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The Effect of Length on Free Vibration Response of Cantilever Beam

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Abstract

The point of this study is to find the best value of frictional and structural damping constant for different length of aluminum cantilever beam when applied 30 mm initial displacement at the tip of the cantilever beam, and get the vibration response of cantilever beam. Initial displacement is used to test dynamic characteristics of cantilever beam. This article is based on cantilever beam under initial displacement. For each length of beam there are specific values of mass (frictional) damping and stiffness (structural) damping. The value of stiffness damping will decrease and mass damping will increase at increasing the length of beam.

Keywords: Cantilever beam, Finite Element Modeling, vibration response, initial displacement, mass damping, stiffness damping.

1. Introduction

The term of vibration points to the limited reciprocation motion of a particle or an object. In mechanical systems, vibrations are dangerous problems that are causing a lot of damage to structures and it is undesirable if the structure vibratory motion becomes excessive.

Beams are members of structure which have smaller dimensions of cross-sections as compared to its length and subjected to transverse loading. Cantilever beam is fixed at one end and free at the other.

A cantilever beam is continuous system, so its mass and elasticity are distributed at all over volume. Cantilever beam has infinite degree of freedom and infinite natural frequencies.

Gergely Takács, and Boris Rohal'-Ilkiv, were studied the classical approximated prototyping which are involves dividing the design process in two stages: differentiate open-loop FEA model and execution a simple system identification procedure, then building a close-loop control system using a simplified mathematical model of the structure [1]. Shibly Ahmed Al-Samarraie, Mohsin N. Hamzah, and Imad Abdulhussein Abdulsahib, Studied vibration suppression of the smart structure and performed the smart structure by using piezoelectric patch structure. The smart structure represented by cantilever beam attached with two piezoelectric as sensor and actuator, and design the controller that based on reduced the order modal of the large scale system. Optimal LQR controller was used as a design control for the smart structure that attenuate the vibration of smart cantilever beam with attached piezoelectric [2]. Sushan Li, Steffen Ochs, Tobias Melz, the design of controller is used to compensate the vibration of the beam. The beam is attached with piezoelectric as a sensor or an actuator is called smart beam (smart structure), that is used to reduce the vibration of the beam. LQR and LC were used as controller of the beam and comparative between them [3]. Deepak Chhabra, Pankaj Chandna , Gian Bhushan, were investigated the active vibration control of cantilever beam that attached with piezoelectric as actuator and sensor at the top and bottom surfaces of the cantilever beam. Used Euler-Bernoulli theory as finite element model. The designing of optimal LQR and the technique of the state/output feedback control by pole placement control scheme is very effective in controlling the vibration [4].

2. Finite Element Method

Finite element method is a technique of a numerical solution that can be used for solution of difficult engineering problems. Finite element computer program Mechanical APDL (ANSYS Parametric Design Language) version 15.0 was used. The finite element analysis that used as design tool has development in the resent years. Cantilever beam was modeled in ANSYS FEA software package. Where the beam was modeled in ANSYS by solid45 which has eight nodes, that each node has three degree of freedom (DOF) only, the beam is made from aluminum in the form of cantilever beam. The displacement only chose as the DOF to the beam. [5]

3. The Finite Element Solution Using ANSYS

In this analysis, beam is created in ANSYS by using BLOCK command. The meshing is done, after assigning appropriate the element type and material properties of beam. The solid45 element type was used to mesh the beam. The meshing is a process to divide the whole structure in to small parts, and the result became more accuracy when the number of elements is increased. After meshing the boundary condition of fixed end is done, where by letting the degree of freedom equal to zero as show in figure (1). During the solution, initial displacement 1mm is applied at tip of cantilever beam to obtain free vibration, and get the response of cantilever beam for different length. The results of response are shown below. [6]

