

# Performance of RS and BCH Codes over Correlated Rayleigh Fading Channel using QAM Modulation Technique

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## Abstract

The quest for a reliable reception of the replica of a transmitted signal over wireless medium has become very important in the transmission and reception of information. In order to mitigate the effect of the degradation of the transmitted signal, error correction techniques are employed to reliably recover the erroneous bits inherent especially when the channel is in deep fading. In this paper, the performance of Reed Solomon (RS) and Bose-Chaudhuri-Hochquenghem (BCH) codes over correlated Rayleigh fading is presented. This work also investigated the effect of error correction code in the variation of the modulation order of the QAM. The simulation was carried out by encoding randomly generated data using both RS and BCH codes and then modulated by varying the modulation order of the quadrature amplitude modulation before transmitted over a correlated Rayleigh distributed channel. At the receiver, the received signal was demodulated and decoded. The bits in error received were detected and corrected to retrieve back the original transmitted signal. The system was evaluated in bit error rate and the result showed that BCH code performs better for higher modulation order.

**Keywords:** Correlated channel, Rayleigh fading, M-QAM, RS code, BCH code.

## 1. Introduction

The process of digital communication involves the transmission and reception of message with no error [1],[2]. Wireless communication systems suffer from channel impairments such as reflection, diffraction, pathloss, shadowing and fading. The signal that is transmitted into the wireless environment arrives at the receiver along a number of distinct paths known as multipath [3]. Also, multipath effects result in the late arrival of the signals at the receiver and the channel being in deep fade, which leads to signal distortion and burst errors [4] [5]. Many techniques have been proposed by researchers to mitigate the effect of multipath such as diversity, multicarrier and coding [6], [7]. The coding technique involves the detection and correction of erroneous bits in the transmitted signal [8], [4] which has become increasingly important with the emergence of new data transmission and storage devices. The use of forward error correcting codes in digital communication systems is an integral part of ensuring a reliable communication [5]. Forward error corrections are most preferred than other error correction techniques due to their great capability for correcting burst errors and simple, does not suffer from delay and additional retransmission cost [9]. RS and BCH codes are one of the powerful forward error correction tools used in different data transmission and storage applications. Some of the work carried out so far that focus only on RS codes includes [5], [10], [11]. On the performance of RS and BCH codes have focused on concatenated BCH code using binary phase shift keying (BPSK) modulation schemes [12], [1] compared the performance of RS and BCH codes over Rayleigh fading channel using BPSK and QAM schemes with BCH performing better than RS in both modulation schemes, others include [13], [14]. This paper addresses the detection and correction of errors propagated in the transmitted signal over correlated Rayleigh fading channel. RS and BCH codes are both evaluated in bit error rate using QAM modulation schemes.

## 2. Materials and Method

### 2.1 System Model

The system model for the simulation of the wireless communication systems is as shown in Figure 1.

