

# Motion Control of an Underactuated 2-DOF Robotic Manipulator

Somer M. Nancy (Corresponding author)

Alkharizmi College of Engineering, University of Baghdad  
Jaderyia, 10071, Baghdad - Iraq

Shahad T. Ibraheem

Alkharizmi College of Engineering, University of Baghdad  
Jaderyia, 10071, Baghdad - Iraq

## Abstract

This paper deals with the motion control of the end effector of a 2-DOF linkage type underactuated robotic manipulator. A simulation procedure is implemented for the motion control in which actuation and braking actions were applied on the two joints of the manipulator subsequently, hence moving the end effector in a point to point manner through the desired path. From the results obtained, it was found that the percentage error in trajectory mainly does not exceed (1%). In some specific points on the trajectory, error reached its maximum value which was found to be (3.64%). In general, these error values are almost acceptable, although an effort will be achieved in future work to reduce this error and improve the design.

**Keywords:** Underactuated Manipulators, Motion Control, Simulation.

## 1. Introduction

Trajectory planning is an important part in industrial robots. Where it calculates how well the manipulator traces a given path from the start point to the end. In industrial robot, there are two main application categories. Position control like those robots used in drilling, spot welding, pick and place, etc., and trajectory planning control such as welding arms, painting, cutting, plotting, and part shaping in CNC machines.

Commonly, to control the motion of a manipulator of any degree of freedom (DOF), every joint in the manipulator is attached to an actuator in order to control its motion directly. Such a system is called actuated robot. On the other hand, underactuated robots are an evolution stage over actuated ones and were paid attention in the nineties of the last century. These systems own joints (one or more) not controlled directly called *passive links* and the actuated ones are called *active links*. The fewer number of actuators, led to less power required to move the system. The cost of this advantage is that controlling and planning the trajectory become harder than that of actuated one due to the lack of actuators. It is an active field due to their wide applications in robotics, marine and aerospace vehicles, mobile robot, walking robot, snake type and swimming robots, acrobatic and grasping robots.

Different strategies and technologies have been adopted to control the motion of underactuated systems, such as position control [1], the use of friction [2] and [3], the use of brakes [4], [5] and [6], implementation of artificial neural network [7], partial feedback linearization [8], bifurcation theory [9], fuzzy control [10], slide mode control [11], energy methods [12] and [13], and finally through the use of smart materials [14].

of a 2-DOF underactuated robotic manipulator. The control procedure was achieved via actuating and braking each of the two joints of the manipulator subsequently, in a manner to move the end effector according to a given path.

## 2. Analysis and Simulation

The simulation model of the 2-DOF underactuated robotic manipulator is shown in Figure 1, for which the dimensions are listed in Table 1.

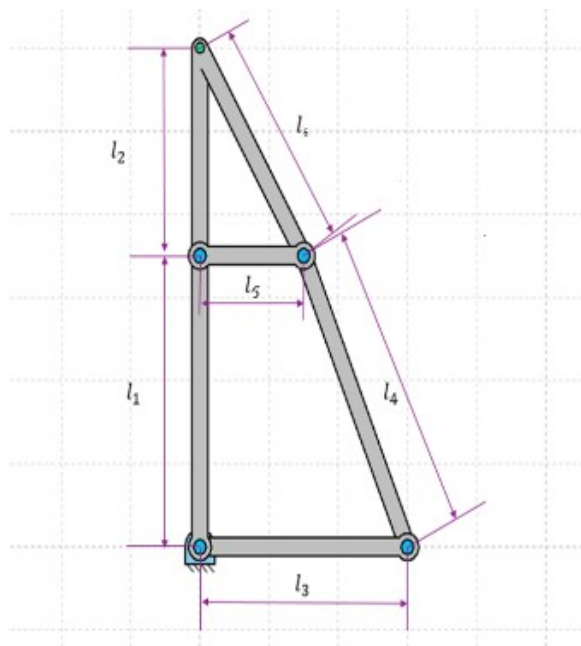


Figure 1. 2-DOF underactuated robotic manipulator

Table 1. Dimensions of the manipulator

$l_1$ (mm)	$l_2$ (mm)	$l_3$ (mm)	$l_4$ (mm)	$l_5$ (mm)	$l_6$ (mm)
350	250	150	380	150	291.5

