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ON COMMON RANDOM FIXED POINTS OF MAPPINGS IN HILBER SPACE

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ABSTRACT: The We find common random fixed point of two random operator in closed convex subset of separable Hilbert space by considering a sequence of measurable function satisfying condition A,B and C.

Key wards: common fixed point, rational expression, hilbert space random variable

1. INTRODUCTION AND PRELIMINARIES

Random nonlinear analysis has grown into an active research area closely associated with the study of random nonlinear operators and their properties needed in solving nonlinear random operator equations (see [9, 14, 17]). The study of random fixed point theory was initiated by the Prague school of probabilists in the 1950's([11, 23]). Random fixed point theorems are of tremendous importance in probabilistic functional analysis as they provide a convenient way of modelling many real life problems and random methods have also revolutionized the financial markets. The survey article by bharucha -Reid [10] in 1976 attracted the attention of several mathematicians and gave wings to this theory. Itoh [13] extended Spacek's and Hans's theorems to random multivalued contraction mappings. In recent years, alot of efforts have been made ([4, 5, 6, 7, 8, 12, 18, 19], and references therein) to show the existence of random fixed points of certain random single valued and multivalued operators and various applications in diverse area from pure mathematics to applied sciences have been explored.

Theorem 1.1 (Banach's contraction principle) Let (X,d) be a complete metric space, $c \in (0,1)$ and f: X→X be a mapping such that for each x, y ∈X, $d(fx,fy) \le cd\, x,y$ Then f has a unique fixed point a ∈X, such that for each x ∈ X, $\lim_{n\to\infty} f^n x = a$.

After the classical result, Kannan [20] gave a subsequently new contractive mapping to prove the fixed point theorem. Since then a number of mathematicians have been worked on fixed point theory dealing with mappings satisfying various type of contractive conditions. The aim of this paper is to generalize some mixed type of contractive conditions to the mapping and then a pair of mappings satisfying general contractive mappings such as Kannan type [20], Zamfirescu type [24], etc.

In recent years, the study of random fixed points has attracted much attention. Some of the recent literatures in random fixed point may be noted in Rhoades [1], and Binayak S. Choudhary [2].In this paper, we construct a sequence of measurable functions and consider its convergence to the common unique random fixed point of two continuous random operators defined on a non-empty closed subset of a separable Hilbert space. For the purpose of obtaining the random fixed point of the two continuous random operators. We have used a rational inequality (from B. Fisher [3] and S.S. Pagey [21]) and the parallelogram law. Throughout this paper, (Ω, Σ) denotes a measurable space consisting of a set Ω and sigma algebra Σ of subsets of Ω, H stands for a separable Hilbert space and C is a nonempty closed subset of H.

Definition 1.2. A function f: Ω → C is said to be measurable if $f^{-1}(B \cap C) \in \sum$ for every Borel subset B of H.

Definition 1.3. A function $F : \Omega \times C \to C$ is said to be a random operator if $F(., x) : \Omega \to C$ is measurable for every $x \in C$

Definition 1.4. A measurable $g : \Omega \to C$ is said to be a random fixed point of the random operator $F : \Omega \times C \to C$ if $F(t, g(t)) = g(t)$ for all $t \in \Omega$

Definition 1.5. A random operator F: Ω×C → C is said to be continuous if for fixed $t \in \Omega$, $F(t,.) : C \to C$ is continuous.

2. MAIN RESULTS

Theorem 2.1. Let X be a Hilbert space and K be non empty bounded closed convex subset of X. $T_1, T_2 : \Omega \times K \to K$ be such that

A. $$\|T_1(t, g_n(t)) - T_2(t, f_n(t))\|^2 \le \alpha_1 \|g_n(t) - f_n(t)\|^2 + \alpha_2 \|g_n(t) - T_1(t, g_n(t)\|^2 + \alpha_3 \| f_n(t) - T_1(t, g_n(t)\|^2 + \alpha_4 \max\left\{\|g_n(t) - T_2(t, f_n(t)\|^2, \| f_n(t) - T_1(t, g_n(t)\|^2\right\}$$

Where $\alpha_i \ge 0, i = 1, 2, 3, 4, 5$ and $2\alpha_1 + 11\alpha_2 + 11\alpha_3 + 8\alpha_4 = 2$

B. $T_1 F \subset F$ iff $T_2 F \subset F$ for every convex subset F of K.

C. Either $\displaystyle\mathop{Sup}_{g_n(t) \in F} \|g_n(t) - T_1(t, g_n(t)\| \prec \delta(F)$ or $\displaystyle\mathop{Sup}_{f_n(t) \in F} \| f_n(t) - T_1(t, f_n(t)\| \prec \delta(F)$

for every non empty bounded closed convex subset F of K which is mapped into itself by T_1 or T_2 then T_1 and T_2 have common random fixed point in X .

