

# Mechatronics Design of Ball and Beam System: Education and Research

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## Abstract

The key element in mechatronics design is the concurrent synergetic integration, modeling, simulation, analysis and optimization of multidisciplinary knowledge through the design process from the very start of the design process. Mechatronics engineer is expected to design engineering systems with synergy and integration toward constraints like higher performance, speed, precision, efficiency, lower costs and functionality. This paper proposes the conception and development of ball and beam system based on mechatronics design approach. A complete overall system and subsystems selection, modeling, simulation, analysis, and integration are presented. The proposed mechatronics design and models were created and verified using MATLAB /Simulink software and are intended for research purposes, as well as, application in educational process.

**Keywords-** Mechatronics, Mechatronics design approach, Ball and beam, modeling/simulation.

## 1. Introduction

Mechatronics is defined as synergistic integration of mechanical engineering, electric engineering, electronic systems, information technology, intelligent control system, computer hardware and software to manage complexity, uncertainty, and communication in design and manufacture of products and processes, a working definition can be; the synergistic integration of sensors, actuators, signal conditioning, power electronics, decision, control algorithms, computer hardware and software to manage complexity, uncertainty, and communication in engineered systems. The key element in mechatronics design is the concurrent synergetic integration (instead of sequential), analysis and optimization of these areas through the design process resulting in products with more synergy [1] and a balance between modeling/analysis and experimentation /hardware implementation. Mechatronics engineer is expected to design engineering systems with synergy and integration toward constraints like higher performance, speed, precision, efficiency, lower costs and functionality.

based on [1-2], Mechatronic system design process can be divided into systematic, simple and clear design steps including; Problem statement; Conceptual design and functional specifications; Parallel (concurrent) design and integration of system and all subsystems as whole and concurrently including; selection, design and synergetic integration of mechanical, electronics, software, control unit, control algorithm and interface sub-systems; Modeling and simulation; Prototyping, Testing and Optimization; and finally Manufacturing, and commercialization.

The significance of the ball and beam system is due to it's complexity, and at the same time it's simplicity, where it is a double integrator system is open loop unstable and simply, since it is a position control system, present a challenging design and control problem, it is often used by many control theorists and engineers as a bench mark problem for testing and analyzing the results of many different theoretical control schemes and new controller and observer methods. This paper proposes the conception and development of ball and beam system based on mechatronics design approach. A complete overall system's and subsystem's components' selection, design, integration, as well as, modeling, simulation and analysis are to be presented.

## 2. Pre-Study Process-problem statement

The ball and beam system is often used as a bench mark problem for many different control schemes. The ball and beam system presents a challenging design and control problem; it is motion control system consisting of three main parts; control system, a servo or electric motor and free rolling back and forth, Ball on top of long V-grooved Beam whose inclination can be adjusted by manipulating the angular position of the servo; ball and beam system present a challenging design and control problem due to fact that it is double integrator (two poles at the origin) system that is open-loop unstable, in such system even at optimal beam position conditions the ball will swing on Beam, up to roll off the end of the beam. Target user of the proposed design; Researchers and Educators. User's requirements; it is required to develop a ball and beam system; linear, one-dimensional single-variable experimental unit, that can be used for testing, analysis, education, and demonstration the effect of various control strategies, mainly SISO systems, precise, cost-efficient, with simple, easy to understand and use interface allowing the users to manipulate a machine, gathering and outputting data, space saving, and with

attractive design. System parameters, requirements and analysis, are listed in Table 1.

### 3. Conceptual design

The purpose of design is to propose the conception and development of ball and beam system based on mechatronics design approach, it is required to propose a design of a ball and beam system that fits user's (researchers and student) needs and requirements, with overall function (intended) to help users to test, visualize and analyze the selected and designed control strategy to continually measure and stabilize the rolling ball position along a given beam by adjusting the rotating angle of a beam by tilting it frontwards, backwards, leftwards, or rightwards, all this accomplished by manipulating the angular position of a selected servo. System's functional structure block diagram is shown in Figure 1. Morphological table, analysis and evaluating the best solution are shown in Table 2. Block diagram representations, preliminary structures including mechanical, electronic and system dynamics are shown in Figure 2 and Figure 3

Table 1 requirements analysis

Requirements		Value/ Unit	Requirements type			
			Fixed Req.	Range or soft Req.	Quantitative Req.	Qualitative Req.
<b>Control system</b>	Compact, cost effective.	-	Fixed			Qual.
<b>Control algorithm</b>	Simple, quick, precise, efficient, easy to program	-				
<b>Beam</b>	V-grooved,	40 cm	Fixed		Quan.	
	Mass, m	1.5 kg				
	Beam length, m	40 cm				
<b>Ball</b>	Radius	0.015 m	Fixed			
	Mass, M	0.111 kg				
<b>Actuators</b>	One actuator for adjusting the beam rotating angle, available simple to interface, and control inexpensive.	12-24 v	-	Soft		Qual.
<b>Sensors</b>	Position sensor, simple to interface, available, inexpensive		-	Soft		Qual.
<b>Transmission</b>	Gears, belt, chain	0.03 m		Soft		
	Output gear radius					
	Overall gear ratio	n				
<b>Overall System dimensions</b>	Height:	40 Cm	Fixed	-		Qual.
	Width	30 Cm				
	Length	50 Cm				
<b>User Interface</b>	Simple and easy to use and understand					Qual.
<b>Machine aesthetics design</b>	Compact, attractive fits user requirements.			Soft	Quan	

Function	Solution 1	Solution 2	Solution 3	Solution4
Control system	Microcontroller	Microcomputer	Analogue	
Control algorithm: <i>Position control</i>	PID	PD : deadbeat, prefilter	Fuzzy	?
Ball : steel	Rigid	hollow		
Beam: aluminium	Two parallel thin bars	V-grooved	U-grooved	
Actuator	DC motor	Stepper motor		
Position Sensor	Potentiometer	Encoder	Ultrasonic	Linear resistive wire
Transmission	Belt	Chains	Flexible cable	Gears and shaft
Interface	Touch screen	Keypad	Switches	
Support	Slides	Linkage-joint	Wheels	
Power	Electric			
Operator	Standing	Seating at front	Seating at rear	Remote control

Table 2 Morphological table, analysis and evaluating the best solution selection

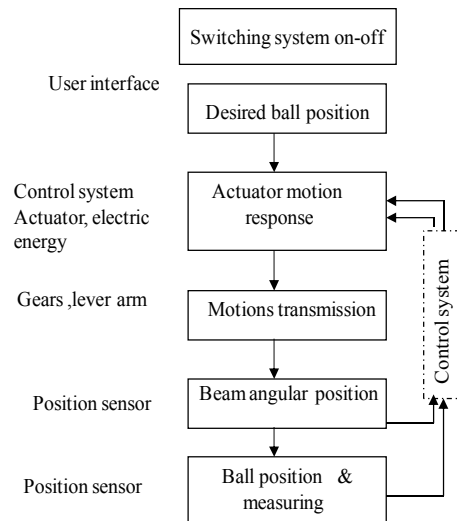


Figure 1 Functional Structure block diagram

#### 4. Parallel (concurrent) selection, design and integration of sub-systems and overall system.

Mechatronics design is the synergistic selection, evaluation and integration of the system and all its components as a whole and concurrently including sensors, actuators, signal conditioning, power electronics, decision, control algorithm, computer hardware and software, all the design disciplines work in parallel and collaboratively throughout the design and development process to produce an overall optimal design. The ball and beam system is to be divided into realizable subsystems (hardware and software; Mechanical system; sensors; actuators; power supplies, controller and Control algorithm, drive, and conditioning circuits), the optimal selection, modeling, simulation, integration, optimization, and the exchange of information between different modules are to be designed concurrently.

