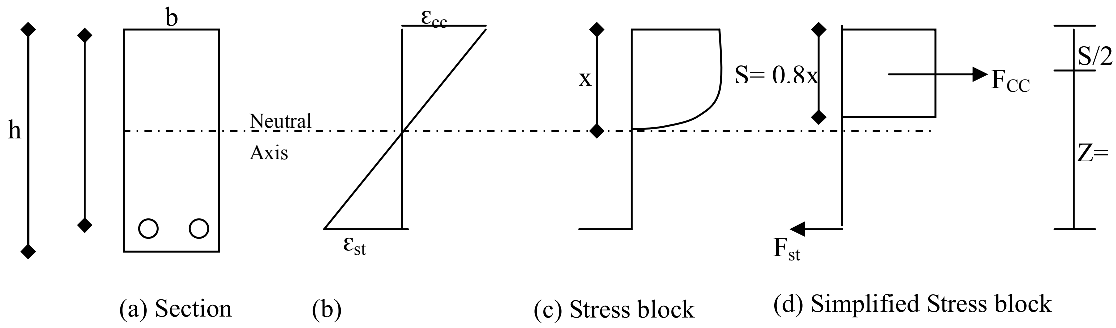


Figure 01: Stress- strain block analysis

In this figure;

h = total depth,
d = effective depth,
b = breadth,
ε_{cc} = maximum compressive strain of the concrete,
ε_{st} = strain in steel, x = depth of compression,
s = depth of the simplified stress distribution,
F_{cc} = resultant compressive force in concrete,
F_{st} = resultant tensile force in reinforcing steel,
z = lever arm,
l_a = z/d factor.

At ultimate limit state, it is important that member sections in flexure be ductile to ensure gradual failure of the structure instead of sudden catastrophic ones. According to Mosley W.H. et al (2007), this is achieved with gradual yielding of the tensile steel.

Hence, to achieve yielding of tension steel and other factors like hardness at ultimate limit state, depth of compression must be less or equals to 0.45d, i.e;

$$X \leq 0.45d \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

At equilibrium of this section, the ultimate design moment, M must be equal to the the moment of resistance of the section, i.e

$$M_R = F_{cc}\, z = F_{st}\, z \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iv)}$$

Where,

F_{cc} = stress x area of action

$= 0.45f_{cu} \times bs$ …………………… (v)

and

$z = d – s/2$

$s = 2(d-z)$ …………………… (vi)

Substituting equation (v) in (iv),

$M_R = 0.45f_{cu} . bs. z$ ………………….. (vii)

Substituting (vi) in(vii),

$M_R = 0.45f_{cu} . b.2(d- z). z$

$M_R = 0.9f_{cu} . b.(d- z). z$ ……………………. (viii)

According to BS 8110 (1999), upper limit of z = 0.95 and lower limit of z = 0.775d at maximum value of x = 0.5d.

Substituting ;

$K = M/ bd^2 f_{cu}$

$M= Kbd^2 f_{cu}$

Considering lower limit of z = 0.775d and substitute in equation (viii);

$M_R = 0.9f_{cu} . b.(d- 0.775d). 0.775d$

Ultimate Moment of Resistance;

$M_R = 0.156f_{cu} bd^2$ …………………………. (ix)

Equation (ix) is the governing equation for the model.

3. Result and Discussion

3.1 Result: Considering the model equation for a reinforced concrete beam made of 20N/mm^2 and 410 N/mm^2 characteristic strength of concrete and steel respectively;

$M_R = 0.156(20)bd^2$

$M_R = 3.12bd^2$ ……………………… (x)

Simulating dimensional implication of 150, 200, 250 and 300mm breadth, b of the element on stability potential in terms of moment of resistance at varying depth, d of 300, 350, 400, 450, 500, 550, 600, 650 and 700mm using equation (x). Detail in Table 01 and 02.

