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Some fixed Point Results for cone metric space

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Abstract

In the Present paper we prove some fixed point theorems in cone metric space our result generalizes the previous result of mathematicians.
Keywords:- fixed point cone, metric space, Altering function.

2. Introduction & Preliminaries

Since the Banach contraction Principles several types of generalization contraction mapping on metric spaces have appeared, one such method of generalization is altering the distances. Delbosco [2] and skof [3] have established fixed point theorems for self maps of complete matric spaces by altering the distances between the points with the use of a positive real valued function Hunage and zhag [1] introduction the concept of cone metric space by replacing the set of real numbers by an ordered Banach space and obtained some fixed point results. Recently Asadi and Soleimani [7] prove some fixed point results on cone metric space by using altering distance function and the (ID) property of partially ordered cone metric space (see[7]). We are giving some new results by introducing a vector valued function in cone metric space which has similarity with altering function it becomes the generalization of altering function in view of cone used in place of positive read numbers as well as the constraints used for self map of cone metric spaces.

Definition-: Let (X,d) be a cone metric space a self mapping T on x is called an almost jaggi contraction if it satisfies the following condition.

$$d(T_x,T_y) \leq \propto.\frac{[d(x,T_x)d(y,T_y)}{d(x,y)} + \beta\, d(x,y)$$

$$+\frac{\gamma[d(x,T_y)+d(y,T_x)+d(x,T_x)+d(y,T_y)}{1+d(x,T_x).d(y,T_y).d(x,T_y).d(y,T_x)}$$

$$+L\, Min\{d(x,T_y).d(y,T_x)$$

$$+j\, Min\{d(x,T_x)d(y,T_y)\}$$

For all $x, y, \in X$ where $L.j \geq 0$ and $\alpha, \beta, \gamma \in [0,1]$ with

Theorem 3.1 Let $(x.d)$ be a complete cone metric space and p a normal cone with normal constant M, Let $T: X \to X$ be on almost jaggi contraction for all $x, y \in X$ where $L, J \geq 0$ and $\alpha, \beta, \gamma \in [0,1]$ with $\alpha+\beta+\gamma > 1$ then T has a unique fixed point in X.

Proof:- choose $x_0 \in X$ set $x_1 = Tx_0.x_n = Tx_{n-1}$

$$d(x_n,x_{n+1}) = d(Tx_{n-1},Tx_n)$$

$$\leq \frac{\propto.d(x_{n-1},Tx_{n-1}).d(x_n,Tx_n)}{d(x_{n-1},x_n)} + \beta d(x_{n-1},x_n)$$

$$+\gamma\frac{d(x_{n-1},Tx_n)+d(x_n,Tx_{n-1})+d(x_{n-1},Tx_{n-1})+d(x_n,Tx_n)}{1+d(x_{n-1},Tx_{n-1})\, d(x_n,Tx_n).d\,(x_{n-1},Tx_n).d(x_n,Tx_{n-1})}$$

$$+L\, Min\{d(x_{n-1},Tx_n).d(x_n,Tx_{n-1})\}$$

$+J\ Min\{d(x_{n-1},Tx_{n-1}).d(x_n,Tx_n)\}$

$d(x_n,x_{n+1}) \leq \propto.\dfrac{d(x_{n-1},x_n)d(x_n,x_{n+1})}{d(x_{n-1},x_n)} + \beta\ d(x_{n-1},x_n)$

$+\gamma \dfrac{d(x_{n-1},x_n) + d(x_n,x_n) + d(x_{n-1},x_n) + d(x_n,x_{n+1})}{1 + d(x_{n-1},x_n).d(x_n,x_{n+1}).d(x_{n-1},x_{n+1}).d(x_n,x_n)}$