In order to command the robot end effector to move in a previously specified trajectory, defined by its points  $(x_n, y_n)$ , inverse kinematic equations have to be implemented hence to produce joint angles  $\alpha_1$  of link  $l_1$  and  $\alpha_2$  of link  $l_2$ . The equations are derived according to [15], as listed below,

$$\alpha_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin \alpha_2}{l_1 + l_2 \cos \alpha_2}\right) \quad (1)$$

$$\alpha_2 = \pm \cos^{-1}(D) \quad (2)$$

where,

$$D = \frac{x^2 + y^2 + l_1^2 + l_2^2}{2l_1 l_2} \quad (3)$$

The above equations are for a fully actuated manipulator, hence some modifications should be made, taking into consideration that the prime mover of the underactuated robotic manipulator is link  $l_3$ , which is the only actuated link.

In the simulation procedure followed in this work, and in order to control the motion of the end effector, it is required to pass the motion of the actuator once to link  $l_1$  then to link  $l_2$  separately in a sequence to produce the desired trajectory. This was achieved by blocking the rotation of link  $l_1$  and calculate the angle of rotation  $\Theta_2$  of link  $l_3$  as shown in Figure 2, then blocking the relative rotation between link  $l_1$  and link  $l_2$  and calculate the angle of rotation  $\Theta_1$  of link  $l_3$  as shown in Figure 3.