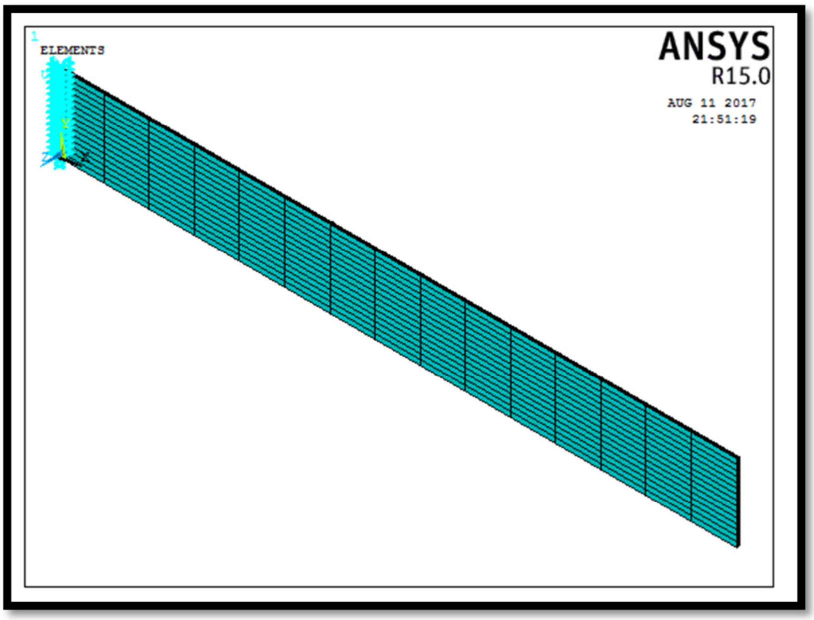

Figure (1): FEA of cantilever beam showing element.

4. Finite Element Formulation of Beam Element

The element of beam is assumed with two nodes at it end. Each of these nodes has two degree of freedom (DOF). The element shape functions are derived by applying boundary conditions and by take into considerations an approximate solution. The matrix of stiffness and mass is derived by using shape functions for the element of beam. The final two row's two element of first matrix are append with first two row's two element of next matrix. The global stiffness and mass matrix is formative. The boundary conditions are applied on the global matrices for the cantilever beam. The first two columns two rows should be removed as one end of the cantilever beam is fixed. The real response of the system, i.e. the displacement $u(x,t)$ is attaining for all the diverse models of the cantilever beam with and without the controller by assuming the first prevailing vibratory modes. [2]

The displacement u is given by $(x) = [N]^T[P]$

$$X = [N1(x)N2(x)N3(x)N4(x)]\begin{bmatrix} u_1 \\ \theta_1 \\ u_2 \\ \theta_2 \end{bmatrix}$$

Where $N1(x)$, $N2(x)$, $N3(x)$, and $N4(x)$ are the shape functions and u_1, θ_1, and u_2, θ_2 are DOF's at each node.

Where $N1(x) = 1 - \frac{3x^2}{l_b^2} + \frac{2x^3}{l_b^3}$

$$N2(x) = x - \frac{2x^2}{l_b} + \frac{x^3}{l_b^3}$$

$$N3(X) = \frac{3x^2}{l_b^2} - \frac{2x^3}{l_b^3}$$

$$N4(x) = \frac{-x^2}{l_b} + \frac{x^3}{l_b^2}$$

The governing differential of motion for the beam element can be represented as:

$$M\ddot{P} + C\dot{P} + KP = q$$

Where M, C, K, and q are mass, damping, stiffness, and the force co-ordinate vectors of beam element. The shapely mass matrix and stiffness matrix are obtained as:-

$$[M] = \rho A \int_0^l [\ddot{N}]^T \; [N] \; dx$$

$$[M] = \frac{\rho A l_b}{420}\begin{bmatrix} 156 & 22l_b & 54 & -13l_b \\ 22l_b & 4l_b^2 & 13l_b & -3l_b^2 \\ 54 & 13l_b & 156 & -22l_b \\ -13l_b & -3l_b^2 & -22l_b & 4l_b^2 \end{bmatrix}$$