Proof: Let Y be the family of all non empty bounded closed convex subsets of K ordered by set inclusion which are mapped into itself by T_2 .Since X is a hilbert space, it is a reflexive banach space and hence by Smulian result [22] every decreasing sequence of non empty bounded closed convex subset of X has non empty intersection and by Zorn's lamma, it follows that X possesses a minimal element F, say. If F contains only one element then that element becomes a fixed point of T_2 . We shall show that F contains only one element. We suppose on the contrary that F contains more than one point, which will show implies a contradiction.

$$\text{Let } A = \underset{f_n(\text{t})\in \text{F}}{Sup} \left\| T_2(t, f_n(\text{t}) - f_n(\text{t}) \right\| \text{ by the condition C, } \text{A} \succ \delta(\text{F})$$

We now defined the following terms for $g_n(\text{t}) \in \text{F}$

$$\gamma_{g_n(\text{t})}(\text{F}) = \max \left\{ \underset{g_n(\text{t})\in \text{F}}{Sup} \left\| \text{g}_n(\text{t}) - f_n(\text{t}) \right\|, A \right\}, \gamma(\text{F}) = \inf \left\{ \gamma_{g_n(\text{t})}(\text{F}), g_n(\text{t}) \in F \right\}$$

$$\text{F}_c = \left\{ g_n(\text{t}) \in F; \gamma_{g_n(\text{t})}(\text{F}) = \gamma(\text{F}) \right\}$$

We know that F_c is non empty closed and convex. For a positive integer n and for $g_n(\text{t}) \in F$,let $F(g_n(\text{t}),\text{n}) = \left\{ f_n(\text{t}) \in F; \left\| g_n(\text{t}) - f_n(\text{t}) \right\| \le \gamma(\text{F}) + \frac{1}{n} \right\}$ and $C_n = \bigcup_{g_n(t)\in F} F(g_n(t), n)$

We show first C_n is non empty. If possible let $C_n = \phi$ than there exit $g_n(\text{t})$ and $f_n(\text{t}) \in F$ such that $F(g_n(t),n) \cap F(f_n(t),n) = \phi$.

By contradiction

$$F(g_n(\text{t}),\text{n}) = \left\{ f_n(\text{t}) \in F; \left\| g_n(\text{t}) - f_n(\text{t}) \right\| \le \gamma(\text{F}) + \frac{1}{n} \right\} \text{and similarly}$$

$$F(f_n(t),m) = \left\{ g_n(\text{t}) \in F; \left\| f_n(t) - g_n(t) \right\| \ge 2\gamma(F) + \frac{2}{n} \right\}$$

$$\text{-----------------------(1)}$$

Now for $g_n(\text{t}) \in F$, $\underset{g_n(\text{t})\in \text{F}}{Sup} \left\| \text{g}_n(\text{t}) - f_n(\text{t}) \right\| \ge \frac{\delta(F)}{2}$ and so $\gamma_{g_n(t)}(F) \ge \frac{\delta(F)}{2}$ and this implies

$\frac{\delta(F)}{2} \le \gamma(F)$ .there for $\delta(F) \le 2\gamma(F) + \frac{2}{n}$ .So Form (1) $\|f_n(t) - g_n(t)\| \succ \delta(F)$ which is contradiction because $g_n(\text{t})$ and $f_n(\text{t}) \in F$.

Therefore C_n is non empty set.

It may further be verified that C_n is closed ,convex and that $C_{n+1} \subset C_n$.

We wish to show that $F_c = \bigcap_{n=1}^{\infty} C_n$

For this let $f_n(t) \in F_c$ .then $\gamma_{f_n(t)}(F) = \gamma(F)$

So $\max\left\{ \underset{g_n(t)\in F}{Sup} \|f_n(t) - g_n(t)\|, A \right\} = \gamma(F)$

And so $\underset{g_n(t)\in F}{Sup} \|f_n(t) - g_n(t)\| < \gamma(F)$ -----------------------------(2)

We verified that $f_n(t) \in F(g_n(t), n)\ \forall g_n(t) \in F$ and for all n.