##### 4.1 The optimal mechanical design:

The mechanical design is the skeleton of mechatronics systems, there are several arrangements of constructing ball and beam system, three system's arrangements are shown in Figure 2. In Figure 2 (a), one end of the beam is coupled to electric motor (Figure 2(b)) through lever arm and gears, the other end is fixed, and supported with vertical beam. In Figure 2(c) one end of the beam is coupled to electric motor through lever arm and gears, the other end is free to move up and down, the beam is supported with vertical bar at its center. In Figure 2(d) the beam at it center, is mounted on the output shaft of an electric motor. System CAD model is shown in Figure 2(e). The whole system layout is shown in Figure 2(a), and the mechanical system parts, materials and optimal dimensions are listed in Table 3. Preliminary block diagram and layout representations of proposed system and

main components are shown in Figure 2(f),

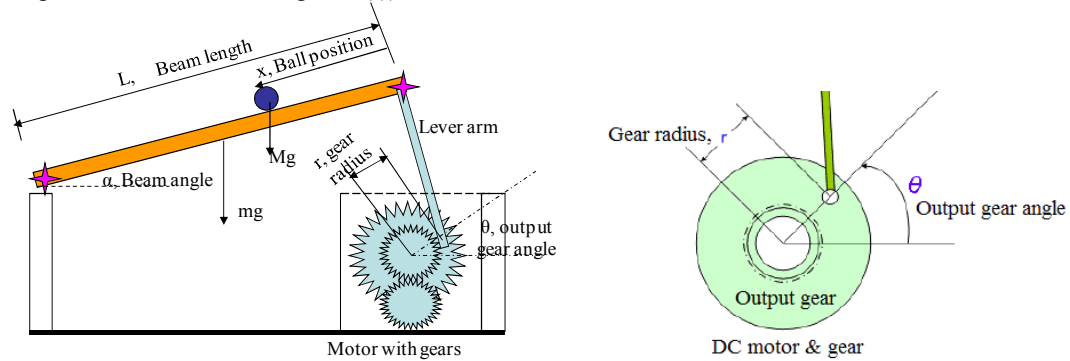


Figure 2(a) one end of the beam is coupled to electric motor through lever arm and gears, the other end is fixed.

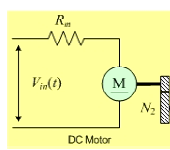


Figure 2 (b) Electric motor with gears

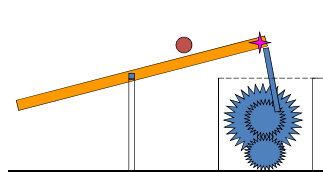


Figure 2 (c) support at beam's center

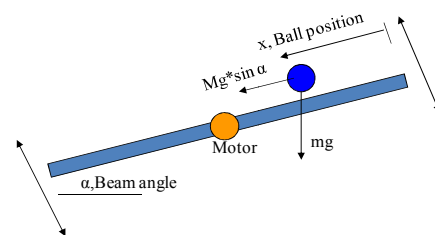


Figure 2 (d) The beam at its center, is mounted on the output shaft of an electric motor.

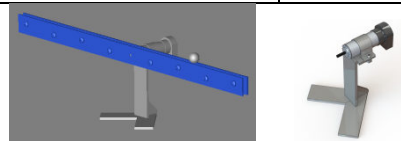


Figure 2(e) 3-D CAD model

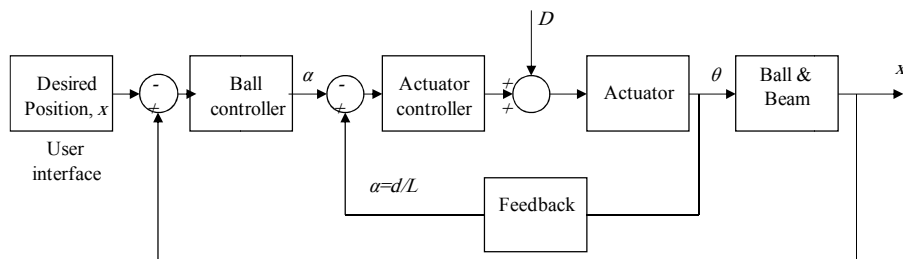


Figure 2 (f) system block diagram

Table 3 mechanical system parts, type of joints materials and dimensions

	<b>Value:</b>	<b>Unit</b>
<b>System dimensions</b>	Height	40 Cm
	Width	30 Cm
	Length	50 Cm
<b>Beam</b>	Shape & material	V-grooved ,steel
	Beam mass, m	1.5 kg
	Beam length, L	40 cm
<b>Ball</b>	Shape & material	Steel
	Radius	0.015 m
	Mass, M	0.111 kg
<b>Transmission</b>	Outer gear radius	0.03 m
	Overall gear ratio	n
<b>Joints</b>	Revolute joint	-
<b>Support</b>	Steel bar	-
<b>Type of joints</b>	Revolute joints	3J
<b>Base, Frame</b>		

#### 4.2 Controller and control algorithm selection

The behavior of moving ball on a beam is behavior of open loop unstable system . The control of unstable systems is critically important to many of most difficult control problems, and must be studied in laboratory, the problem is that real unstable systems are usually dangerous and can not brought in laboratory, such unstable systems include; the control model in the rocket or aircraft toppling control system during vertical take-off, it is the control where a feedback system is used to prevent rocket or aircraft to topple out of balance during launch by forces and moments that could perturb the vertical motion, the angle of thruster jets or diverters must be continually controlled to prevent the rocket tumbling or aircraft tipping [2-4]. The ball and beam system was developed to resolve this paradox, it is simple, safe mechanism and yet it has the important dynamic features of unstable system [2], due to fact that it is double integrator (two poles at the origin) system and is open-loop unstable, the ball and beam system is often used by many control theorists and engineers as a bench mark problem for testing and analyzing the results of many different theoretical control schemes and new controller and observer methods, In order to stabilize the double integrator and open-loop unstable ball and beam system, a special control system is to be designed.

Control algorithm selection is the most critical decision in the Mechatronics design process, there are number of alternative control algorithm strategies that can be applied to the ball and beam system, including but not limited to ; PD control, lead compensator, Fuzzy control, Robust control, Linear quadratic Gaussian, Linear quadratic regulator. The control system task, in ball and beam system, is to continually measure and stabilize the rolling ball position a long the beam by adjusting the rotating angle of a beam ( tilting it frontwards or backwards) by manipulating the angular position of the servo, in such a way that the actual position of the ball reaches desired position, this is difficult control task, because even at optimal beam position conditions the ball will swing on Beam with acceleration proportional to tilt angle, this behavior is behavior of open loop unstable system, where the system output (ball position on beam) increases without limits for fixed input (beam angle) and is a challenging design and control problem. In [5] W. Yu and F. Ortiz studied complete nonlinear model of beam and ball system and stability analysis of PD control. In [6] a designed and implemented both conventional pole-placement and neural network methods for beam and ball system are introduced. In [7] the robust nonlinear servomechanism theory was applied to design a tracking controller for beam and ball system. In [8] switching control for beam and ball system is studied. In [9] fuzzy logic controller and real time implementation of beam and ball balancing system is studied. In this paper will be introduce PD controller with deadbeat response, PID, and Lead integral compensator.