$+L\ Min\{d\ (x_{n-1},x_{n+1}).d(x_n,x_n)\}$

$+J\ Min\ \{d(x_{n-1},x_n)\ d(x_n,x_{n+1})\}$

$d(x_n,x_{n+1}) \leq \propto.d(x_n,x_{n+1}) + \beta.d\ (x_{n-1},x_n)$

$+\gamma.[d(x_{n-1},x_{n+1}) + d(x_{n-1},x_n) + d(x_n,x_{n+1})$

$+J\ Min\{d(x_{n-1},x_n)d(x_n,x_{n+1})\}$

$d(x_n,x_{n+1}) \leq \alpha.d(x_n,x_{n+1}) + \beta\ d(x_{n-1},x_n)$

$+\gamma.[d(x_{n-1},x_n) + d(x_n,x_{n+1}) + d(x_{n-1},x_n) + d(x_n,x_{n-1})]$

$+J\ Min\ \{d(x_{n-1},x_n)\ d(x_n,x_{n+1})\}$

$\leq\ \alpha.d(x_n,x_{n+1}) + \beta\ d\ (x_{n-1},x_n)$

$+\gamma.[2d(x_{n-1},x_n) + 2d(x_n,x_{n+1})$

$+J\ Min\ \{d\ (x_{n-1},x_n).d(x_n,x_{n+1})$

$(1-\alpha-2\gamma)\ d(x_n,x_{n+1}) \leq (\beta+2\gamma)\ d(x_{n-1},x_n)$

$+J\ Min\ \{d\ (x_{n-1},x_n)\ d(x_n,x_{n+1})$

Case I[st] when $J\ Min\{\ d(x_{n-1},x_n).d(x_n,x_{n+1})$

$= J\ d(x_{n-1},x_n)$ Then

$(1-\alpha-2\gamma)\ d(x_n,x_{n+1}) \leq (\beta+2\gamma+J)\ d\ (x_{n-1},x_n)$

$d(x_n,x_{n+1})\ \leq \dfrac{(\beta+2\gamma+J)}{(1-\alpha-2\gamma)}.d\ (x_{n-1},x_n)$

$R_1 = \dfrac{(\beta+2\gamma+J)}{1-\alpha-2\gamma}, \alpha+\beta+4\gamma+J < 1$

$d(x_n,x_{n+1})\ \leq\ R_1\ (x_{n-1},x_n)$

Case-II when $J\ Min\{d(x_{n-1},x_n),d(x_n,x_{n+1})$

$= d(x_n,x_{n+1})$ Then.

$d(x_n,x_{n+1})\ (1-\alpha-2\gamma) \leq (\beta+2\gamma)\ d\ (x_{n-1},x_n) + Jd\ (x_n,x_{n+1})$

$(1-\alpha-J-2\gamma)\ d\ (x_n,x_{n+1}) \leq (\beta+2\gamma)d\ (x_{n-1},x_n)$

$d(x_n,x_{n+1}) \leq \dfrac{\beta+2\gamma)}{(1-\alpha-J-2\gamma)}.d\ (x_{n-1},x_n)$

$$R_2 = \frac{(\beta+2\gamma)}{(1-\alpha-J-2\gamma)}, \alpha+\beta+J+4\gamma < 1.$$

We get Then both case, $R = R_1 = R_2 = \alpha+\beta+J+4\gamma < 1$

and by induction. $d(x_n,x_{n+1}) \leq R\ d\ (x_{n-1},x_n)$

$\leq R^n\ d(x_0\ ,x_1)$

$d(x_n,x_m) \leq d(x_n,x_{n+1}) + d\ (x_{n+1},x_{n+2}) \ldots\ldots\ldots.. + d(x_{n+m-1},x_m)$

$\leq (R^n, R^{n+1} + \cdots + R^{n+m-1})d(x_0,x_1)$

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$d(x_n,x_m) \leq \frac{R^n}{1-R} d\,(x_0,x_1)$

We get II $d(x_n,x_m)II \leq M\frac{R^n}{1-R}II(x_0,x_1)II$ which implies that $d(x_n,x_m) \to 0 \quad as\ n \to \infty$ hence x_n is a Cauchy sequence so by completeness of X this sequence must be convergent in X

$d(u,Tu) \leq d(u,x_{n+1}) + d(x_{n+1},Tu)$

$\leq d(u,x_{n+1}) + d\,(Tx_n,Tu)$

$\leq d(u,x_{n+1}) + \frac{\alpha\,.[d\,(x_n,Tx_n)d\,(u,Tu)]}{d\,(x_n,u)} + \beta, d(x_n,u)$

$+\gamma.\frac{[d\,(x_n,Tu) + d(u,Tx_n) + d(x_n,Tx_n) + d(u,Tu)]}{[1 + d(x_n,Tx_n)\,d(u,Tu).d(x_n,Tu).d\,(u,Tx_n)]}$