$$[K] = EI \int_0^l [N^{\cdot\cdot}]^T \, [\dot{N}] \, dx$$

$$[K] = \frac{E_b I_b}{l_b^3}\begin{bmatrix} 12 & 6l_b & -12 & 6l_b \\ 6l_b & 4l_b^2 & -6l_b & 2l_b^2 \\ -12 & -6l_b & 12 & -6l_b \\ 6l_b & 2l_b^2 & -6l_b & 4l_b^2 \end{bmatrix}$$

$$C = \alpha M + \beta K$$

Where α is the mass (frictional) damping constant, β is the stiffness (structural) damping constant. And its calculated from

$$\xi i = \frac{\alpha}{2wi} + \frac{\beta wi}{2}$$

Where ξi *and* wi are the damping ratio and natural frequency of ith mode.

5. Problem Statement

Long and thin Cantilever beams (one end of the beam is fixed and the other is free) were studied in this paper. The origin of coordinate axis is at the fixed end of the cantilever beam. The material of cantilever beam is aluminum. A series of FE modal are generated for the beam with length from 200mm to 400mm, and the width and the thickness of beam stay constant. The three-dimensional FEM of beam is performed in ANSYS 15.0 and calculate the vibration response of beams. The geometric and material properties of cantilever beam are taken from the Table 1

Table 1: Geometric Parameter and Material Property

Geometric parameter of beam	Material property of beam
$L = from\ 200\ to\ 400\ mm$	$E = 70x10^9 N/M^2$
$B = 40\ mm$	$\rho = 2700\ kg/M^3$
$H = 1.4\ mm$	$v = 0.34$

A. 200 mm Length

The vibration response of the beam is show in figure 2. The mass damping α that represents the frictional damping constant is defined as α *equal* 0 as in reference [3]. Then the stiffness damping β that represents the structural damping constant is defined as β *equal* 10^{-5} as in reference [3]

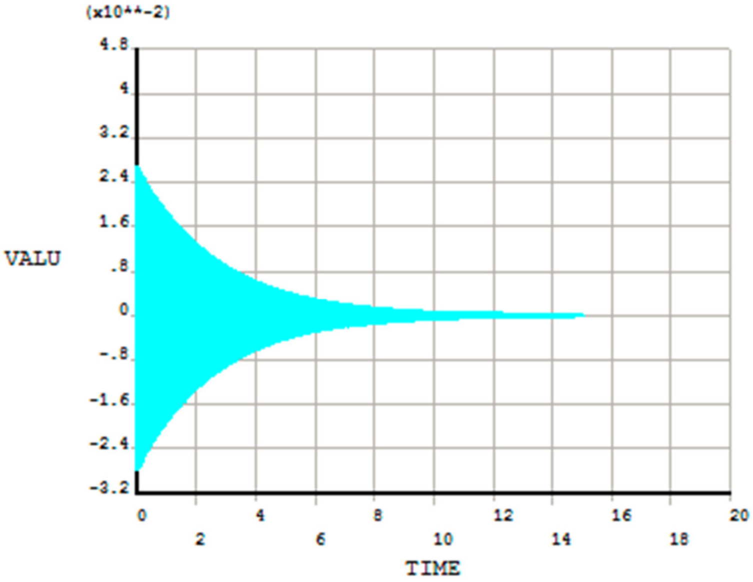

Figure 2: Free Vibration Response

B. 250 mm Length

The vibration response of the beam is show in figure 3. The mass damping α that represents the frictional damping constant is defined as α *equal* 0.001 as in reference [4]. Then the stiffness damping β that represents the structural damping constant is defined as β *equal* 0.0001 as in reference [4].