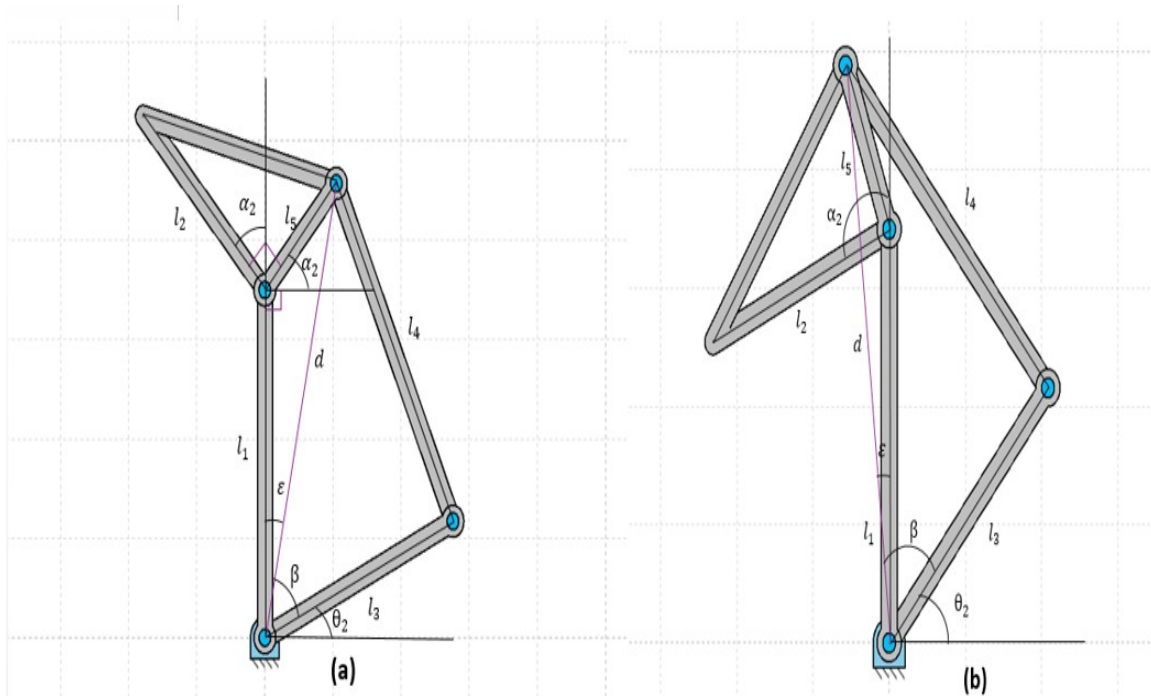


Figure 2. Blocking link  $l_1$  from rotation

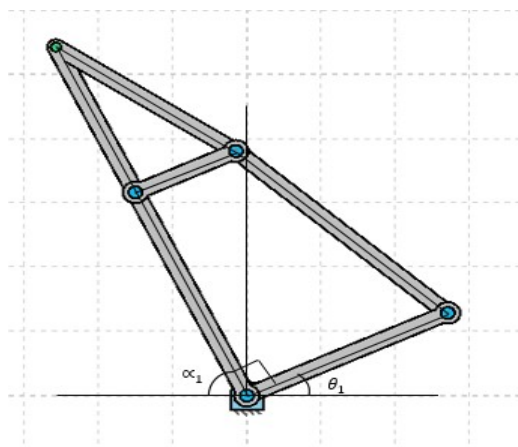


Figure 3. Blocking relative rotation between link  $l_1$  and link  $l_2$

Referring to Figure 2, the following geometrical relations can be obtained,

$$d = \sqrt{l_1^2 + l_5^2 - 2l_1l_5 * \cos(90 + \alpha_2)} \quad (4)$$

$$l_5^2 = l_1^2 + d^2 - 2l_1d * \cos(\epsilon) \quad (5)$$

$$l_4^2 = l_3^2 + d^2 - 2l_3d * \cos(\beta) \quad (6)$$

Rearrange terms to obtain,

$$(\epsilon) = \cos^{-1} \left( \frac{l_1^2 + d^2 - l_5^2}{2 * l_1 * d} \right) \quad (7)$$

$$(\beta) = \cos^{-1} \left( \frac{l_3^2 + d^2 - l_4^2}{2 * l_3 * d} \right) \quad (8)$$

Angles ( $\varepsilon$ ) and ( $\beta$ ) are related to ( $d$ ) which starts to increase as ( $\alpha_2$ ) increases till  $90^\circ$ , then ( $d$ ) decreases as ( $\alpha_2$ ) increases over  $90^\circ$ . Hence referring to Figure 2,

(a) ( $\alpha_2 < 90$ ) then:  

$$\theta_2 = 90 - \varepsilon - \beta \quad (9)$$

(b) ( $\alpha_2 > 90$ ) then:  

$$\theta_2 = 90 + \varepsilon - \beta \quad (10)$$

On the other hand, referring to Figure 3, it can be stated that,

$$\theta_1 = 90 - \alpha_1 \quad (11)$$

### 3. Simulation Procedure and Case Study

The simulation procedure adopted in this work was achieved via GIM software [16]. Values of  $\Theta_1$  and  $\Theta_2$  were calculated for each ( $x,y$ ) point on the required trajectory, then motion is applied by the actuator to the manipulator in two separate steps, as shown in Figure 4.