$+L\,Min\,\{d\,(x_n,u).d(u,Tx_n)$

$+J\,Min\{d(x_n,Tx_n).d(u,Tu)$

$\leq d(u,x_{n+1}) + \alpha.\frac{d(x_n,x_{n+1}).d\,(u,u)}{d(x_n,u)} + \beta.d(x_n,u)$

$+\frac{\gamma.[d(x_n,u) + d(u,x_{n+1}) + d\,(x_n,x_{n+1}) + d(u,u)]}{1 + d(x_n,x_{n+1}).d\,(u,u).d(x_n,u).d(u.x_{n+1})}$

$+L\,Min\,\{d(x_n,u).d(u,x_{n+1})\}$

$+J\,Min\,\{d\,(x_n,x_{n+1}).d\,(u,u)\}$

$\leq d(u,x_{n+1}) + \beta.d\,(x_n,u) + \gamma.[d(x_n,x_{n+1}) + d(x_n,x_{n+1})$

$+L\,Min\,\{d(x_n,u).d\,(u,x_{n+1})$

$\leq d(u,x_{n+1}) + \beta.d(x_n,u) + 2\gamma.[d(x_n,x_{n+1})$

$+L.Min\,\{d(x_n,u)d(u,x_{n+1})\}$

$\leq d(u,x_{n+1}) + \beta.d(x_n,u) + 2\gamma[d(x_n,u) + (u,x_{n+1})$

$+\,L\,Min\,\{d(x_n,u)d(u,x_{n+1})$

$(u,Tx) \leq (1+2\gamma)d(u,x_{n+1}) + (\beta+2\gamma)d\,(x_n,u)$

$+L\,Min\{d(x_n,u)d(u,x_{n+1})\}$

So using the condition of normality of cone

$IId(u,Tu)II \leq M(1+2\gamma)IId(u,x_{n+1})II + (\beta+2\gamma)IId(x_n,u)$

$+L\,Min\,IId\,(x_n,u)d\,(u,x_{n+1})II$

As $n \to 0$ we have II $d(u,Tu)II \leq 0$ hence we get

$u = Tu,\ u\ is\ a\ fixed\ point\ T$

Theorem (3.2) Let (X,d) be a complete cone metric space and P a normal cone with normal constant M, suppose the mapping F,G, is called on almost jaggi contraction if it satisfies the following condition

$d(Fx,Gy) \leq \frac{\alpha.d(x,Fx).d(y,Gy) + \beta.d(x,y)}{d(x,y)}$

$+\gamma\frac{[d(x.Gy) + d(y,Fx) + d(x,Fx) + d(y,Gy)]}{1 + d(x,Fx).d(Y,Gy).d(Y,Fx).d(x,Gy)}$

$+L\,Min\{d(x,Gy).d(Y_3,Fx)$

$+J\,Min\{d(x,Fx).d(Y,Gy)$

For all $x, y \in X$ $where\ L \geq 0\ and\ \alpha, \beta, \gamma\ \in (0,1)\ with\ \alpha + \beta + \gamma < 1$ then each of F,G has a; unique fixed point and these two fixed points coincide

$Proof: -\ Let\ x, \in F(x_0)\ and\ x_2 = G(x_1)$ such that

$x_{2n+1} = F(x_{2n}), \qquad x_{2n+2} = G(x_{2n+1})$

$d(x_{2n+1}, x_{2n+2}) = d(Fx_{2n}, Gx_{2n+1})$

$$\leq \frac{\alpha. d(x_{2n}, Fx_{2n}).d(x_{2n+1}, Gx_{2n+1})}{d(x_{2n}, x_{2n+1})} + \beta. d(x_{2n}, x_{2n+1})$$

$$\frac{+\gamma[d(x_{2n}, Gx_{2n+1}) + d(x_{2n+1}, Fx_{2n}) + d(x_{2n}, Fx_{2n}) + d(x_{2n+1}, Gx_{2n+1})}{1 + d(x_{2n}, Fx_{2n}).d(x_{2n+1}, Gx_{2n+1})d(x_{2n}, Gx_{2n+1})d(x_{2n+1}, Fx_{2n})}$$