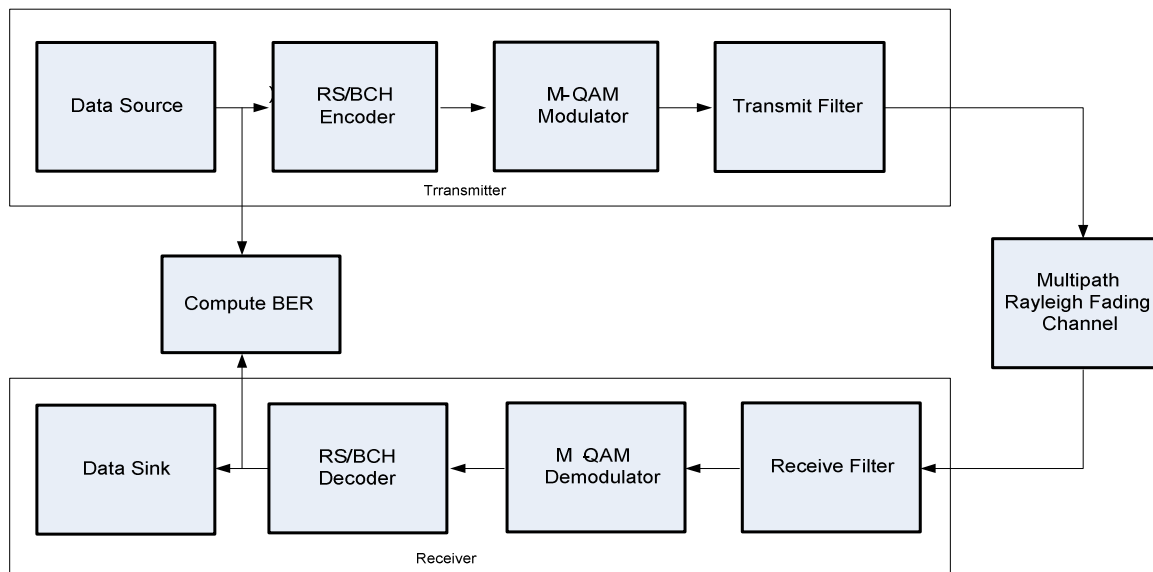


Figure 1: System model

## 2.2 Encoding and Decoding for Reed Solomon Codes

The Reed Solomon code named after its inventors; Irving S. Reed and Gustave Solomon [15]. Reed Solomon code is a member of the cyclic codes which are capable of correcting errors which appear in bursts and are commonly used in concatenated coding systems. The code described a systematic way of building codes that could detect and correct multiple random symbol errors. Furthermore, RS codes are suitable as multiple burst bit error correcting codes.

Reed-Solomon codes are non-binary cyclic codes with symbols made up of  $m$ -bit sequences, where  $m$  is any positive integer having a value greater than 2. R-S  $(n, k)$  codes on  $m$ -bit symbols exists for all  $n$  and  $k$  for which  $0 < k < n < 2^m + 2$ , where  $k$  is the number of data symbols being encoded, and  $n$  is the total number of code symbols in the encoded block [16],[17].

For the most conventional R-S  $(n, k)$  code,

$$(n, k) = (2^m - 1, 2^m - 1 - 2t) \quad (1)$$

where  $t$  is the symbol error correcting capability of the code, and

$$n - k = 2t \text{ is the number of parity symbols.}$$

An extended RS code can be made up with  $n = 2^m$  or  $n = 2^m + 1$ , but not any further.

Reed-Solomon codes achieve the largest possible code minimum distance for any linear code with the same encoder input and output block lengths [18]. For non-binary codes, the distance between two codewords is defined as the number of symbols in which the sequences differ. For Reed Solomon codes, the code minimum distance is given by [1],[4], [19]

$$D_{min} = n - k + 1 \quad (2)$$

The code is capable of correcting any combination of  $t$  or fewer errors, where  $t$  can be expressed as

$$t = \left\lfloor \frac{d_{min}-1}{2} \right\rfloor = \left\lfloor \frac{n-k}{2} \right\rfloor \quad (3)$$

Equation (3) shows that for RS codes, correcting  $t$  symbol errors requires no more than  $2t$  symbol parity symbols. Hence, one can say that the decoder has  $n - k$  redundant symbol to spend, which is twice the amount of correctable errors. For each error, one redundant symbol is used to locate the error, and another redundant symbol is used to find its correct value.

The erasure-correcting capability of the code is

$$\rho = d_{min} - 1 = n - k \quad (4)$$

Simultaneous error-correction and erasure-correction capability can be expressed as follows:

$2\alpha + \gamma < d_{min} < n - k$ , where  $\alpha$  is the number of symbol-error patterns that can be corrected and  $\gamma$  is the number of symbol erasure patterns that can be corrected.

### 2.2.1 Reed Solomon Encoder

Reed Solomon codes are based on Galois's field. If a finite field of  $q$  elements is chosen, whose  $GF(2^m)$  as a result of message  $f$  to be transmitted consists of  $k$  elements of  $GF(2^m)$  which is given by [1]

$$f = (f_0, f_1, \dots, f_{(k-1)}) \quad (5)$$

The message polynomial is calculated by multiplying the coefficients of the message with appropriate power of

$x$  which is expressed as

$$f(x) = f_0 + f_1 x + \dots + f_{(k-1)} x^{(k-1)} \quad (6)$$

The remaining polynomial is known as parity check polynomial

$$b(x) = b_0 + b_1 x + \dots + b_{(2t-1)} x^{(2t-1)} \quad (7)$$

The codeword is formed by adding (6) and (7)

$$v(x) = f(x) + b(x) \quad (8)$$

### 2.2.2 Reed Solomon Decoder

After the transmitted signal has been propagated through the channel, the received message at the receiver can be given as

$$r(x) = c(x) + e(x) \quad (9)$$

where  $c(x)$  is the original transmitted codeword and  $e(x)$  is the error which is expressed as

$$e(x) = e_{(n-1)} x^{(n-1)} + \dots + e_1 x + e_0 \quad (10)$$

Using RS code  $t = ((n - k) / 2)$  errors can be corrected, if errors are more than  $t$  then the code fails [1],[10].