A vast variety of controllers (physical controller) are available in the market, embedded Microcontroller is optimal selection, since it is inexpensive single chip computer, easy to embed into larger electronic circuit designs, also, because of their versatility, Microcontrollers add a lot of power, control, and options at little cost; capable of storing and running programs, programmed to perform a wide range of control tasks. Optimal microcontroller is PICmicro Microcontroller, supplied with 5VDC.

#### 4.3 Actuator (electric machine) selection and integration

Actuator converts an information signal from the microcontroller control unit, into energy acting on the basic system. Mechatronic systems often use electric motors to drive their work loads, the electric actuator most suitable to be used for ball and beam is DC motor, due to simple principle of working, quick instantaneous and accurate torque generation, available, inexpensive, reversible, and ease designing and implementing controller to achieve optimal instantaneous, precise motion control performance. PMDC motor turns electrical energy into mechanical energy and produces the torque required to move (rotate) the beam to the desired position,  $\theta_L$ .

**Actuator placement and integration** (see Figure 3): to physically integrate mechatronics system components, and to give the overall system safe, compact and attractive design, the selected DC motor and gears, are embedded within mechanical design and to be located on adjacent (right side, a whole system base) cuboide housing, with slots for digital user interface and input/output ports.

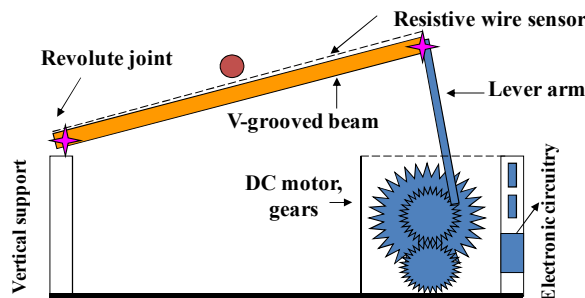


Figure 3(a)

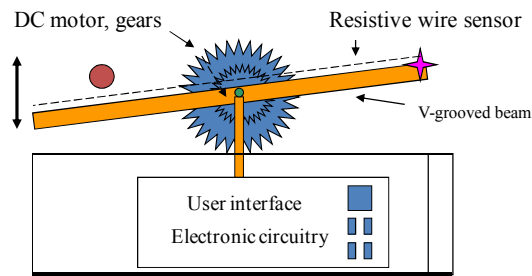


Figure 3(b)

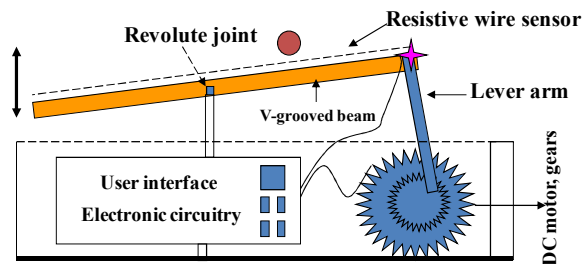


Figure 3(c)

Figure 3(a)(b)(c) Three arrangements of overall system design, components and integration

#### 4.4 Sensor selection

Sensor converts a state variable of the basic system, into an information signal to the control unit. To calculate the error between the actual and desired positions, we need to convert the actual position, into voltage,  $V$ , and then compare this voltage with the input voltage; the difference between both is the error signal in volts. Ball and beam system, provides two outputs measurements and correspondingly two sensors, one to measure ball translational position,  $x$  along the beam and other to measure motor shaft angular position  $\theta$ , correspondingly, the optimal control system design requires two control loops designs, inner for motor angle control, to make motor follow the input angle and other outer loop to control the ball position on the beam, to make ball position follow the reference input position. The two angular quantities the beam angle,  $\alpha$  and DC motor output position angle,  $\theta$  are not the same, but are correlated in simple relationship, relating these two quantities result in that only one sensor can be used to measure the position. The position of the ball on the beam can be measured using a special sensor, e.g. linear Potentiometer, infrared, and a linear resistive wire, the optimal selection is linear resistive wire, to used as a position sensor and the conductive ball moving along it acting as a voltage divider (see Figure 3).

#### 4.5 Output signal, Conditioning and Interfacing selection, design and integration

A most suitable and simple to implement drives for PMDC motor are H-bridge and IC circuit e.g.L93D. The H-bridge circuit is supplied with 12VDC and the four bits outputs of microcontroller to drive the desire conditions of electric Motor. By H-bridge four NPN Power transistors are used as switch to control the direction of current flows to the Motor, and correspondingly control the angular position of beam, selected microcontroller type is supported with ADC pins, to convert the analog input of sensor readings to digital value, fed to microcontroller. A common carrier (see Figure 3-4) for electronics, data outputting and control circuitry to be located inside the cuboide housing, that will include slots for user interface and input/output ports

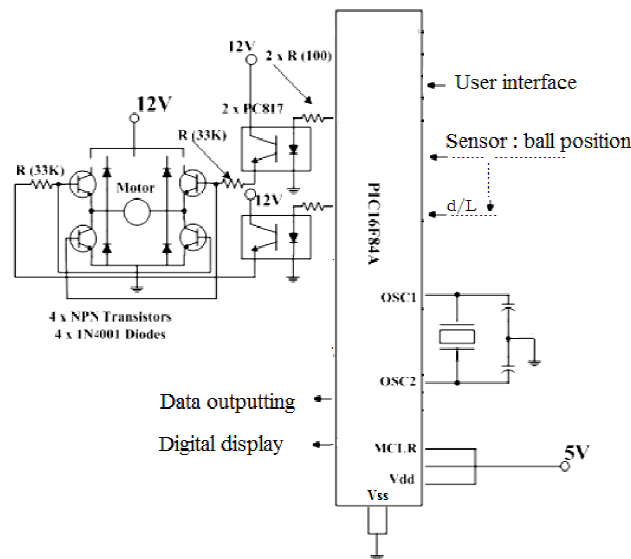


Figure 4 Microcontroller based control Circuitry and interfacing diagram

## 5. Modeling, simulation, analysis and evaluation

The key essential characteristics of a mechatronics engineer are a balance between modeling/analysis skills and experimentation/hardware implementation skills. Modeling, simulation, analysis and evaluation processes in mechatronics design consists of two levels, sub-systems models and whole system model with various sub-system models interacting similar to real situation, all engineering subsystems should be included in whole system model. Based on the specification of requirements and design, the subsystems models and the whole system model, are to be tested and analyzed, specifications to test and check whether the given design specifications are satisfied, If the specification are not satisfied, modifications can be made, if the specifications are satisfied the model can be Optimized. The derivation of complete and accurate ball and beam system's dynamics is quit complicated, in this paper, different system designs and dynamic models will be introduced, simplified and more actual models and corresponding control system strategies selection and design. Ball and beam dynamics are shown in Figure 5.