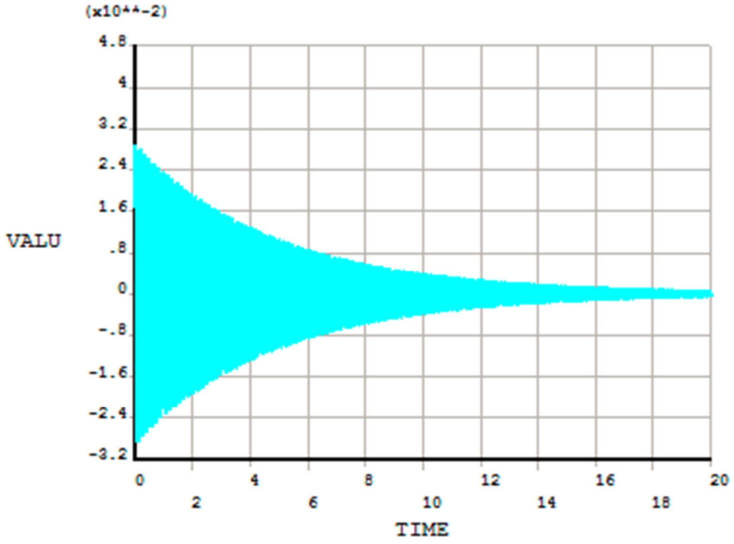

Figure 3: Free Vibration Response.

C. 350 mm Length

The vibration response of the beam is show in figure 5. By using the trial and error method find that the best value of mass damping and stiffness damping for this length is mass damping $\alpha\ equal\ 0$, and stiffness damping $\beta\ equal\ 0.0001$.

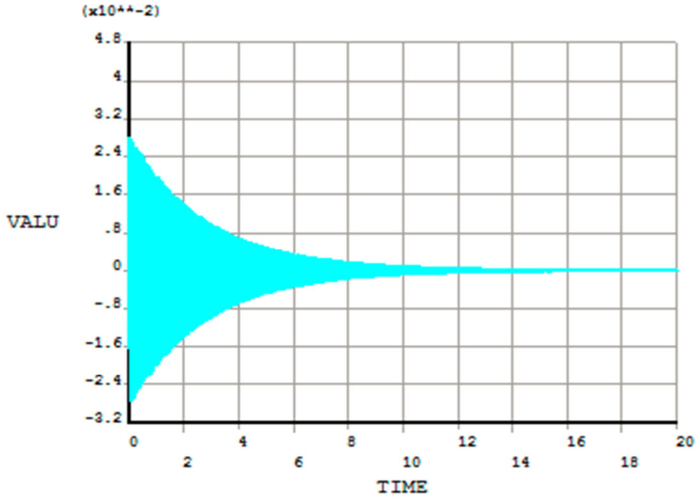

Figure 5: Free Vibration Response

D. 400 mm Length

The vibration response of the beam is show in figure 6. The mass damping α that represents the frictional damping constant is defined as $\alpha\ equal\ 0$ as in reference [1]. Then the stiffness damping β that represents the structural damping constant is defined as $\beta\ equal\ 0.00015$ as in reference [1].

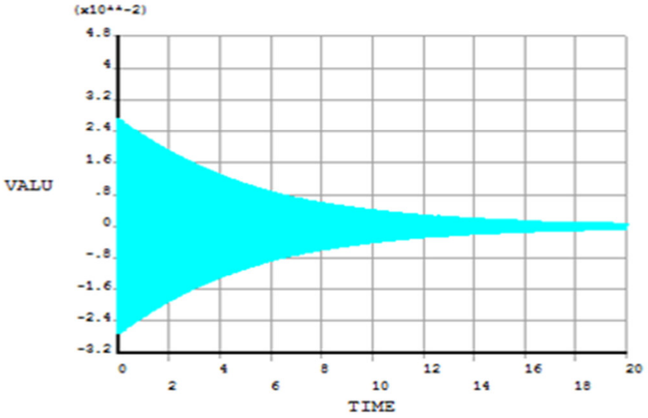

Figure 6: Free Vibration Response

6. Free Vibration Response Verification

The Finite Element Analysis of mechanical APDL ANSYS program is used to calculate tip displacement response for cantilever beam used in reference [1] to verification the program that used in this research. The tip displacement response of reference [1] is shown in figure 7 (a) for cantilever beam. The verification tip displacement response is shown in figure 7 (b) for cantilever beam. It was observed from the result there is good matching between the results of APDL ANSYS and the result of reference [1] is completely identical.