It possible let $f_n(t) \in F(g_n(t), n)$ for some $g_n(t) \in F$ and for some n. then $\|g_n(t) - f_n(t)\| \succ \gamma(F) + \frac{1}{n}$ -------------------------------(3) From (2) we see that $\|f_n(t) - g_n(t)\| \le \gamma(F)$ which contradiction to (3),

So $f_n(t) \in \bigcap_{n=1}^{\infty} C_n$ and so $F_c \subset \bigcap_{n=1}^{\infty} C_n$

Next let $f_n(t) \in \bigcap_{n=1}^{\infty} C_n$ then $f_n(t) \in F(g_n(t), n)\ \forall g_n(t) \in F$ and for all n. and this implies that $\underset{g_n(t)\in F}{Sup} \|g_n(t) - f_n(t)\| \le \gamma(F)$ .also $A \le \gamma(F)$ .these two give $\gamma_{f_n(t)}(F) \le \gamma(F)$. But $\gamma(F) \le \gamma_{f_n(t)}(F)$ always and then $\gamma(F) = \gamma_{f_n(t)}(F)$ and this gives $f_n(t) \in F_c$. So $\bigcap_{n=1}^{\infty} C_n \subset F_c$ .thus $\bigcap_{n=1}^{\infty} C_n = F_c$.

This equality further gives that F_c is closed convex and by smulian's result[22] non empty. Next we show that $\delta(F_c) < \delta(F)$.

Since K has normal structure and $A < \delta(F)$ there exists a point $g_n(t) \in F$ such that $\gamma_{g_n(t)}(F) \prec \delta(F)$.

If $g_n(t), f_n(t) \in F_c$, then $\|g_n(t) - f_n(t)\| \le \gamma_{g_n(t)}(F) = \gamma(F)$.

So

$$\delta(F_c) = \sup\{\|g_n(t) - f_n(t)\| : g_n(t), f_n(t) \in F_c\} \le \gamma(F) \le \gamma_{g_n(t)}(F) < \delta(F) ----------(4)$$

If $g_n(t) \in F_c$ and $f_n(t)$ is an arbitrary of F we obtained

$$\|T_2(t, f_n(t)) - T_1(t, g_n(t))\|^2 \le \alpha_1 \|g_n(t) - f_n(t)\|^2 + \alpha_2 \|f_n(t) - T_2(t, f_n(t))\|^2 + \alpha_3 \|g_n(t) - T_1(t, g_n(t))\|^2$$
$$+ \alpha_4 \max\{\|f_n(t) - T_1(t, g_n(t))\|^2, \|g_n(t) - T_2(t, f_n(t))\|^2\}$$

<u>*If* $\|f_n(t) - T_1(t, g_n(t))\|^2 > \|g_n(t) - T_2(t, f_n(t))\|^2$</u> then by (A)

$$\|T_2(t, f_n(t)) - T_1(t, g_n(t))\|^2 \le \alpha_1 \|g_n(t) - f_n(t)\|^2 + \alpha_2 \|f_n(t) - T_2(t, f_n(t))\|^2 + \alpha_3 \|g_n(t) - T_1(t, g_n(t))\|^2$$
$$+ \alpha_4 \|f_n(t) - T_1(t, g_n(t))\|^2$$

$$\le \alpha_1 \|g_n(t) - f_n(t)\|^2 + \alpha_2 \|f_n(t) - T_2(t, f_n(t))\|^2 + \alpha_3 \left[2\|g_n(t) - f_n(t)\|^2 + 2\left\{ \begin{matrix} 2\|f_n(t) - T_2(t, f_n(t))\|^2 + \\ 2\|T_2(t, f_n(t)) - T_1(t, g_n(t))\|^2 \end{matrix} \right\} \right]$$

$$+ \alpha_4 \left\{ 2\|f_n(t) - T_2(t, f_n(t))\|^2 + 2\|T_2(t, f_n(t)) - T_1(t, g_n(t))\|^2 \right\}$$

$$\therefore (1 - 2\alpha_4 - 4a_3)\|T_2(t, f_n(t)) - T_1(t, g_n(t))\|^2 \le (\alpha_1 + 2\alpha_3)\|g_n(t) - f_n(t)\|^2$$
$$+ (\alpha_2 + 4\alpha_3 + 2\alpha_4)\|f_n(t) - T_2(t, f_n(t))\|^2$$
$$-----------------(*)$$