### 5.1 Modeling of basic physical sub-system model with no control involved; Ball and beam balancer System Modeling

#### 5.1.1 Ball and beam system's dynamics, basic mathematical model derivation: sliding ball.

In this paper, different approaches for deriving the mathematical model of ball and beam will be introduced. Based on [1] and on system arrangement shown in Figure 2(a), dynamics are shown in Figure 5. In the absence of friction or other disturbances and assuming the ball is sliding, the dynamics of the ball and beam system can be obtained as follows; The ball moves a long the beam with acceleration, the forces that accelerate the ball as it rolls comes from the component of gravity that acts parallel to the beam. The equation of motion for the Ball is given by the following equation:

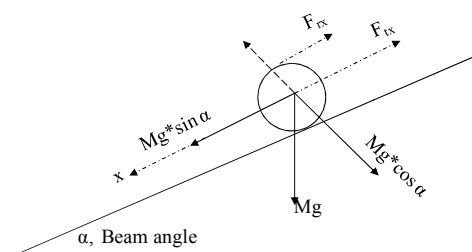


Figure 5 (a) Ball dynamics

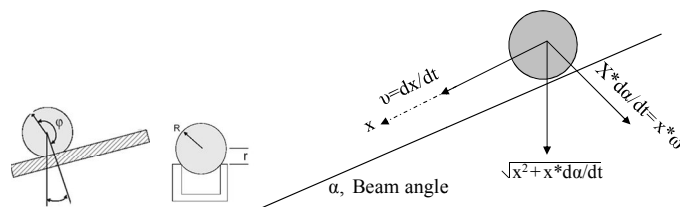


Figure 5 (b) Ball dynamics

$$\sum F = M * a \Leftrightarrow \sum F = M * g * \sin(\alpha)$$

Substituting and manipulating, gives:

$$M * \frac{d^2 x}{dt^2} = M * g * \sin(\alpha) \Rightarrow \frac{d^2 x}{dt^2} = g * \sin(\alpha)$$

This model can be linearized, where for small angles,  $\sin(\alpha) \approx \alpha$ , the model becomes:



$$\frac{d^2x}{dt^2} = \frac{g}{L} \alpha \quad (1)$$

This is the basic mathematical model of the ball and beam system, this simple mathematical model shows that, the acceleration of the ball is proportional to the beam angle  $\alpha$  and to gravity  $g$ . Where:  $x$  : the ball position on the beam,  $M$  : the ball mass. to write an approximated overall transfer function of the ball and beam system, that relates the input voltage and measured output acceleration, we need to replace the beam angle  $\alpha$ , by control voltage,  $V_{in}$ , and replace the ball position  $x$ , by the position sensor output  $y$ , and finally Combine the actuator and sensor constants with the gravity constant, we have a single constant  $b$ , Substituting, performing Laplace transform and rearranging, we have the transfer function that relates the input voltage and measured output acceleration

$$\frac{d^2y}{dt^2} = b V_{in} \Leftrightarrow Y(s)s^2 = b V_{in}(s) \quad (2)$$

$$\frac{Y(s)}{V_{in}(s)} = \frac{b}{s^2} \Leftrightarrow \frac{Y(s)}{V_{in}(s)} = \frac{g}{s^2}$$

Where:  $b$ : the actuator and sensor with the gravity constants. The state space representation of this approach is given by:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The linear model given above is good approximation of true system dynamics, all what we need is to measure the constant parameter,  $b$ , it can be obtained by measured the time taken for the ball to accelerate from one end of the beam to the other at fixed angle of the beam, the laws of motion can be then used to integrate Eq.(1) and calculate constant  $b$ . The real ball and beam system has additional dynamic components due to the motor, plus non-linear and noise components that influence its control behavior, (coulomb friction, dead zone saturation in the motor input amplifier)[4].

### 5.1.2 Ball and beam system's dynamics, second approach: rolling ball without slipping.

Based on system arrangement shown in Figure 2(a), Ball dynamics shown in Figure 5(b), and in the absence of friction or other disturbances and assuming the ball is rolling without slipping, the dynamics of the ball and beam system can be obtained as follows;

There are two forces influencing the motion of the rolling ball, these forces are:  $F_{tx}$  : the force due to translational motion and  $F_{rx}$  : the force due to ball rotation. The equation of motion for the Ball is given by:

$$\Sigma F = M \cdot a = M \cdot g \cdot \sin(\alpha)$$

Substituting forces influencing the motion of the rolling ball, gives:

$$\Sigma F = F_{tx} + F_{rx} = M \cdot g \cdot \sin(\alpha) \quad (3)$$

Now, to find forces influencing the motion of the rolling ball, and substituting in Eq.(3). Force  $F_{tx}$  due to translational motion is given by:

$$F_{tx} = m\ddot{x} \quad (4)$$

The force due to ball rotation,  $F_{rx}$  is found as follows: The torque produced by the ball's rotational motion is equal to the radius of the ball,  $R$ , multiplied by the rotational force, and is given by

$$T_r = F_{rx} R$$

Also, the torque is equal to the ball's moment of inertia  $J$ , multiplied by its angular acceleration,  $d\omega_b/dt$ , which then can be written as its moment of inertia multiplied by the double-derivative of its translational motion ( $x_b$ ) divided by its radius, this is as shown next:

$$T_r = F_{rx} R = J \frac{d\omega_b}{dt}$$

$$T_r = F_{rx} R = J \frac{d\omega_b}{dt} = J \frac{d(v_b / R)}{dt} = J \frac{d^2(x_b / R)}{dt^2} = \frac{J}{R} \ddot{x}$$

$$F_{rx} = (J \cdot a) / R^2$$

Substituting, the moment of inertia,  $J$ , for the ball and given by ( $J = (2/5)MR^2$ ), we have:

$$F_{rx} = (2/5)M \frac{d^2x}{dt^2} \quad (5)$$

Substituting Eq.(5) and Eq. (4), in Eq.(3) we have the basic mathematical model of ball and beam system that



relates the ball displacement and angular position of the motor shaft, and given as:

$$\Sigma F = F_{tx} = M * g * \sin(\alpha)$$

$$M * g * \sin(\alpha) = \left(\frac{2}{5}\right) M \frac{d^2 x}{dt^2} + M \frac{d^2 x}{dt^2}$$

Manipulating and rearranging, gives:

$$M * g * \sin(\alpha) = d^2 x / dt^2 [ (2/5) M + M ]$$

$$g * \sin(\alpha) = d^2 x / dt^2 [ (7/5) ]$$

$$(5/7) * g * \sin(\alpha) = d^2 x / dt^2$$

This model can be linearized, since for small values of angle  $\alpha$ ,  $\sin(\alpha) = \alpha$ , this approximation is good for angle values  $-\pi/4 < \alpha < \pi/4$ , sine of the angle is approximately the angle itself. Taking Laplace transform and rearranging input displacement,  $X(s)$ , over output angular position,  $\alpha(s)$ , of the motor shaft gives :

$$(5/7) g \alpha(s) = X(s) s^2$$

$$\frac{X(s)}{\alpha(s)} = \frac{(5/7)g}{s^2} \quad (6)$$

The state space representation of for this approach is given by:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 5/7 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The transfer function can be derived based on system arrangement shown in Figure 2(b), as follows; the beam angle controls acceleration, not position, from gravity the equation of motion for the Ball is given by:

$$\Sigma F = M * a = M * g * \sin(\alpha)$$

Assuming the ball is rolling without slipping, we have  $r\omega = dx/dt$ , The moment of inertia of the solid ball is given by: ( $J = (2/5)MR^2$ ). Given centripetal acceleration with rotation of the rod as  $a_{\text{centripetal}} = x\alpha^2$ , altogether we have:

$$\frac{d^2 x}{dt^2} - x \frac{d^2 \alpha}{dt^2} = -g \sin \alpha - \frac{2}{5} \frac{d^2 x}{dt^2}$$