Table 01

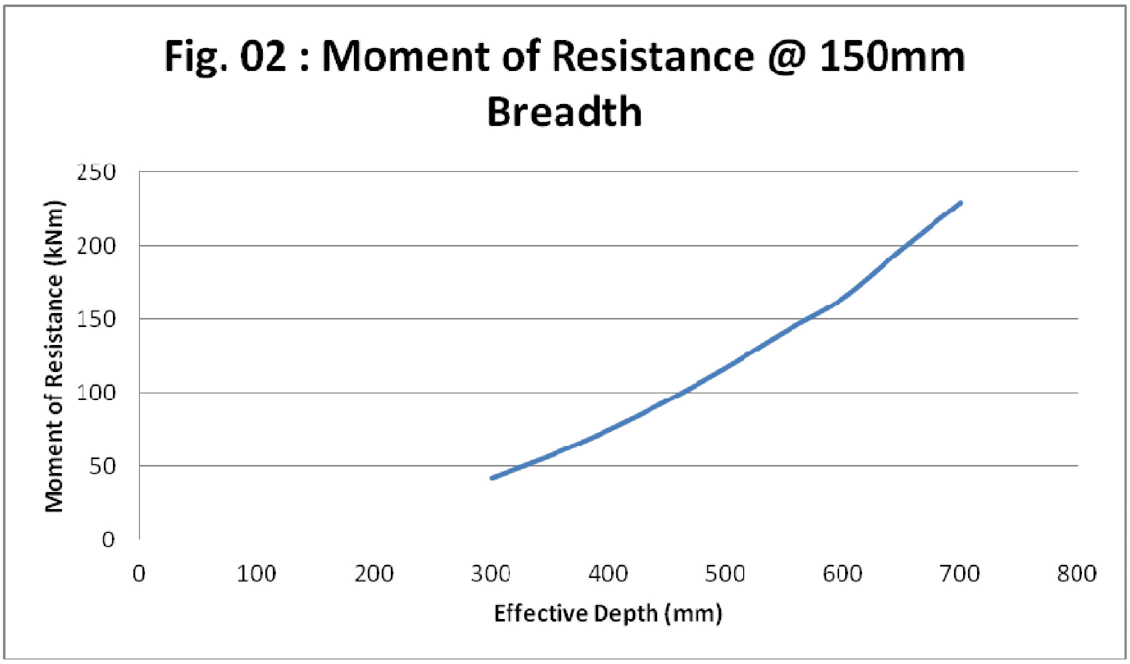
Effective Breadth, b (mm)	Effective Depth, d (mm)	d^2 (mm^2)	$M_R = 3.12bd^2$ (kNm)
150	300	90000	42.12
	350	122500	57.33
	400	160000	74.88
	450	202500	94.77
	500	250000	117.00
	550	302500	141.57
	600	360000	168.48
	650	422500	197.73
	700	490000	229.32
200	300	90000	56.15
	350	122500	76.44
	400	160000	99.84
	450	202500	126.36
	500	250000	156.00
	550	302500	188.76
	600	360000	224.64
	650	422500	263.64
	700	490000	305.76
250	300	90000	70.20
	350	122500	95.55
	400	160000	124.80
	450	202500	157.95
	500	250000	195.00
	550	302500	235.95
	600	360000	280.80
	650	422500	329.55
	700	490000	382.20
300	300	90000	84.24
	350	122500	114.66
	400	160000	149.76
	450	202500	189.54
	500	250000	234.00
	550	302500	283.14
	600	360000	336.96
	650	422500	395.46
	700	490000	458.64

Table 02

Breadth, b (mm)	Mmt. Of Resist. M_R (kNm)	M_R. Diff. (kNm)
150	42.12	-
200	56.15	14.03
250	70.21	14.04
300	84.21	14.00

Table 03

Eff. Depth , d (mm)	Mmt. Of Resist. M_R (kNm)	M_R. Diff. (kNm)
300	42.12	-
350	57.35	15.21
400	74.88	17.55 (2.34)
450	94.77	19.89(2.34)
500	117.00	22.23(2.34)
550	141.57	24.57(2.34)
600	168.48	26.91(2.34)
650	197.73	29.25(2.34)
700	229.32	31.59(2.34)