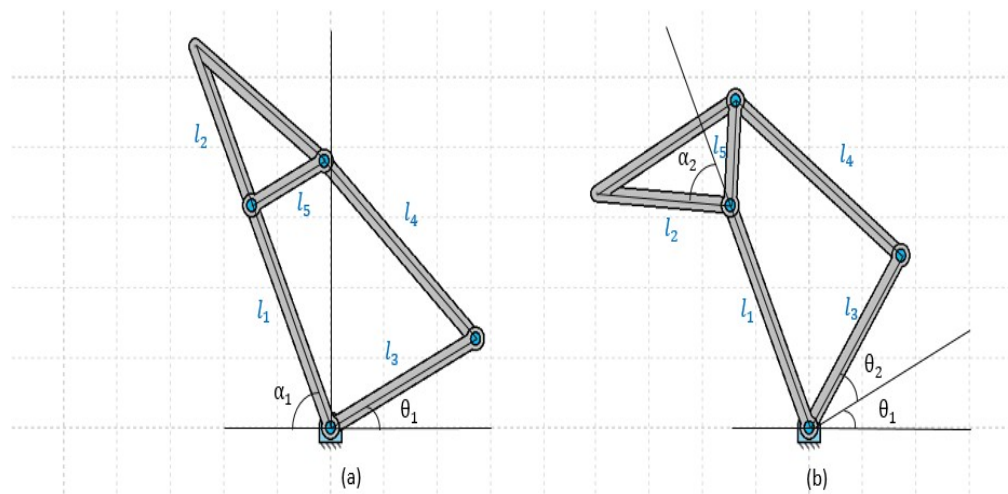


Figure 4. The two steps of motion of the manipulator  
 (a) first step (b) second step

The first step is by rotating link  $l_3$  by an angle of  $\Theta_1$  while blocking the relative rotation between links  $l_1$  and  $l_2$ . The second step is by rotating link  $l_3$  by an angle of  $\Theta_2$  while blocking the rotation of link  $l_1$ . By the application of these two steps of motion, it is obvious that the end effector of this manipulator will move to the desired point ( $x,y$ ) on the trajectory. Repetition of this procedure will induce motion of the end effector on the required trajectory.

This procedure was applied on two cases, the first is a line shape trajectory, while the second is a V shape trajectory. For the line shape trajectory, two sub-cases were considered, namely, vertical and horizontal line. For each one of them, four different locations of the trajectory inside the workspace of the manipulator were considered. Each line is 10 cm in length, segmented into 20 equi-spaced points  $[(x_n, y_n), n=20]$ , to be fed to the simulator hence to make the manipulator follows the desired trajectory.

### 4. Results and Discussions

The first and simplest case taken into consideration is to plan a line trajectory to be obtained by the movement of the end effector of the underactuated robotic manipulator. Two subcases were conducted, namely a vertical line trajectory and a horizontal line trajectory. In order to cover the workspace of the robotic manipulator, four different locations inside this workspace (a,b,c,d) were considered for both vertical and horizontal line trajectories, as shown in Figures 5 and 6. Certain points on each line trajectory were selected, to calculate the error obtained in the simulation process related to the desired line trajectory. The results of these errors are tabulated in Tables 2 and 3 for vertical and horizontal line trajectory respectively.

It can be seen that minimum percentage error occurred at the mid-space of the workspace, which is to say at

locations b and c, for both vertical and horizontal line trajectories. The zigzag motion amplitude at these locations is smaller than locations a and d. This is due to a fact observed from this simulation procedure, in which the relative motion between the two links of the manipulator,  $l_1$  and  $l_2$ , at locations b and c is less than that at locations a and d, hence producing less fluctuation at the end effector which in turn induces less errors.