$+L\ Min\{d(x_{2n}, Gx_{2n+1})d(x_{2n+1}, Fx_{2n})$

$+J\ Min\{d(x_{2n}, Fx_{2n}). d(x_{2n+1}, Gx_{2n+1})$

$$\leq \frac{\alpha. d(x_{2n}, x_{2n+1}).d(x_{2n+1}, x_{2n+2})}{d(x_{2n}, x_{2n+1})} + \beta. d(x_{2n}, x_{2n+1})$$

$$+\frac{\gamma[d(x_{2n}, x_{2n+2}) + d(x_{2n+1}, x_{2n+1}) + d(x_{2n}, x_{2n+1}) + (x_{2n+1}, x_{2n+2})}{1 + d(x_{2n+1}, x_{2n+1}).d(x_{2n+1}, x_{2n+2}). d(x_{2n}, x_{2n+2}).d(x_{2n+1}, x_{2n+2})}$$

$+L\ Min\ \{d(x_{2n}, x_{2n+2}).d(x_{2n+1}, x_{2n+1})$

$+J\ Min\{d(x_{2n}, x_{2n+1})d(x_{2n+1}, x_{2n+2})$

$\leq \alpha. d(x_{2n+1}, x_{2n+2}) + \beta d(x_{2n}, x_{2n+1})$

$+\gamma[d(x_{2n}, x_{2n+2}) + d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})$

$+J\ Min\{d(x_{2n}, x_{2n+1}).d(x_{2n+1}, x_{2n+2})$

$\leq\ \alpha. d(x_{2n+1}, x_{2n+2}) + \beta. d(x_{2n}, x_{2n+1})$

$+\gamma[d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2}) + d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})]$

$+J\ Min\ \{d(x_{2n}, x_{2n+1}).d(x_{2n+1}, x_{2n+2})\}$

$\leq \alpha. d(x_{2n+1}, x_{2n+2}) + \beta. d(x_{2n}, x_{2n+1})$

$+2\gamma[d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})]$

$+J\ Min\ \{d(x_{2n}, x_{2n+1})d(x_{2n+1}, x_{2n+2})$

Case I when. $Min\{d(x_{2n}, x_{2n+1}).d(x_{2n+1}, x_{2n+2}) = d(x_{2n}, x_{2n+1})$

Than

$d(x_{2n+1}, x_{2n+2}) \leq \alpha. d(x_{2n+1}, x_{2n+2}) + \beta. d(x_{2n}, x_{2n+1})$

$+2\gamma[d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})] + Jd(x_{2n}, x_{2n+1})$

$(1 - \alpha - 2\gamma)d(x_{2n+1}, x_{2n+2}) \leq (\beta + J + 2\gamma)d(x_{2n}, x_{2n+1})$

$$d(x_{2n+1}, x_{2n+2}) \leq \frac{(\beta + J + 2\gamma)}{1 - \alpha - 2\gamma} d(x_{2n}, x_{2n+1})$$

$$where\ R_1 = \frac{(\beta + J + 2\gamma)}{1 - \alpha - 2\gamma}, \alpha + \beta + J + 4\gamma < 1$$

$d(x_{2n+1}, x_{2n+2}) \leq R_1 d(x_{2n}, x_{2n+1})$

Case II. When $Min\{d(x_{2n}, x_{2n+1}).d(x_{2n+1}, x_{2n+2})\} = d(x_{2n+1}, x_{2n+2})$

$d(x_{2n+1}, x_{2n+2}) \leq \alpha. d(x_{2n+1}, x_{2n+2}) + \beta. d(x_{2n}, x_{2n+1})$

$+2\gamma[d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})] + Jd(x_{2n+1}, x_{2n+2})$

$(1 - \alpha - J - 2\gamma)d(x_{2n+1}, x_{2n+2}) \leq (\beta + 2\gamma)d(x_{2n}, x_{2n+1})$

$d(x_{2n+1}, x_{2n+2}) \leq \frac{\beta + 2\gamma}{(1 - \alpha - J - 2\gamma)} d(x_{2n}, x_{2n+1})$

Where $R_2 = \frac{\beta+2\gamma}{(1-\alpha-J-2\gamma)}$, $\alpha + \beta + J + 4\gamma < 1$

In both case we get $R_1 = R_2 = R$.