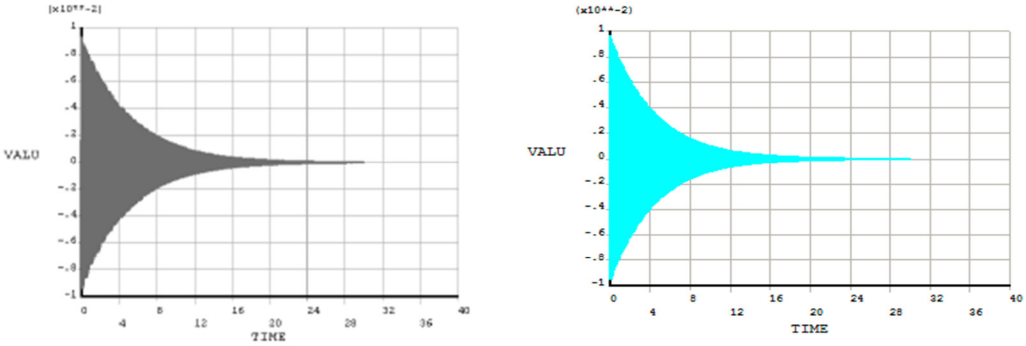

Figure 7: (a) Free Vibration Response of reference [1]: (b) Free Vibration Response of APDL ANSYS.

7. Conclusion

The shape of free vibration response of cantilever beam depended on the mass (frictional) damping and stiffness (structural) damping, where these values change from length to length. The mass and stiffness damping depended on property and dimensions of cantilever beam. In this study, the properties of cantilever beam are constant and the dimensions are variable. For each length of cantilever beam there are specific values of mass damping and stiffness damping. From this study, the main conclusion is when the length of cantilever beam is increase the stiffness damping is decreased, and mass damping is increased.

Reference

[1] Gergely Takács, and Boris Rohal'-Ilkiv, "DIRECT CLOSED-LOOP ACTIVE VIBRATION CONTROL SYSTEM PROTOTYPING IN ANSYS", Noise and Vibration: Emerging Methods, NOVEM 2012, 1-4 April 2012, Sorrento.

[2] Shibly Ahmed AL-Samarraie, Moshin N. Hamzah, Imad Abdulhussein Abdulsahib, " Vibration Suppression Control for a Flexible Beam with Sliding Mode Observer", Al-Nahrain University, College of Engineering Journal Vol.9= No.2, 619: pp.327 – 341.

[3] Sushan Li, Steffen Ochs, Tobias Melz, "DESIGN OF CONTROL CONCEPTS FOR A SMART BEAM STRUCTURE WITH REGARD TO SENSITIVITY ANALYSIS OF THE SYSTEM", 7th ECCOMAS Thematic Conference on Smart Structures and Materials SMART 2015.

[4] Deepak Chhabra, Pankaj Chandna , Gian Bhushan, "Design and Analysis of Smart Structures for Active Vibration Control using Piezo-Crystals" International Journal of Engineering and Technology Volume 1 No. 3, December, 2011.

[5] DAVID V. HUTTON, "FUNDAMENTALS OF FINITE ELEMENT ANALYSIS", BOOK 1st ed. ISBN 0-07-239536-2, 2014.

[6] ANSYS Structural Analysis Guide, ANSYS Release 15, ANSYS Inc., South Point, Canonsburg, 2004