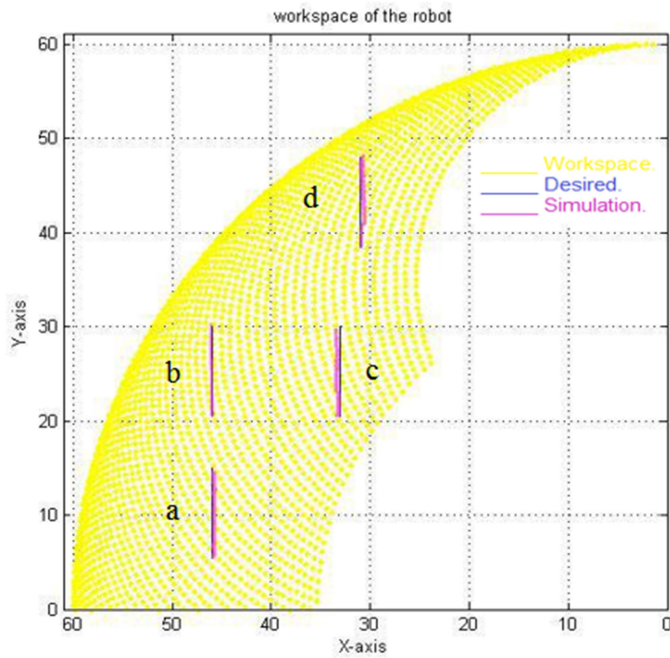


Figure 5. Vertical line trajectory

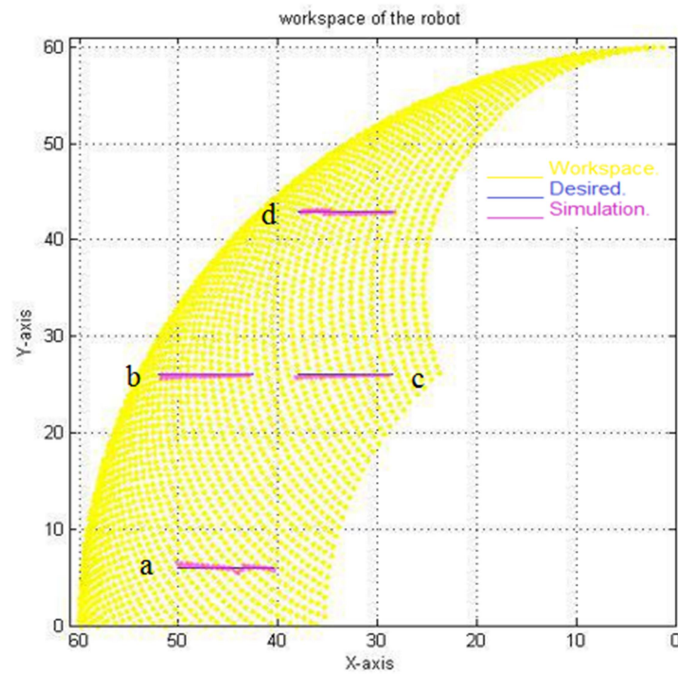


Figure 6. Horizontal line trajectory

Table 2. Percentage error of vertical line trajectory (a, b, c, d), all dimensions in cm.

(a) P	Desired		Simulation		Percentage error	
	x	y	x	y	x	y
1	46	5.5	46.1	5.3	0.22	3.64
2	46	10.5	46.1	10.3	0.22	1.90
3	46	15.5	46.1	15.6	0.22	0.65

(b) P	Desired		Simulation		Percentage error	
	x	y	x	y	x	y
1	46	20.5	46.0	20.2	0.00	1.46
2	46	25.5	46.2	25.4	0.43	0.39
3	46	30.5	46.4	30.2	0.87	0.98

(c) P	Desired		Simulation		Percentage error	
	x	y	x	y	x	y
1	33	20.5	33.2	20.6	0.61	0.49
2	33	25.5	33.3	25.5	0.91	0.00
3	33	30.5	33.3	30.4	0.91	0.33

(d) P	Desired		Simulation		Percentage error	
	x	y	x	y	x	y
1	31	38.5	30.9	38.4	0.32	0.26
2	31	43.5	30.8	43.3	0.65	0.46
3	31	48.5	30.9	48.3	0.32	0.41

Table 3. Percentage error of horizontal line trajectory (a, b, c, d), all dimensions in cm

(a) P	Desired		Simulation		Percentage error	
	x	y	x	y	x	y
1	40	6	40.0	5.9	0.00	1.67
2	45	6	45.5	5.9	1.11	2.50
3	50	6	51.6	6.1	3.20	1.67

(b) P	Desired		Simulation		Percentage error	
	x	y	x	y	x	y
1	42	26	42.0	26.0	0.00	0.00
2	47	26	46.9	25.8	0.21	0.77
3	52	26	51.5	26.0	0.96	0.00