By linearization, taking Laplace transform, rearranging for transfer function, gives:

$$\frac{X(s)}{\alpha(s)} = \frac{(5/7)g}{s^2} \quad (7)$$

### 5.1.3 The Lagrangian approach to the Ball and Beam system's dynamics:

In the absence of friction or other disturbances, the dynamics of the ball and beam system can be obtained by Lagrangian method based on the energy balance of the system, The Lagrangian method is utilized to derive the equation of motions for the ball and beam system in the most model based research works on the ball and beam system [6][11-13]. In order to derive the Euler-Lagrange equation, the first step is to define the kinetic and potential energy for the ball and beam. The kinetic energy of the system is given by:

$$T = T_1 + T_2 \quad (8)$$

Where  $T_1$  and  $T_2$  are kinetic energies of the beam and the ball, these kinetic energies include radial and circular motions. The rotational kinetic energy of the beam is given by:

$$T_1 = 0.5J_1 \dot{\alpha}^2$$

Where:  $J_1$ , is the moment of inertia of the beam and  $\alpha$  is the angle velocity of the frame. The ball has kinetic energy, given by:

$$T_2 = 0.5(Mx^2) * \dot{\alpha}^2 + 0.5Mv^2 + 0.5J_b \omega^2$$

Substituting  $v = R\omega$  and  $J_b = (2/5) * M * R^2$ , the rotational kinetic energy of the ball is given by:  $0.5((2/5) * M * R^2)$ , substituting, the kinetic energy of the system in Eq.( 8) will give:

$$T = 0.5 \left( (J + Mx^2) * \dot{\alpha}^2 + (7/5) Mx^2 \right)$$

The potential energy of the system is exhibited by the rolling ball alone, and given by:

$$P = M * g * x * \sin \alpha + 0.5m * g * L * \sin \alpha$$

$$P = (M * g * x + 0.5 * m * g * L) \sin \alpha$$

Where  $m$ : the mass of the beam,  $M$ : the mass of the ball,  $L$ : the beam length. We can now write the Lagrangian, where the difference between kinetic and potential energy is called the Lagrange function, which is defined by (L) equation and given by:

$$L = T - P$$

Where : T is the kinetic energy and P is the potential energy in the system.

$$L = \frac{1}{2} \left[ 0.5 \left( (J + Mx^2) \dot{\alpha}^2 + (7/5) Mv^2 \right) \right] - \left[ (M * g * x + 0.5 * m * g * L) \sin \alpha \right]$$

The dynamics equation representing the variation effect of system variable is given by:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = Q \quad (9)$$

$$0 = \left( \frac{J}{R^2} + M \right) \frac{d^2 x}{dt^2} + Mg * \sin \alpha - m * x * \alpha^2 \quad (10)$$

Where  $\theta$ : the motor output position angle.  $\alpha$ : the beam angle.  $x$ : ball coordinate position,  $m$ : beam mass. As shown in Figure 2 (a) and Figure 7, It should be considered, that the beam angle,  $\alpha$ , and DC motor output shaft angle,  $\theta$ , are not the same, but there is also relationship between these two angles that can be seen from Figure 6, this relation can be approximated as linear, where the distance traveled by the beam at radius equals to  $L$ , is equal to distance traveled by gear at radius,  $r$ , that is:

$$r\theta = L\alpha \Leftrightarrow \alpha = \frac{r}{L}\theta \quad \text{for } \theta \ll L \quad (11)$$

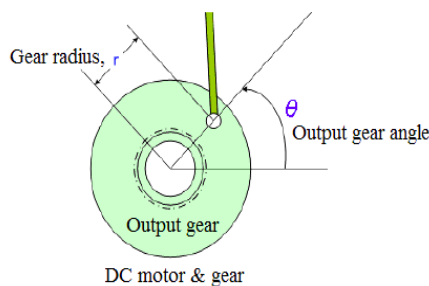


Figure 2 (a)

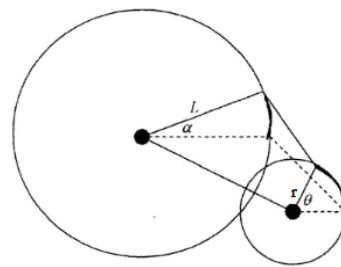


Figure 7 the beam angle,  $\alpha$ , and DC motor output position angle,  $\theta$

**Ball and beam system's dynamics in terms of ball radius  $R$ , and  $r$ :** based on system arrangement shown in Figure 2 (a), dynamics shown in Figure 5, and in the absence of friction or other disturbances, we have:

$$x = r\phi \Leftrightarrow \omega = \dot{\phi} + \dot{\theta} = \frac{\dot{x}}{r} + \dot{\theta} \Leftrightarrow v = \sqrt{\dot{x}^2 + (x\dot{\theta})^2}$$

$$U = \frac{1}{2} m v^2 + \frac{1}{2} I_b \omega^2 + \frac{1}{2} I_a \dot{\theta}^2$$

Writing and substituting the Lagrangian equation, gives:

$$L = \frac{1}{2} \left\{ m[\dot{x}^2 + (x\dot{\theta})^2] + I_b \left[ \frac{\dot{x}}{r} + \dot{\theta} \right]^2 + I_a \dot{\theta}^2 \right\}$$

$$\frac{d}{dt} \left( \frac{d}{dx} L \right) - \frac{d}{dx} L = mg \sin \theta$$

$$\left( m + \frac{I_b}{r^2} \right) \ddot{x} + \left( \frac{I_b}{r^2} \right) \ddot{\theta} - mx\dot{\theta}^2 = mg \sin \theta$$

$$\left( m + \frac{I_b}{r^2} \right) \ddot{x} = mg \theta$$

Taking Laplace transform, Substituting  $I_b = (2/5)MR^2$ , and manipulating for transfer function gives:

$$G(s) = \frac{X(s)}{\Theta(s)} = \frac{mg}{m + \frac{I_b}{r^2}} \frac{1}{s^2} = \frac{mg}{m + \frac{2}{5} \frac{MR^2}{r^2}} \frac{1}{s^2} = \frac{g}{1 + \frac{2}{5} \left( \frac{R}{r} \right)^2} \frac{1}{s^2}$$

The transfer function in terms of input gear angle,  $\theta$ , to output ball position,  $X$ , also can be written as follows: Linearization of Eq.(10) about the beam angle,  $\alpha=0$ , gives

$$0 = \left( \frac{J}{R^2} + M \right) \frac{d^2 x}{dt^2} + Mg * \sin \alpha - m * x * \alpha^2$$

$$\left( \frac{J}{R^2} + M \right) \frac{d^2 x}{dt^2} = -M * g * \alpha$$

Linearization of this equation, substituting Eq.(11) taking Laplace transforms, and manipulating for transfer function gives:

$$\left(\frac{J}{R^2} + M\right) \frac{d^2x}{dt^2} = -M * g * \frac{d}{L} \theta \Rightarrow \frac{Y(s)}{\theta(s)} = \frac{M * g * d}{L \left(\frac{J}{R^2} + M\right)} * \frac{1}{s^2}$$