Then $d(x_{2n+1}, x_{2n+2}) \leq R.\, d(x_{2n}, x_{2n+1})$

$d(x_{2n+3}, x_{2n+2}) = d(Fx_{2n+2}, Gx_{2n+1})$

$\leq \alpha \left[\frac{d(x_{2n+2}, Fx_{2n+2}) d(x_{2n+1}, Gx_{2n+1})}{d(x_{2n+2}, x_{2n+1})}\right] \beta.\, d(x_{2n+2}, x_{2n+1})$

$+\frac{\gamma[d(x_{2n+2}, Gx_{2n+1}) + d(x_{2n+1}, Fx_{2n+2}) + d(x_{2n+2}, Fx_{2n+2}) + d(x_{2n+1}, Gx_{2n+1})]}{1 + d(x_{2n+1}, Fx_{2n+2}).d(x_{2n+1}, Gx_{2n+1}).d(x_{2n+1}, Fx_{2n+2}).d(x_{2n+2}, Gx_{2n+1})}$

$+L\ Min\{d(x_{2n+2}, Gx_{2n+1}).d(x_{2n+1}, Fx_{2n+2})\}$

$+J\ Min\{d(x_{2n+2}, Fx_{2n+2}) d(x_{2n+1}, Gx_{2n+1})\}$

$\leq \alpha. \frac{[d(x_{2n+2}, x_{2n+3}) d(x_{2n+1}, x_{2n+2})]}{d(x_{2n+2}, x_{2n+1})} + \beta d(x_{2n+2}, x_{2n+1})$

$+\gamma \frac{.[d(x_{2n+2}, x_{2n+2}) + d(x_{2n+1}, x_{2n+3}) + d(x_{2n+2}, x_{2n+3}) + d(x_{2n+1}, x_{2n+2})]}{1 + d(x_{2n+1}, x_{2n+3}).d(x_{2n+1}, x_{2n+2}) d(x_{2n+2}, x_{2n+2}).d(x_{2n+1}, x_{2n+3})}$

$+L\ Min\{d(x_{2n+2}, x_{2n+2}).d(x_{2n+1}, x_{2n+3})\}$

$+J\ Min\{d(x_{2n+2}, x_{2n+3}).d(x_{2n+1}, x_{2n+2})$

$\leq \alpha. \frac{d(x_{2n+2}, x_{2n+3}).\, d(x_{2n+2}, x_{2n+1})}{d(x_{2n+2}, x_{2n+1})} + \beta d(x_{2n+2}, x_{2n+1})$

$+\gamma.[d(x_{2n+1}, x_{2n+3}) + d(x_{2n+2}, x_{2n+3}) + d(x_{2n+1}, x_{2n+2})$

$+J\ Min\{d(x_{2n+2}, x_{2n+3}).d(x_{2n+1}, x_{2n+2})$

$\leq \alpha.\, d(x_{2n+3}, x_{2n+2}) + \beta d(x_{2n+2}, x_{2n+1})$

$+\gamma[d(x_{2n+3}, x_{2n+2}) + d(x_{2n+2}, x_{2n+1}) + d(x_{2n+3}, x_{2n+1})]$

$+J\ Min\ \{d(x_{2n+3}, x_{2n+2}).d(x_{2n+1}, x_{2n+2})$

$\leq \alpha.\, d(x_{2n+3}, x_{2n+2}) + \beta d(x_{2n+2}, x_{2n+1})$

$+\gamma[d(x_{2n+3}, x_{2n+2}) + d(x_{2n+2}, x_{2n+1}) + d(x_{2n+3}, x_{2n+2}) + d(x_{2n+2}, x_{2n+1})]$

$+J\ Min\{d(x_{2n+3}, x_{2n+2}).\, d(x_{2n+1}, x_{2n+2})$

$\leq \alpha.\, d(x_{2n+3}, x_{2n+2}) + \beta.\, d(x_{2n+2}, x_{2n+1})$

$+\gamma.\, 2[d(x_{2n+3}, x_{2n+2}) + d(x_{2n+2}, x_{2n+1})]$

$+J\ Min\ \{d(x_{2n+3}, x_{2n+2}) d(x_{2n+1}, x_{2n+2})$

Case-I when $Min\{d(x_{2n+3}, x_{2n+2}).d(x_{2n+1}, x_{2n+2})\} = d(x_{2n+1}, x_{2n+2})$

Then $d(x_{2n+3}, x_{2n+2}) \leq \alpha.\, d(x_{2n+3}, x_{2n+2}) + \beta.\, d(x_{2n+2}, x_{2n+1})$

$+2\gamma[d(x_{2n+3}, x_{2n+2}) + d(x_{2n+2}, x_{2n+1})]$

$+J.d(x_{2n+1}, x_{2n+2})$

$(1 - \alpha - 2\gamma) d(x_{2n+3}, x_{2n+2}) \leq (\beta + J + 2\gamma) d(x_{2n+2}, x_{2n+1})$