(c) P	Desired		Simulation		percentage error	
	x	y	x	y	x	y
1	28.5	26	28.5	25.9	0.00	0.38
2	33.5	26	33.6	25.8	0.30	0.77
3	38.5	26	38.6	25.7	0.26	1.15

(d) P	Desired		Simulation		Percentage error	
	x	y	x	y	x	y
1	28.5	43	28.5	43.0	0.00	0.00
2	33.5	43	33.5	42.9	0.00	0.23
3	38.5	43	38.5	43.1	0.00	0.23

The next case considered is the V-shape trajectory, which is tested to experiment the behavior of the manipulator with sharp edge trajectory. Figure 7 shows the difference between the desired trajectory and the results of the simulation, and Table 4 shows the percentage errors between them. It can be seen that the manipulator treats well with sharp edge trajectory.

Table 4. Percentage error of V-shape trajectory, all dimensions in cm

P	Desired		Simulation		Percentage error	
	x	y	x	y	x	y
1	31.75	30	32.0	30.0	0.79	0.00
2	32.95	27.6	33.6	26.9	1.97	2.54
3	34.45	24.6	35.0	24.1	1.60	2.03
4	36.25	26	36.5	26.7	0.69	2.69
5	38	29.6	37.5	29.5	1.32	0.34



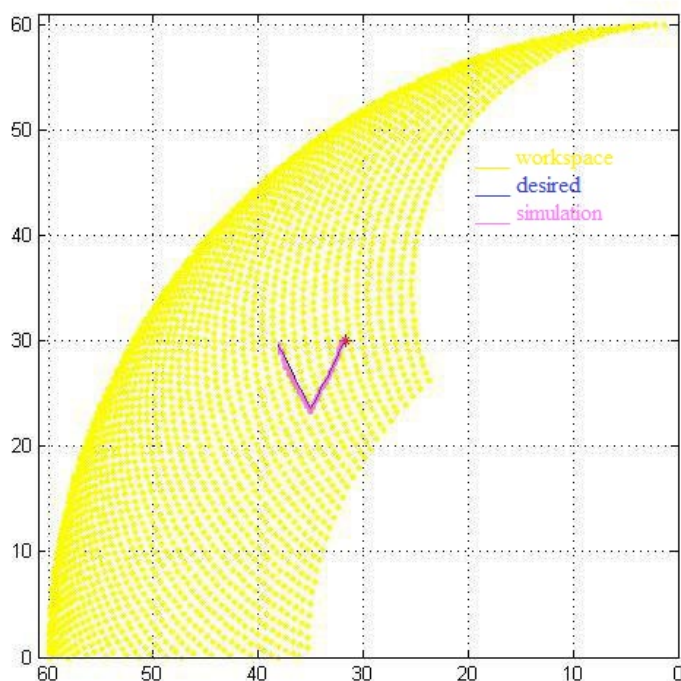


Figure 7. V-shape trajectory

## 5. Conclusion

The trajectory control of mechanical linkage type underactuated manipulator was demonstrated through a simulation procedure, from which the following can be concluded. The main goal of this work is to control the motion of the end effector through a required trajectory by using the only torque available at the active joint. The adopted control method succeeded in obtaining the desired trajectories with the linkage type manipulator. The accuracy obtained was acceptable. Due to using the linkage type mechanism, the end effector of the manipulator can not reach to the points near the base. Trajectories located in the middle of the workspace were found to be obtained with higher accuracy than those on other places inside the workspace.