The state space representation of this model is given below, here it is important to notice in these models, that by combining the ball radius R, and r constants with the gravity constant, g, we have a single constant b, this will give the basic transfer function model previously obtained, this is shown in below comparison of different approaches for model derivations:

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{mgd}{L(\frac{J}{R^2} + m)} \end{bmatrix} \theta \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} u \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ g \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ \frac{X(s)}{\theta(s)} &= \frac{M * g * d}{L \left(\frac{J}{R^2} + M\right)} * \frac{1}{s^2} \Rightarrow \frac{Y(s)}{\theta(s)} = \frac{b}{s^2} \Rightarrow \frac{Y(s)}{\theta(s)} = \frac{g}{1 + \frac{2}{5} \left(\frac{R}{r}\right)^2} * \frac{1}{s^2} \Rightarrow \frac{Y(s)}{\theta(s)} = \frac{b}{s^2} \end{aligned}$$

The transfer function between the ball position and rotor angle can be obtained based on basic transfer function

$$\frac{Y(s)}{V_{in}(s)} = \frac{b}{s^2} = \frac{g}{s^2} \quad \text{and on relation given by Eq.(11), and given by}$$

$$\frac{Y(s)}{V_{in}(s)} = \frac{r}{L} \frac{g}{s^2} = \frac{r * g}{L} \frac{1}{s^2}$$

## 5.2 MATLAB/Simulink representation, simulation and analysis

The loop system simulink models for the simplified models is shown in Figure 8. The simulink model of mathematical model given by Eq.(6) is the same as simulink model shown in Figure 8, but the constant b, now is more accurately defined and is given by (5/7)\*g, the simulink model of open loop ball and beam system is shown in Figure 8(c), The system response using MATLAB, can be found using the following codes

```
>> g=9.8; num=[5*g/7]; den=[1 0 0]; G_open=tf(num,den); printsys(num,den); step(num,den); [A,B,C,D]=tf2ss(num,den); Figure , step(A,B,C,D)
```

Now , Based on [14], the nonlinear Lagrangian equation of motion is directly modeled, where Eq.(10) gives  $d(x)/dt$  as a function of the state and input variables , x,  $d(x)/dt$  , alpha, and  $d(alpha)/dt$ . The reference made use of the nonlinear function block to express this function; the function takes the input vector  $u=[x \ dx/dt \ \alpha \ da/dt]$  and returns  $d^2x/dt^2$  , where each component is referred to as  $u[1]$ ,  $u[2]$ , etc, and correspondingly,  $u[1]=x$ ,  $u[2]=d(x)/dt$ ,  $u[3]=\alpha$ , and  $u[4]=d(\alpha)/dt$ , the function is given by:

$$\frac{d^2x}{dt^2} = (-1 / (J / R^2 + M)) (Mg \sin \alpha - Mx \frac{d^2\alpha}{dt^2})$$

And can be written in MATLAB as:

$$(-1/(J/(R^2)+m))*(m*g*\sin(u[3])-m*u[1]*(u[4]^2))$$

This model is shown in Figure 9 (a), first it is required, to define ball mass m, Ball radius, R, beam length, L, gear radius, r, and ball's moment of inertia J, (e.g. m= 0.111; R= 0.015; g= -9.8; L= 0.6; d= 0.03; J= 9.99e-6). Running all simulink models for derived system's models, when subjected to random input, will result in the same position, speed and acceleration response curves shown in Figure 10(a). In Figure 10(b) is shown ball position and acceleration open loop response when subjected to step input.

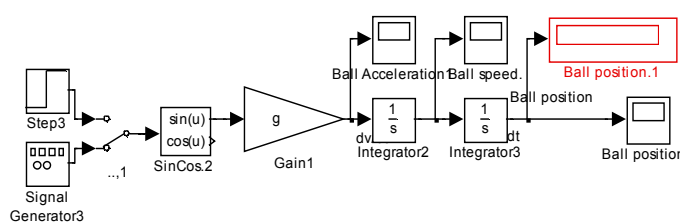


Figure 8(a) Simulink model of simplified representation

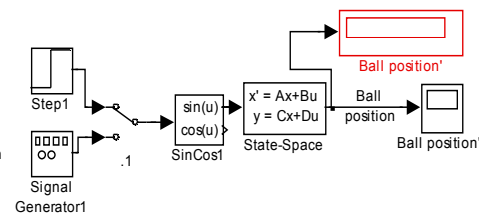


Figure 8(b) Simulink model of state space representation

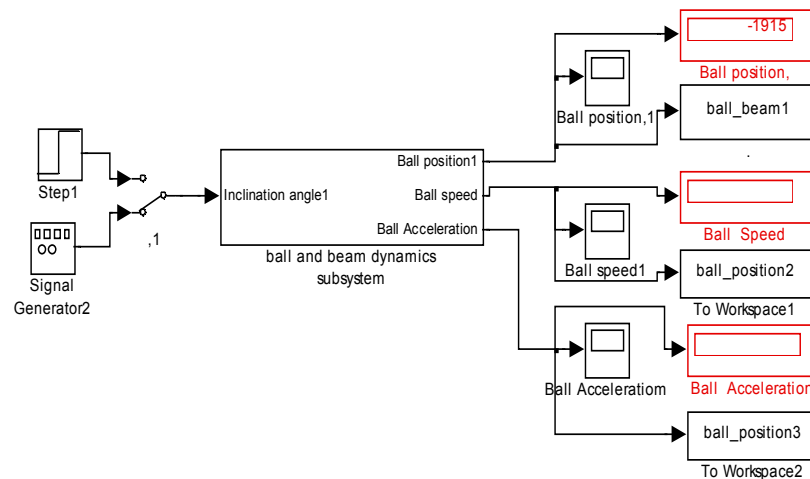
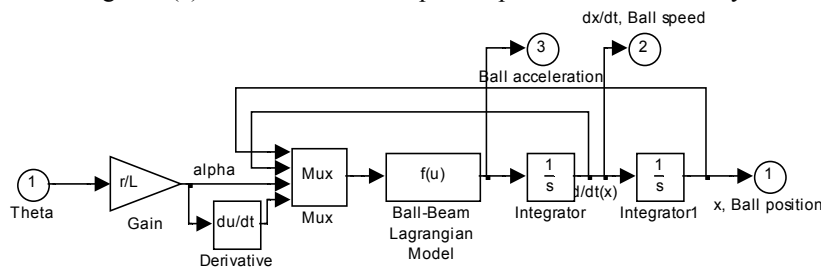


Figure 8(c) Simulink mode of open loop ball and beam sub-system



Fig, 9 (a) The nonlinear Lagrangian equation of ball and beam open loop motion sub-system model