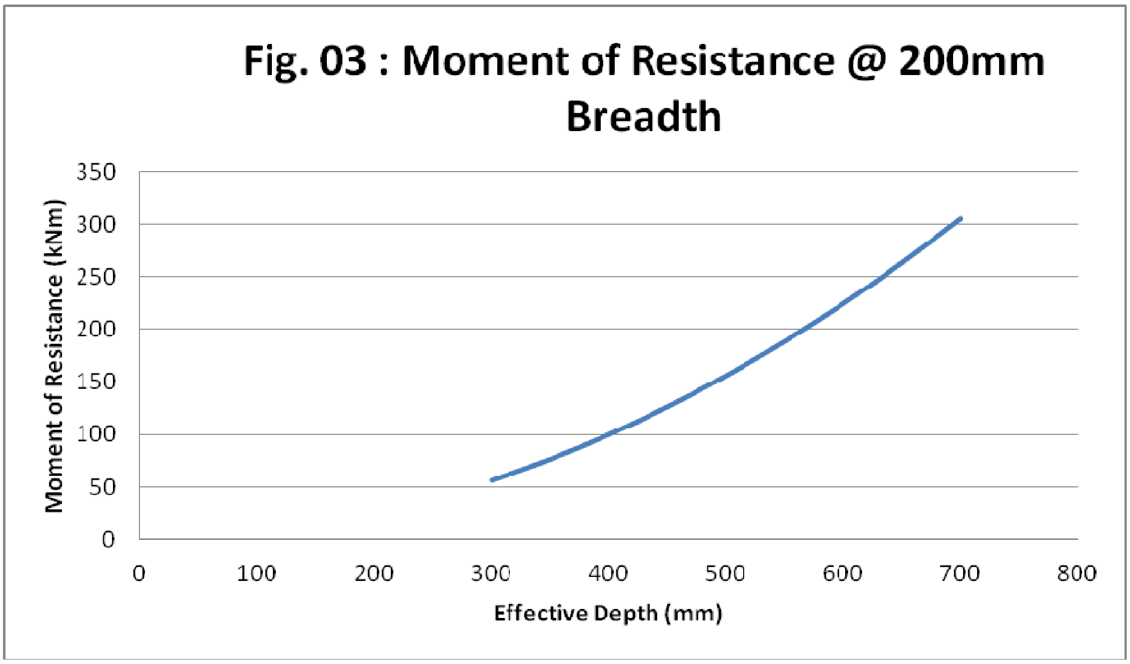


Fig. 03 : Moment of Resistance @ 200mm Breadth
Moment of Resistance (kNm)
0
50
100
150
200
250
300
350
0
100
200
300
400
500
600
700
800
Effective Depth (mm)