$d(x_{2n+3}, x_{2n+2}) \leq \frac{(\beta + J + 2\gamma)}{(1 - \alpha - 2\gamma)}.\, d(x_{2n+2}, x_{2n+1})$

$$where\ \ R_1 = \frac{(\beta + J + 2\gamma)}{(1 - \alpha - 2\gamma)}, \alpha + \beta + J + 4\gamma < 1$$

$$d(x_{2n+3,}x_{2n+2}) \leq R_1 d(x_{2n+2},x_{2n+1})$$

Case II – when $Min\{d(x_{2n+3,}x_{2n+2}).d(x_{2n+1},x_{2n+2})\} = d(x_{2n+3,}x_{2n+2})$

$$d(x_{2n+3,}x_{2n+2}) \leq \alpha.d(x_{2n+3,}x_{2n+2}) + \beta.d(x_{2n+2},x_{2n+1})$$

$$+2\gamma[d(x_{2n+3,}x_{2n+2}) + d(x_{2n+2},x_{2n+1})]$$

$$+Jd(x_{2n+3,}x_{2n+2})$$

$$(1 - \alpha - J - 2\gamma)d(x_{2n+3,}x_{2n+2}) \leq (\beta + 2\gamma)d(x_{2n+2},x_{2n+1})$$

$$d(x_{2n+3,}x_{2n+2}) \leq \frac{(\beta + 2\gamma)}{(1 - \alpha - J - 2\gamma)} d(x_{2n+2},x_{2n+1})$$

$$where\ \ R_2 = \frac{(\beta + 2\gamma)}{(1 - \alpha - J - 2\gamma)}, \alpha + \beta + J + 4\gamma < 1\ \ ,Then$$

$$d(x_{2n+3,}x_{2n+2}) \leq R_2 d(x_{2n+2},x_{2n+1})$$

In both case we get $R_1 = R_2 = R$.

(B) $\qquad d(x_{2n+3,}x_{2n+2}) \leq Rd(x_{2n+2},x_{2n+1})$

Add Equation (A) and (B) we get

$$\sum_{n=1}^{\infty} d(x_n,x_{n+1}) \leq \sum_{n=1}^{\infty} R^n\ d(x_0,x_1)$$

$$= \frac{R}{1-R} d(x_0,x_1)$$

We get $\parallel d(x_n,x_{n+1}) \parallel \leq M.\frac{R}{1-R} \parallel d(x_0,x_1) \parallel$ *which implies that*

$d(x_n,x_{n+1}) \to 0\ \ as\ n \to \infty$ *Hence* $\{x_n\}$ is a Cauchy sequence, so by completeness of X this sequence must be convergent in X , we shall prove that u is a common fixed of F and G.

$$d(u,Gu) \leq d(x_{2n+1},Gu)$$

$$\leq d(u,x_{2n+1}) + d(Fx_{2n},Gu)$$

$$\leq d(u,x_{2n+1})\frac{+[\alpha.d(x_{2n},Fx_{2n}).d(u,Gu)]}{d(x_{2n},u)} + \beta(x_{2n},u)$$

$$\frac{+\gamma[d(x_{2n},Gu) + d(u,Fx_{2n} + d(x_{2n},Fx_{2n}) + d(u,Gu)]}{[1 + d(u,Fx_{2n}).d(u,Gu).d(x_{2n},Gu).d(u,Fx_n)]}$$

$$+L\ Min\{d(x_{2n},Gu).d(y,Fx_{2n})$$

$$+JMin\{d(x_{2n},Fx_{2n}).d(u,Gu)$$

$$\leq d(u,x_{2n+1}) + \alpha.\frac{d(x_{2n},x_{2n+1}).d(u,u)}{d(x_{2n},u)} + \beta(x_{2n},u)$$

$$\frac{+\gamma[d(x_{2n},u) + d(u,x_{2n+1}) + d(x_{2n},x_{2n+1}) + d(u,u)]}{1 + d(u,x_{2n+1})d(u,u).d(u,x_{2n+1}).d(x_{2n},u)}$$

$$+L\ Min\{d(x_{2n},u),d(u,x_{2n+1})$$

$$+J\ Min\{d(x_{2n},x_{2n+1}).d(u,u)\}$$

$$\leq d(u,x_{2n+1}) + \beta(x_{2n},u) + \gamma[d(x_{2n},u) + d(u,x_{2n+1}) + d(x_{2n},x_{2n+1})$$

$$+L\ Min\{d(x_{2n},u)d(u,x_{2n+1})\}$$

$\leq d(u,x_{2n+1})+\beta d(x_{2n},u)+\gamma[d(x_{2n},u)+d(u,x_{2n+1})+d(x_{2n},u)+d(u,x_{2n+1})]$