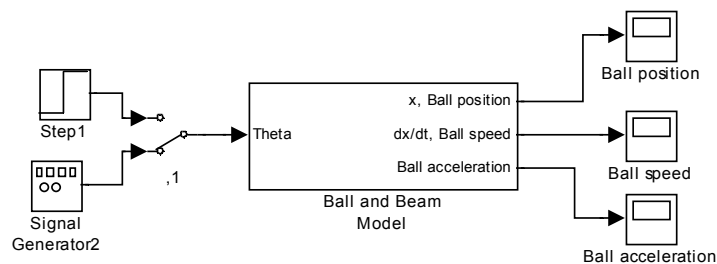
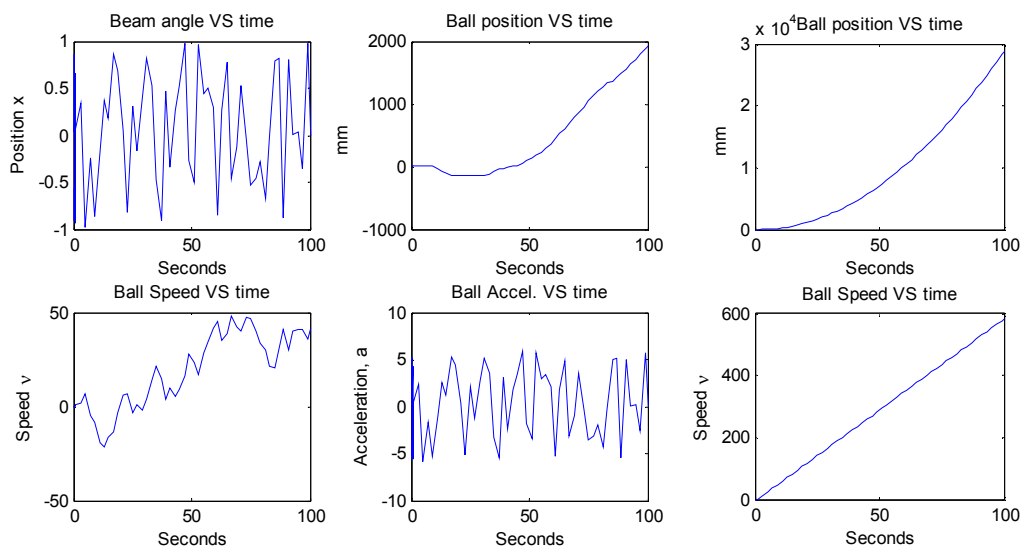


Figure 9 (b) The ball and beam sub-system open loop model and subsystem.



(a) Response for randomly changing input (b) Response for step input  
Figure 10 open loop, ball position/time, speed/time and acceleration/time response curves

The next MATLAB code can be used to define system's parameters in MATLAB and return transfer function and open loop step response

clc, clear all

M=input('Enter mass of the ball, M = ');

m=input('Enter mass of the beam, m = ');

R=input('Enter radius of the ball, R = ');

d=input('Enter lever arm offset , d = ');

L=input('Enter length of the beam , L = ');

J=input('Enter ball's moment of inertia , J = ');

g=-9.8; % gravitational acceleration

s = tf([1 0],1); G\_open1 = -(M\*(g\*d)/(L\*((J/R^2)+ M)\*s^2); step(G\_open1)

% another simplified way of obtaining the transfer function

K = (M\*(g\*d)/(L\*(J/R^2+M))); num = [-K]; den = [1 0 0]; G\_open2=tf(num,den); step(G\_open2); [A,B,C,D]=tf2ss(num,den); step(A,B,C,D);

% Obtaining closed loop transfer function with unity feedback , no gain

G\_close=feedback(G\_open1,1); axis([1,100,0,2]);

title ('step response of closed loop system with H(s)=1 ')

### 5.3 Modeling of actuator sub-system dynamics

The DC motor is an example of electromechanical systems with electrical and mechanical components, a simplified equivalent representation of DC motor's two components with gears attached are shown in Figure 2(c) up . DC motor turns electrical energy into mechanical energy and produces the torque required to move the beam to the desired angular position,  $\theta_L$ , [15]. In [16] a detailed derivation of DC motor electric and mechanical parts dynamics are introduced. Based on the Newton's law combined with the Kirchoff's law, the mathematical model in the form of differential equations describing dynamics characteristics of the armature controlled PMDC motor can be derived, and correspondingly DC motor transfer function open loop transfer function without any load attached relating the input voltage,  $V_{in}(s)$ , to the motor shaft output angle,  $\theta_m(s)$ , to be given by:

$$G_{angle}(s) = \frac{\theta(s)}{V_{in}(s)} = \frac{K_t / n}{\{s[(L_a s + R_a)(J_m s + b_m) + K_t K_b]\}} \quad (12)$$

$$\frac{\theta(s)}{V_{in}(s)} = \frac{K_t / n}{\{[L_a J_m s^3 + (R_a J_m + b_m L_a)s^2 + (R_a b_m + K_t K_b)s]\}}$$

The PMDC motor open loop transfer function relating the input voltage,  $V_{in}(s)$ , to the angular velocity,  $\omega(s)$ , given by:

$$G_{speed}(s) = \frac{\omega(s)}{V_{in}(s)} = \frac{K_t / n}{\{[(L_a J_m s^2 + (R_a J_m + b_m L_a)s + (R_a b_m + K_t K_b))]\}} \quad (13)$$

Modeling nonlinearities: Coulomb friction is a non-linear element in which forces tend to appose the motion of bodies in contact in mechanical systems, it acts as disturbance torque feedback for the mechanical system, Coulomb friction is considered to be a constant retarding force but is discontinuous over zero crossings, that is, when a DC motor reverses direction it must come to a stop at which point Coulomb friction drops to zero and then opposes the reversed direction. In effect Coulomb friction is constant when rotational velocity is not zero. Based on differential equations describing mechanical characteristics of DC motor; the sum of the torques must equal zero, we have:

$$\sum T = J \ddot{\alpha} = J \frac{d^2 \theta}{dt^2}$$

Coulomb friction and dead zone friction, where ( $T_{load}=0$ ), we have:

$$K_t i_a = T_a + T_\omega + T_{load} + T_f$$

Where:  $i_a$  armature current,  $K_t$  : torque constant,  $T_f$  : Coulomb friction torque, taking Laplace transform:

$$K_t I(s) - J_m s^2 \theta(s) - b_m s \theta(s) - T_f = 0$$

At steady state conditions,  $d/dt=0$ , gives:

$$K_t i_a = -b \omega - T_f$$

### 5.4 Simplification of open loop PMDC motor system transfer functions models.

Referring to [16] and based on the fact that, the PMDC motor response is dominated by the slow mechanical time constant, where the electric time constant is much faster (e.g. ten times) than the mechanical time constant, this can motivate us to assume that the armature inductance,  $L_a$  is low compared to the armature resistance,  $R_a$ . neglecting motor inductance, ( $L_a=0$ ), will result in the following simplified first order form of PMDC motor transfer function in terms input voltage,  $V_{in}(s)$  and output speed,  $\omega_m(s)$  given by:

$$G_{speed}(s) = \frac{\omega(s)}{V_{in}(s)} = \frac{K_t}{\{(R_a J_m)s + (R_a b_m) + K_t K_b\}}$$

Rearranging this first order equation into standard first order transfer function form yields:

$$G_{\text{speed}}(s) = \frac{\omega(s)}{V_{\text{in}}(s)} = \frac{K_b K_t}{(R_a b + K_t K_b)} \left/ \left[ \left( \frac{R_a J}{(R_a b + K_t K_b)} \right) s + 1 \right] \right. = \frac{K_b}{\tau s + 1} \quad (14)$$