similarly,

$$(1\text{-}4\alpha_2)\|T_2(t, f_n(t)) - T_1(t, g_n(t))\|^2 \le (\alpha_1 + 2\alpha_2)\|g_n(t) - f_n(t)\|^2$$
$$+ (4\alpha_2 + \alpha_3 + 4\alpha_4)\|f_n(t) - T_2(t, f_n(t))\|^2$$
$$----------------(**)$$

adding (*) and (**), we get

$$(2 - 4\alpha_2 - 4\alpha_3 - 2\alpha_4)\|T_2(t, f_n(t)) - T_1(t, g_n(t))\|^2$$
$$\le (2\alpha_1 + 2\alpha_2 + 2\alpha_3)\|g_n(t) - f_n(t)\|^2 + (5\alpha_2 + 5\alpha_3 + 6\alpha_4)\|f_n(t) - T_2(t, f_n(t))\|^2$$

$$\leq (2\alpha_1 + 7\alpha_2 + 7\alpha_3 + 6\alpha_4)\max\left\{ \sup_{f_n(t)\in F} \|g_n(t) - f_n(t)\|^2, \sup_{f_n(t)\in F} \| f_n(t) - T_2(t, f_n(t))\|^2 \right\}$$

$$or, \|T_2(t, f_n(t)) - T_1(t, g_n(t))\|^2 \leq \frac{(2\alpha_1 + 7\alpha_2 + 7\alpha_3 + 6\alpha_4)}{(2 - 4\alpha_2 - 4\alpha_3 - 2\alpha_4)}\max\left\{ \begin{array}{l} \sup_{f_n(t)\in F} \|g_n(t) - f_n(t)\|^2, \\ \sup_{f_n(t)\in F} \| f_n(t) - T_2(t, f_n(t))\|^2 \end{array} \right\}$$

Using the condition $2\alpha_1 + 11\alpha_2 + 11\alpha_3 + 8\alpha_4 = 2$

We can show that

$$\frac{(2\alpha_1 + 7\alpha_2 + 7\alpha_3 + 4\alpha_4)}{(2 - 4\alpha_2 - 4\alpha_3 - 2\alpha_4)} \leq 1$$

$\underline{now\ \ If\ \| f_n(\mathrm{t}) - T_1(t, \mathrm{g}_n(\mathrm{t}))\|^2 < \|\mathrm{g}_n(\mathrm{t}) - T_2(t, f_n(\mathrm{t}))\|^2}$ then by (A)

$$\|T_2(t, f_n(\mathrm{t})) - T_1(t, \mathrm{g}_n(\mathrm{t}))\|^2 \leq \alpha_1 \|\mathrm{g}_n(\mathrm{t}) - f_n(\mathrm{t})\|^2 + \alpha_2 \| f_n(\mathrm{t}) - T_2(t, f_n(\mathrm{t}))\|^2 + \alpha_3 \|\mathrm{g}_n(\mathrm{t}) - T_1(t, \mathrm{g}_n(\mathrm{t}))\|^2$$
$$+ \alpha_4 \|\mathrm{g}_n(\mathrm{t}) - T_2(t, f_n(\mathrm{t}))\|^2$$

$$\leq \alpha_1 \|g_n(t) - f_n(t)\|^2 + \alpha_2 \| f_n(t) - T_2(t, f_n(t))\|^2 + \alpha_3 \left[2\|g_n(t) - f_n(t)\|^2 + 2\left\{ \begin{array}{l} 2\| f_n(t) - T_2(t, f_n(t))\|^2 + \\ 2\|T_2(t, f_n(t)) - T_1(t, g_n(t))\|^2 \end{array} \right\} \right]$$
$$+ \alpha_4 \left\{ 2\|g_n(t) - f_n(t)\|^2 + 2\| f_n(t) - T_2(t, f_n(t))\|^2 \right\}$$
$$\therefore (1 - 4\alpha_3)\|T_2(t, f_n(t)) - T_1(t, g_n(t))\|^2 \leq (\alpha_1 + 2\alpha_3 + 2a_4)\|g_n(t) - f_n(t)\|^2$$
$$+ (\alpha_2 + 4\alpha_3 + 2\alpha_4)\| f_n(t) - T_2(t, f_n(t))\|^2$$

---------------------------------(C)

similarly,

$$(1\text{-}4\alpha_2 - 2\alpha_4)\|T_2(t, f_n(t)) - T_1(t, g_n(t))\|^2 \leq (\alpha_1 + 2\alpha_2)\|g_n(t) - f_n(t)\|^2$$
$$+ (4\alpha_2 + \alpha_3 + 2\alpha_4)\| f_n(t) - T_2(t, f_n(t))\|^2$$