## References

- [1] Jun-ichi Imura, Keigo Kobayashi, and Tsuneo Yoshikawa., "Nonholonomic Control of 3 Link Planner Manipulator with a Free Joint". Proceedings of the 35<sup>th</sup> Conference on Deciriion and Control Kobe, Japan December 1996.
- [2] Kee-Ho Yu, Yoshinobu Shito and Hikaru Inooka., " POSITION CONTROL OF AN UNDERACTUATED MANIPULATOR USING JOINT FRICTION". International jornal Non-Linear Mechanics,Japan, Vol. 33, No. 4, pp. 607-614, 1998.
- [3] Arun D. Mahindrakar, Shodhan Rao, R.N. Banavar., "Point-to-point control of a 2R planar horizontal underactuated manipulator"., ELSEVER Mechanism and Machine Theory 41 (2006) 838–844., doi:10.1016/j.mechmachtheory.2005.10.013.
- [4] H.Arai and S.Tachi. "Position Control of a Manipulator with Passive Joints Using Dynamic Coupling". IEEE Trans. on Robotics and Automation, vol.7, no.4, pp. 528-534, August 1991.
- [5] Wei Chen, Yueqing Yu, Xinhua Zhao, Lianyu Zhao and Qiyuan Sun., "Position Control of a 2DOF Underactuated Planar Flexible Manipulator ". Proceedings of the 2011 IEEE International Conference on Mechatronics and Automation August 7 - 10, Beijing, China.
- [6] Marcel Bergerman, Christopher Lee and Yangsheng Xu., " EXPERIMENTAL STUDY OF AN UNDERACTUATED MANIPUL -ATOR". Report CMU-RI-TR-95-16, Robotics Institute, Carnegie Mellon University, April, 1995.



- [7] Ali T. Hasan., "Under-Actuated Robot Manipulator Positioning Control Using Artificial Neural Network Inversion Technique ". Hindawi Publishing Corporation., Advances in Artificial Intelligence Volume 2012 (2012), Article ID 927905, 6 pages.
- [8] Alessandro De Luca and Giuseppe Oriolo., "Stabilization of the Acrobot via Iterative State Steering". Proceedings of the 1998 IEEE International Conference on Robotics & Automation Leuven, Belgium. May 1998, pp 3581-3587.
- [9] Hiroshi Yabuno, Takashi Matsuda, and Nobuharu Aoshima., " Reachable and Stabilizable Area of an Underactuated Manipulator without State Feedback Control". IEEE/ASME TRANSACTIONS ON MECHATRONICS, VOL. 10, NO. 4, AUGUST 2005, pp 397-403.
- [10] Nenad Mus`kinja and Boris Tovornik. "Swinging Up and Stabilization of a Real Inverted Pendulum",. IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, Manuscript received March 12, 2004; revised June 10, 2005. Abstract published on the Internet January 25, 2006,. VOL. 53, NO. 2, APRIL 2006, pp 631-639.
- [11] Mun-Soo Park, DongKyoung Chwa and Suk-Kyo Hong., " Decoupling Control of a Class of Underactuated Mechanical Systems Based on Sliding Mode Control".SICE-ICASE International Joint Conference 2006 Oct. 18-21, 2006 in Bexco, Busan, Korea, pp 806-810.
- [12] Naoko Miyashita, Masashi Kishikawa and Masaki Yamakita., " 3D Motion Control of 2 links (5 D.O.F.) Underactuated Manipulator named AcroBOX". Proceedings of the 2006 American Control Conference Minneapolis, Minnesota, USA, June 14-16, 2006, pp 5614-5619.
- [13] A.D. Luca, S. Iannitti, "A simple STLC test for mechanical systems under-actuated by one control", in Proceedings of the IEEE International Conference on Robotics and Automation, Washington, DC, May 2002, pp. 1735–1740.
- [14] R. Bansevicius, K.K. Sarkauskas and R.T. Tolocka, "Underactuated Manipulator with Control Based on Variable Dynamic Properties of Joints", Electronics and Electrical Engineering - Kaunas:Technological, No. 7 (79), 2007, pp. 3-6.
- [15] Industrial Robotics: Theory, Modelling and Control Edited by Sam Cubero, ISBN 3-86611-285-8, 964 pages, Publisher: Pro Literatur Verlag, Germany / ARS, Austria , Chapters published December 01, 2006 under CC BY-NC-SA 3.0 license. DOI: 10.5772/44 Edited Volume.
- [16] <http://www.ehu.es/compmech/software/>.