### 2.3 Bose-Chaudhuri-Hocquenghem Codes

BCH codes named after its inventors; Hocquenghem, Bose and Ray-Chaudhuri was invented in the year 1959 [20]. This code forms a class of cyclic error-correcting codes that are constructed using finite fields. One of the key features of BCH codes is that during code design, there is a precise control over the number of symbol errors correctable by the code. Another advantage is the ease with which they can be decoded [21]. BCH code is a multi-level cyclic variable length digital error correcting code used to correct multiple random error patterns.

#### 2.3.1 BCH Encoder and Decoder

BCH codes are generally characterized for any positive integers  $m \geq 3$  and  $t$  error as  $(k \leq 2^m - 1)$  by the following parameters

Block length:  $n = 2^m - 1$  (11)

Parity check bits:  $n - k \leq mt$  (12)

Minimum distance:  $d_{min} \geq 2t + 1$  (13)

In BCH codes,  $\alpha$  is a primitive element of  $GF(2^m)$ . The generator polynomial is the lowest degree polynomial over  $GF(2)$  which has  $\alpha, \alpha^2, \alpha^3, \dots, \alpha^{2t}$ . The generator of a binary BCH codes is found to be least common multiples of the minimal polynomial of each  $\alpha^i$  term ( $0 < i < 2t$ ). To simplify the generator polynomial, the even power of a primitive element is considered. The generating polynomial  $g(x)$  of BCH codes for  $t$  correctable error and length of codeword  $2^m - 1$  is given by [10]

$$g(x) = LCM\{\phi_1(x), \phi_2(x), \dots, \phi_{(2t-1)}(x)\} \quad (14)$$

### 2.4 M-ary Quadrature Amplitude Modulation

One of the widely used schemes is the Quadrature Amplitude Modulation (QAM) where phase and amplitude modulation techniques are combined. In QAM, incoming bits are mapped into two base signals  $S_1(t)$  and  $S_2(t)$ .  $S_1(t)$  is then multiplied by  $\cos \llbracket (2\pi f_o t) \rrbracket$  and  $S_2(t)$  by  $\sin \llbracket (2\pi f_o t) \rrbracket$ . The sum of these products forms the transmitted QAM signal. At the receiver however, the incoming waveform is separately multiplied by  $\cos \llbracket (2\pi f_o t) \rrbracket$  and  $\sin \llbracket (2\pi f_o t) \rrbracket$  respectively. When these multiplied waveforms are filtered,  $(S_1(t)) / 2$  and  $(S_2(t)) / 2$  are gotten, from which the bit stream is reconstructed. QAM is widely used in high speed modems [4] and [19].

$$s(t) = I(t) \cos \llbracket (2\pi f_o t) \rrbracket + Q(t) \sin \llbracket (2\pi f_o t) \rrbracket \quad (15)$$

where  $I(t)$  and  $Q(t)$  are the modulating signals and  $f_o$  is the carrier frequency.

Like all modulation schemes, QAM conveys data by changing some aspect of a carrier signal, or the carrier wave in response to a carrier signal. In the case of QAM, the amplitude of two waves,  $90^\circ$  out of each other are changed to represent the data signal. Amplitude modulating two carriers in quadrature can be equivalently viewed as both amplitude modulating and phase modulating a single carrier.

### 2.5 Rayleigh Fading Distribution

This is one of the distributions encountered in multipath propagation, this occurs when the envelope of the received signal follows a Rayleigh distribution. It is used to model locations that are heavily shadowed by surrounding buildings as Rayleigh Distribution. Rayleigh distribution is statistically used to model a faded signal when there is no dominant path. The envelope of the received signal with Rayleigh distribution has the probability density function (pdf) given by [4],[19] and [22]

$$f(r) = r/\sigma^2 e^{((-r^2)/(2\sigma^2))} \quad (16)$$

where  $r$  is the received signal envelope.  
 $r^2/2$  is the instantaneous power.  
 $\sigma$  is the root mean square (r.m.s) value of the received signal  
 $\sigma^2$  is the local average power of the received signal before envelope detection.

### 3. Simulation Parameters

The simulation parameters are as presented in Table 1.