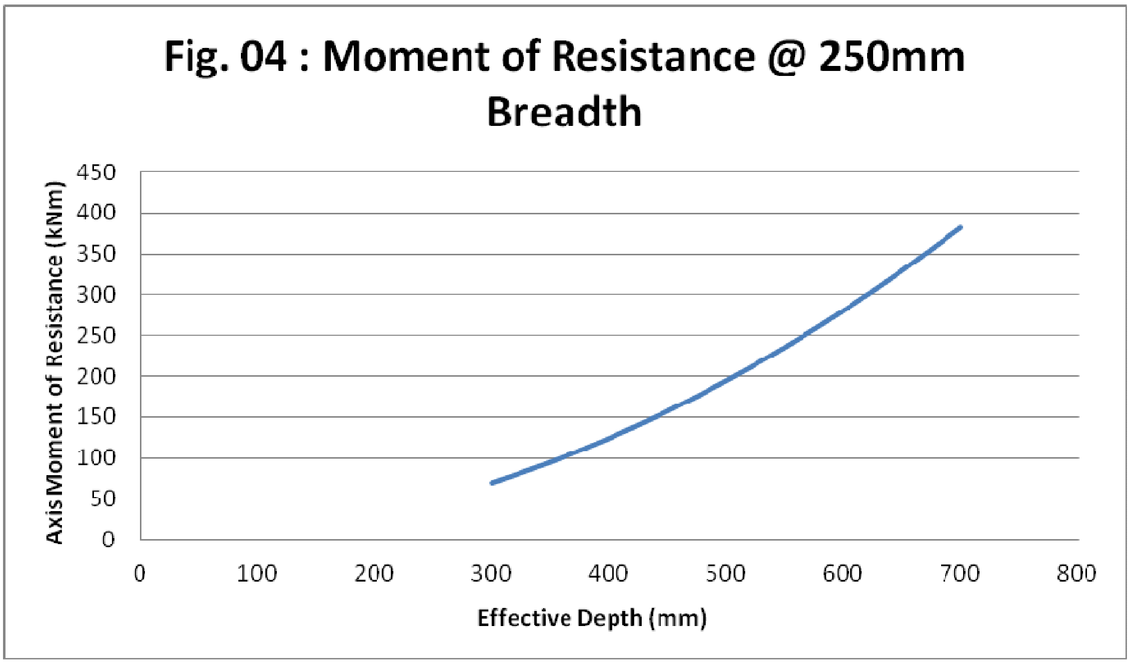


Fig. 04 : Moment of Resistance @ 250mm Breadth
Axis Moment of Resistance (kNm)
0
50
100
150
200
250
300
350
400
450
0
100
200
300
400
500
600
700
800
Effective Depth (mm)

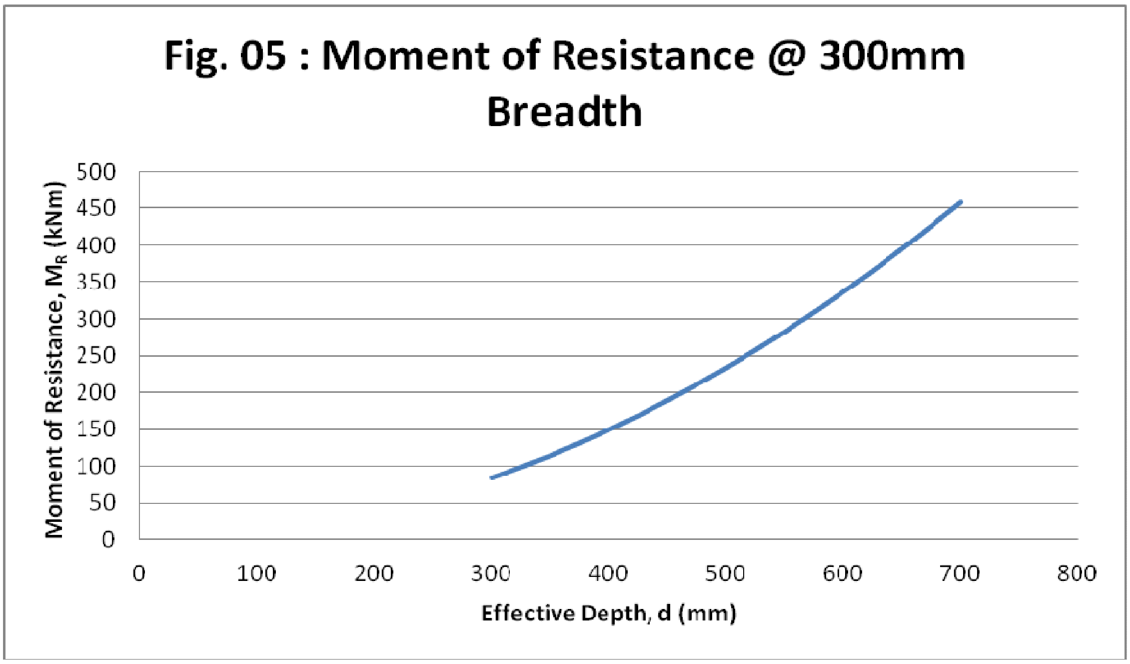


Fig. 05 : Moment of Resistance @ 300mm Breadth
Moment of Resistance, M_R (kNm)
500
450
400
350
300
250
200
150
100
50
0
0
100
200
300
400
500
600
700
800
Effective Depth, d (mm)