$+L\ Min\{d(x_{2n},u)d\ (u,x_{2n+1})\}$

$d(u,Gu)\leq d(u,x_{2n+1})+\beta.d(x_{2n},u)+2\gamma[d(x_{2n},u)+d(u,x_{2n+1})]$

$+L\ Min\{d(x_{2n},u).d(u,x_{2n+1})$

So using the condition of normality of cone

$\| d(u,Gu)\leq M(\| d(u,x_{2n+1})\| +\beta \| d(x_{2n},u)\| +2\gamma[\| d(x_{2n},u)+d(u,x_{2n+1})\|]$

$+L\ Min \| d(x_{2n},u).d(u,x_{2n+1})\|$

$As\ n\rightarrow 0\ we\ have\ \| d(u,Gu)\| \leq 0\ Hence\ we\ get\ u=Gu$

U is a fixed point of G.

Similarly

$d(u,Fu)\leq d(u,x_{2n+2})+d(x_{2n+2},Fu)$

$\leq d(u,x_{2n+2})+d(Fu,Gx_{2n+1})$

$\leq d(u,x_{2n+2})+\frac{[\alpha.d(u,Fu).d(x_{2n+1},Gx_{2n+1})]}{d(u,x_{2n+1})}$

$+\beta.d(u,x_{2n+1})$

$+\gamma\frac{[d(u,Gx_{2n+1})+d(x_{2n+1},Fu)+d(u,Fu)+d(x_{2n+1},Gx_{2n+1})]}{1+d(u,Fu).d(x_{2n+1},Gx_{2n+1})d(x_{2n+1},Fu).d(u,Gx_{2n+1})}$

$+L\ Min\{d(u,Gx_{2n+1}).d(x_{2n+1},Fu)\}$

$+J\ Min\{d(u,Fu).d(x_{2n+1},Fu)\}$

$\leq d(u,x_{2n+2})\frac{+[\alpha.d(u,u).d(x_{2n+1},x_{2n+2})]}{d(u,x_{2n+1})}+\beta.(u,x_{2n+1})$

$\frac{+\gamma[d(u,x_{2n+2})+d(x_{2n+1},u)+d(u,u)+d(x_{2n+1},x_{2n+2})]}{1+d((u,u).d(x_{2n+1},x_{2n+2}).d(x_{2n+1},u).d(u,x_{2n+2})}$

$L\ Min\{d(u,x_{2n+2}).d(x_{2n+1},u)\}$

$J\ Min\{d(u,u).d(x_{2n+1},u)\}$

$\leq d(u,x_{2n+2})+\beta(u,x_{2n+1})+\gamma[d(u,x_{2n+2})+d(x_{2n+1},u)+d(x_{2n+1},x_{2n+2})]$

$+L\ Min\{d(u,x_{2n+2})d(x_{2n+1},u)\}$

$\leq d(u,x_{2n+2})+\beta(u,x_{2n+1})+\gamma[d(u,x_{2n+2})+d(x_{2n+1},u)$

$+d(x_{2n+1},u)+(u,x_{2n+2})]$

$+L\ Min\{d(u,x_{2n+2})d(x_{2n+1},u)$

$\leq d(u,x_{2n+2})+\beta d(u,x_{2n+1})+2\gamma[d(u,x_{2n+2})+d(x_{2n+1},u)]$

$+L\ Min\{d(u,x_{2n+2})d(u,x_{2n+1})\}$

So using the condition of normality of cone

$\leq M(\| d(u,x_{2n+1})\| +\beta \| d(u,x_{2n+1})\|$

$+2\gamma \| [d(u,x_{2n+2})+d(x_{2n+1},u)\|$

$+L\ Min \| d(u,x_{2n+2})d(ux_{2n+1})\|$

$As\ n\rightarrow 0\ we\ have\ \| d(u,Fu)\| \leq 0\ \ Hence\ we\ get$

$u=Fu, u\ is\ a\ fixed\ point\ of\ F.$

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