Table 1: Simulation Parameters for the Coding System

Parameters	Values
Message Length	$1 \times 10^4$
Modulation Order	16, 32, 64
Carrier Frequency	900MHz
Speed of Light	$3 \times 10^8$ m/s
Propagation Environment	Rayleigh Fading Channel
SNR	0dB-14dB
Correlation Coefficient	0.15

### 4. Results and Discussion

The results obtained from the performance of M-QAM over correlated Rayleigh fading using RS and BCH codes are presented herein. Figure 2 shows the modulation schemes for 16-QAM, 32-QAM and 64-QAM for the uncoded signals. Comparing the three modulation order, the BER values of 0.1331, 0.1083 and 0.0810 were obtained for 16-QAM, 32-QAM and 64-QAM respectively at an initial SNR of 0dB. However, as the SNR increases, the BER value for the 16-QAM 32-QAM and 64-QAM decreases slightly for the same correlation coefficient of 0.15. Thus, it was observed that for the uncoded signals, as the modulation order increases, with an increase in SNR, the BER value decreases. Figure 3 shows the performance of the RS coded signal. The result obtained shows that 16-QAM has a better performances compared to 32-QAM and 64-QAM with BER values of 0.0231, 0.0239 and 0.0243 respectively at SNR of 10dB. The result shown in Figure 4 depicts the performance for the 16-QAM, 32-QAM and 64-QAM for the BCH coded signals. From the graph, it was observed that for the BCH coded 16-QAM, at SNR of 10dB, the BER value of 0.0471 was obtained. Also, the BER values of 0.0625 and 0.0123 were obtained for 32-QAM and 64-QAM respectively. However, it was observed that the 64-QAM has a higher significant decrease in BER as the SNR increases. Therefore, the 64-QAM has a better performance than the others when coded with BCH. The performance of the RS and BCH coded signals over correlated Rayleigh fading channel is shown in Figure 5. The result obtained shows that the BCH 64-QAM has the lowest BER value at all SNR. This means that transmitting BCH 64-QAM signal would be advisable. However, the result obtained for the RS M-QAM shows a very significant and better performance over BCH 16-QAM and BCH 32-QAM in that the BER values obtained in RS coding at all SNRs are lower than the result obtained in BCH coding.

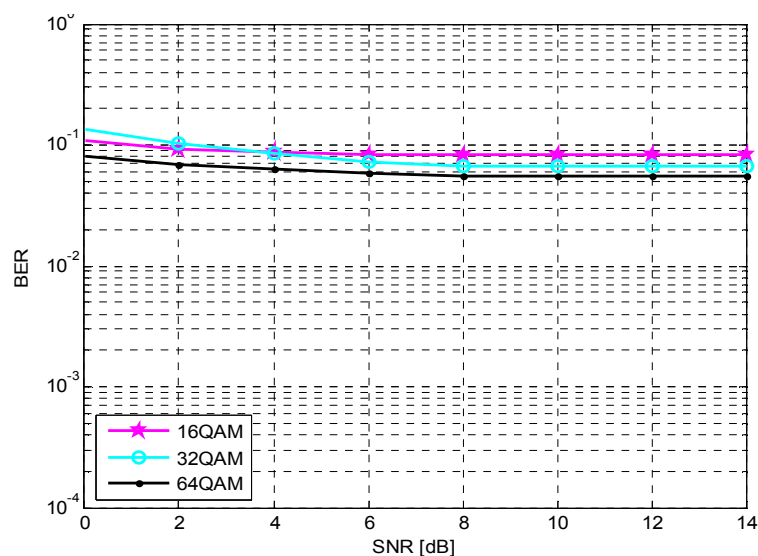


Figure 2: BER Performance of uncoded M-QAM signal over correlated Rayleigh channel

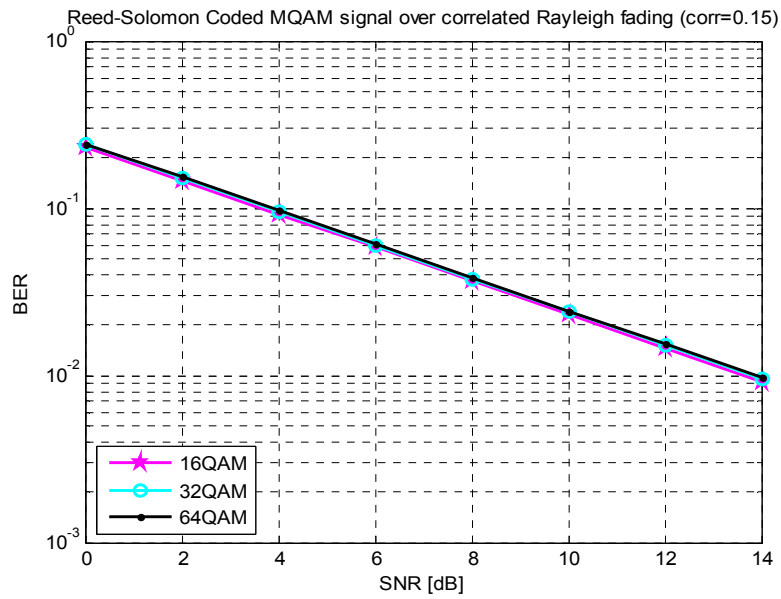


Figure 3: BER Performance of RS coded M-QAM signal over correlated Rayleigh channel