------------------------------(D)

adding (C) and (D), we get

$$(2 - 4\alpha_2 - 4\alpha_3 - 2\alpha_4)\|T_2(t, f_n(\mathrm{t})) - T_1(t, \mathrm{g}_n(\mathrm{t}))\|^2$$

$$\leq (2\alpha_1 + 2\alpha_2 + 2\alpha_3 + 2\alpha_4)\|\mathrm{g}_n(\mathrm{t}) - f_n(\mathrm{t})\|^2 + (5\alpha_2 + 5\alpha_3 + 4\alpha_4)\| f_n(\mathrm{t}) - T_2(t, f_n(\mathrm{t}))\|^2$$

$$\leq (2\alpha_1 + 7\alpha_2 + 7\alpha_3 + 6\alpha_4)\max\left\{ \sup_{f_n(t)\in F} \|g_n(t) - f_n(t)\|^2, \sup_{f_n(t)\in F} \| f_n(t) - T_2(t, f_n(t))\|^2 \right\}$$

$$or, \|T_2(t, f_n(t)) - T_1(t, g_n(t))\|^2 \le \frac{(2\alpha_1 + 7\alpha_2 + 7\alpha_3 + 6\alpha_4)}{(2 - 4\alpha_2 - 4\alpha_3 - 2\alpha_4)} \max \left\{ \begin{array}{l} \sup_{f_n(t)\in F} \|g_n(t) - f_n(t)\|^2, \\ \sup_{f_n(t)\in F} \|f_n(t) - T_2(t, f_n(t))\|^2 \end{array} \right\}$$

using the condition $2\alpha_1 + 11\alpha_2 + 11\alpha_3 + 8\alpha_4 = 2$

we can show that $\frac{(2\alpha_1 + 7\alpha_2 + 7\alpha_3 + 6\alpha_4)}{(2 - 4\alpha_2 - 4\alpha_3 - 2\alpha_4)} \le 1$

Taking the positive square root we get

$$\|T_2(t, f_n(t)) - T_1(t, g_n(t))\|^2 \le \max \left\{ \begin{array}{l} \sup_{f_n(t)\in F} \|g_n(t) - f_n(t)\|^2, \\ \sup_{f_n(t)\in F} \|f_n(t) - T_2(t, f_n(t))\|^2 \end{array} \right\}$$

$$= \max \left\{ \sup_{f_n(t)\in F} \|g_n(t) - f_n(t)\|^2, A \right\} = \gamma_{g_n(t)}(F) = \gamma(F)$$

So the set $T_2(F)$ is contained in a closed sphere with centre at $T_1(t, g_n(\mathrm{t}))$ and radius

$\gamma(F)$ .We denote this sphere by $\overline{U}$.

Clearly $T_2(F \cap \overline{U}) \subset F \cap \overline{U}$ because F is minimal $F \subset \overline{U}$

And so

$$\sup_{f_n(t)\in F} \|f_n(t) - T_2(t, f_n(t))\|^2 < \gamma(F). \text{ ------------------------------(5)}$$

now $\gamma_{T_1(t, g_n(t))}(F) = \max \left\{ \sup_{f_n(t)\in F} \|f_n(t) - T_2(t, f_n(t))\|^2, A \right\}$

$\le \max \{\gamma(\mathrm{F}), \mathrm{A}\}$ ------ $From (5)$

$= \gamma(\mathrm{F})$, bacause $\gamma(\mathrm{F}) \ge \mathrm{A}$.

Hence $\gamma_{T_1(t, g_n(t))}(F) \le \gamma(F)$ but we always have $\gamma_{T_1(t, g_n(t))}(F) \ge \gamma(F)$.

So $\gamma_{T_1(t, g_n(t))}(F) = \gamma(F)$

This implies that $T_1(t, g_n(t)) \in F_c$ and by (B) $T_2(t, f_n(t)) \in F_c$

There for F_c is a non-empty, closed and convex subset of F which is mapped into itself by T_1 and T_2 and because of (4) $\delta(F_c) < \delta(F)$.

Therefore F_c is proper subset of F .this contradicts the fact that F is minimal. Therefore, F can not contain more than one element, but F is not empty. Hence F contains only one element which clearly a random fixed point of T_1 and T_2 .

This completes the proof of the theorem (2.1)

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