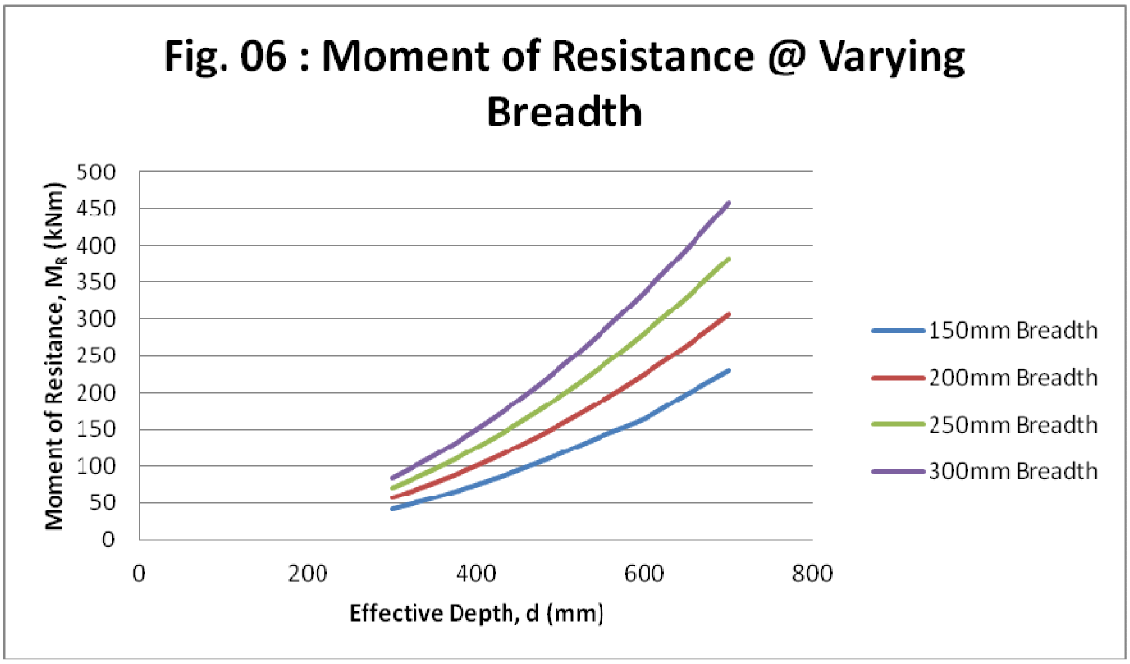
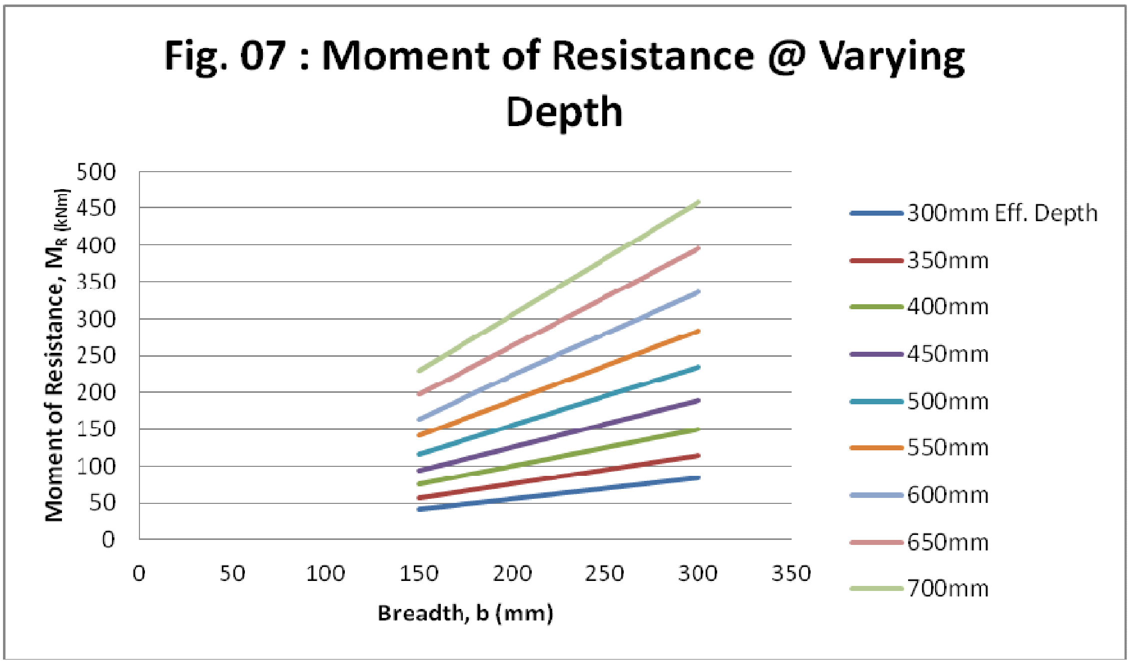