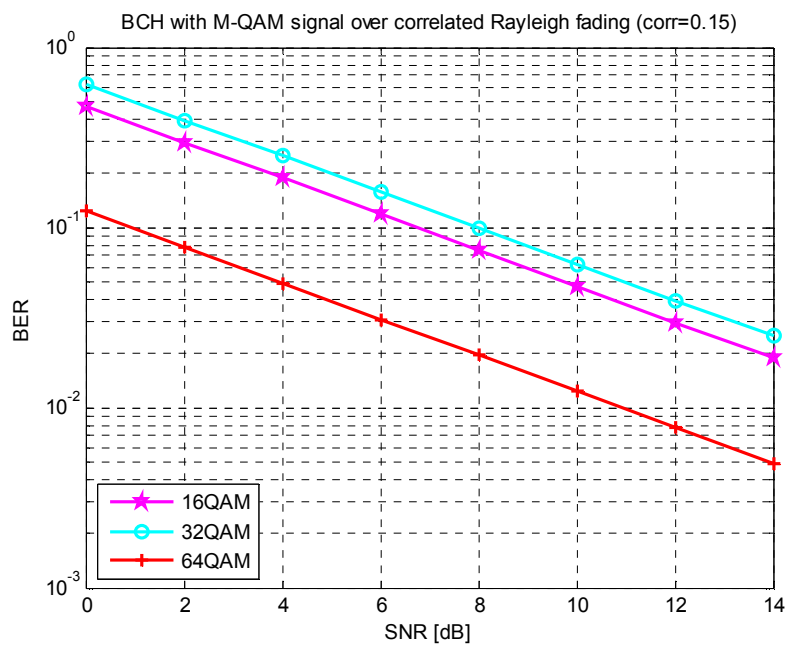


Figure 4: BER Performance of BCH coded M-QAM signal over correlated Rayleigh channel

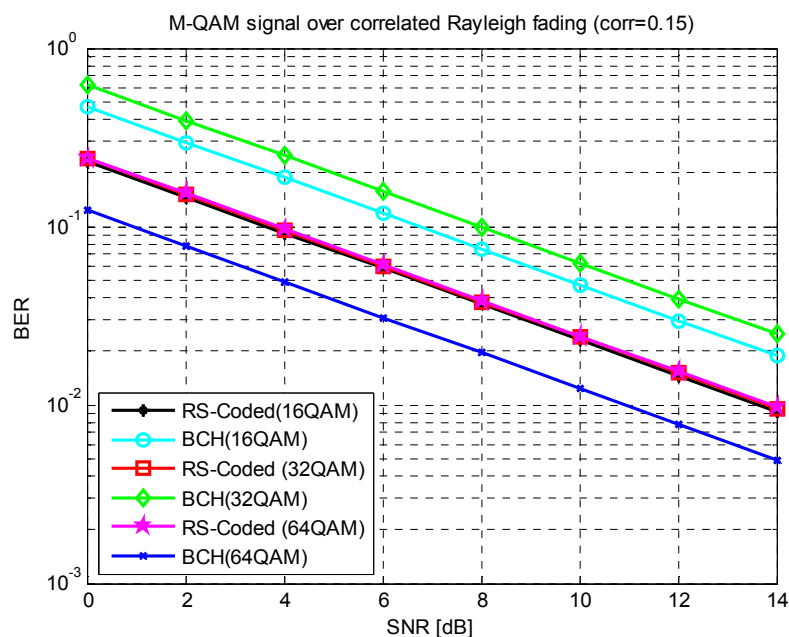


Figure 5: Comparison of BER Performance of RS and BCH coded M-QAM signal over correlated Rayleigh channel

## 5. Conclusion

The simulation results of RS and BCH code over correlated Rayleigh fading channel has been presented. The simulation was performed in MATLAB software environment. The M-QAM modulation scheme was used in the order 16, 32 and 64. The performance of the RS and BCH codes were evaluated in terms of BER using randomly generated binary data. The results obtained showed that for the uncoded signals, as the modulation order increases, the better the performance. When RS and BCH codes, were used, there was a decrease in the BER performance for all the modulation order. However, the erroneous bits obtained for the BCH 64-QAM is lesser compared to that of all other signals at all SNR. This study has further revealed the necessity for error detection and correction scheme. This study is further open for research by exploring other fading channel distributions and adaptability of the error correction code to emerging generation of wireless system.

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