Fig. 06 : Moment of Resistance @ Varying Breadth
Moment of Resitance, M_R (kNm)
500
450
400
350
300
250
200
150
100
50
0
0
200
400
600
800
Effective Depth, d (mm)
150mm Breadth
200mm Breadth
250mm Breadth
300mm Breadth



3.2 Discussion: Experience overtimes had established the fact that reinforced concrete is a durable structural material. The composite material is a combination of concrete and steel. Concrete is poor in tension but good in compression, durable at all conditions of exposure. Steel at the other hand, is good in tension, slender sections are poor in compression but posses good shear strength. Hence, combination of the two gives room for complimentary advantage over the short-comings, Morgan W. (1964) and Buckle I. G. (1979).

On this basis, design of reinforced concrete structures are carried out with assumption that concrete does not resist tensile forces but designed to be transferred to steel through the bond between the interface of the two materials.

Structural members are purposed to carry loads, that which normally induce stresses and strains in materials of concrete and the reinforcing steel. However, consideration of the subjects is applied at the condition of equilibrium of the forces in concrete and steel. While resistance to the imposed moment is taken care of by material strength and sectional dimensions.

Considering the table and figure above, Moment of resistance of rectangular beam increases at every increment in sectional dimension of breadth and depth respectively. Thus, breadth and depth are directly proportional to moment of resistance at varying capacity due to difference in power of degree of the two variables.

It is also noted in table 02 & 03 above that equal magnitude of increment of the two variables produce different additional moment of resistance. As a result of this the figures generated by variable d in the governing equation are curve graphs while that of variable b are straight graphs, see fig. 06 & 07. The difference in the moment of resistance produced by varying breadth, b at a particular eff. Depth is a constant value. For instance, at 300mm eff. Depth, moment of resistance follows a consecutive addition of 14.0kNm. However, in the case of variable d (depth) at a particular value of the breadth b, the difference in moment of resistance is at an increasing trend. For instance, at 150mm breadth, moment of resistance follows an increasing trend of 2.3 kNm.

4. Conclusion

The results and tables above establish the consistency of the basic principle of stress-strain relationship of concrete materials and its sectional dimensional combinations implications of rectangular sections in structural design. By this knowledge, it is very easy to establish the magnitude of stress which can be safely supported by a particular dimensional combination.

Thus in modern design, it is pertinent that a special care be taken at the time of designing a structure. That the same should be able to withstand the stresses under various load conditions in order to avoid failure.

For this reason, it is considered essential to have a complete data about the properties of the selected materials while designing a particular structure and appropriate prediction of the implications of dimensional combinations.

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