A simplified first order form of PMDC motor transfer function in terms of output angle can also be obtained by substituting ( $L_a = 0$ ), and given by:

$$\frac{\theta(s)}{V_{\text{in}}(s)} = \frac{K_t / n}{\left[ L_a J_m s^3 + (R_a J_m + b_m L_a) s^2 + (R_a b_m + K_t K_b) s \right]} \Rightarrow \frac{K_t}{\left[ R_a J_m s^2 + (R_a b_m + K_t K_b) s \right]} = \frac{\frac{K_t}{R_a J_m}}{s \left[ s + \frac{1}{J_m} \left( b_m + \frac{K_t K_b}{R_a} \right) \right]} = \frac{K}{s(s+a)} \quad (15)$$

by substituting ( $L_a = 0$ ), motor equation can be simplified to first and second order system relating input voltage and output angle, as well as equation relating input voltage and output speed, to be given by:

$$G_{\text{angle}}(s) = \frac{\theta(s)}{V_{\text{in}}(s)} = \frac{K_t}{s(R_a J_m s + K_t K_b)} \quad \text{and} \quad G_{\text{speed}}(s) = \frac{\omega(s)}{V_{\text{in}}(s)} = \frac{K_t}{(R_a J_m s + K_t K_b)}$$

The transfer function relating input voltage and output angle can be more simplified to have the next form:

$$G_{\text{angle}}(s) = \frac{\theta(s)}{V_{\text{in}}(s)} = \frac{1}{s \left[ \left( \frac{s J_m R_a}{n K_b} \right) + n K_t \right]}$$

The geometry of the mechanical part determines the moment of inertia, the total equivalent inertia,  $J_{\text{equiv}}$  and total equivalent damping,  $b_{\text{equiv}}$  at the armature of the motor with gears attaches, are given by Eq.(16), The inertias of the gears, sensor, beam and ball have to be included in the calculations of total equivalent inertia and damping, for simplicity are given by Eq.(17):

$$b_{\text{equiv}} = b_m + b_{\text{Load}} \left( \frac{N_1}{N_2} \right)^2 \quad \text{and} \quad J_{\text{equiv}} = J_m + J_{\text{Load}} \left( \frac{N_1}{N_2} \right)^2 \quad (16)$$

$$J_{\text{load}} = J_{\text{beam}} + J_{\text{ball}} = (m l^2) / 12 + (2 M R^2) / 5 \quad (17)$$

#### 5.4.1 DC motor sub-system simulation and analysis.

The derived mathematical models of DC motor, simplified and more actual, can be represented in Simulink, as shown in Fig 11, with applied input voltage and output angular position, angular speed, current and torque; where in Figure 11 (a) is shown simulink model of more accurate DC motor mathematical model and the corresponding function block window is shown in 11 (b). The simulink model of simplified mathematical model of DC motor is shown Fig 11(c) and the corresponding function block window is shown in Figure 11 (d). These models can be used to analyze and evaluate DC motor sub-system performance, as well as, designed control system. Defining in MATLAB DC motor parameters used, and running the simulink model will return angular position/time, angular speed/time, current/time and torque/time response curves shown in Figure 11 (e). The following code can be used to plot response curves:

```
load Motor ,load Motor1,load Motor2,load Motor3,subplot(2,2,1),plot( motor_angle ), xlabel(' Seconds'),ylabel(' Position \theta'),title(' Shaft angular position VS time '), grid,subplot(2,2,2),plot( motor_speed ), xlabel(' Seconds'),ylabel(' Speed \omega'),title(' Shaft angular Speed VS time '), grid, subplot(2,2,3),plot( motor_torque ), xlabel(' Seconds'),ylabel(' Torque N.m'), title(' Torque VS time '), grid, subplot(2,2,4),plot( motor_current ), xlabel(' Seconds'),ylabel(' Current Amp'),title(' Current VS time '), grid,
```

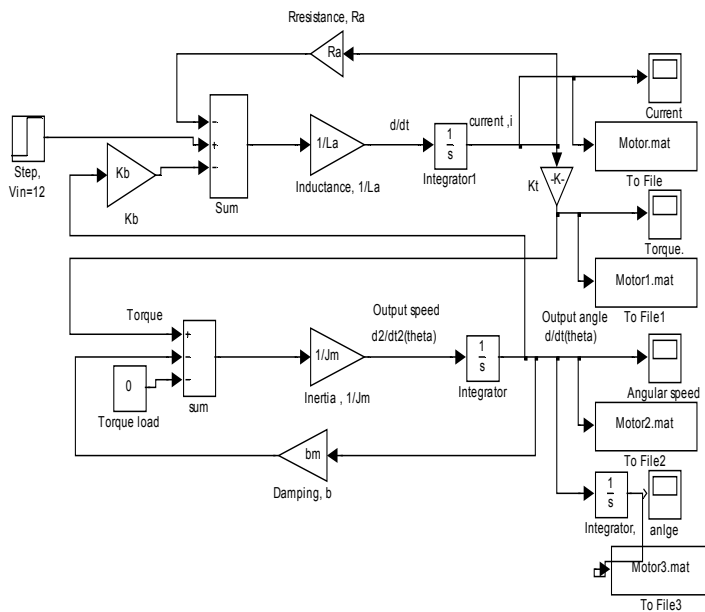


Figure 11 (a) DC motor simulink model

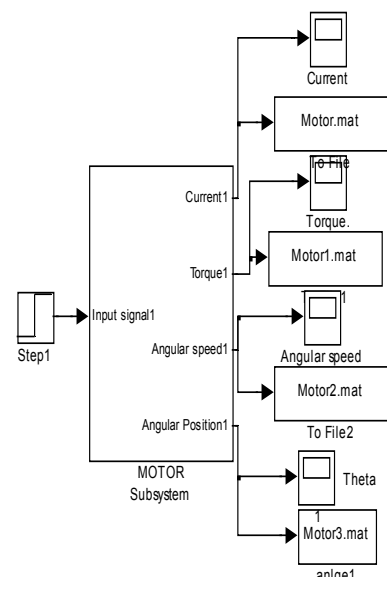


Figure 11 (b) function block model

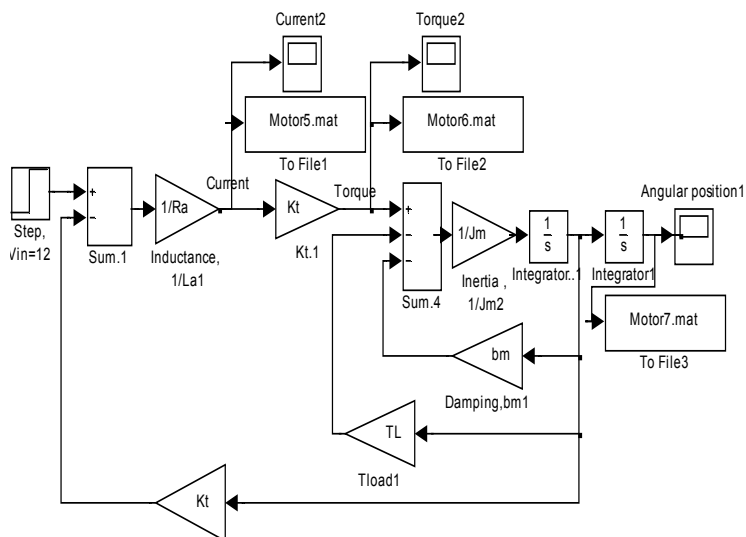


Figure 11 (c) Simplified DC motor simulink model

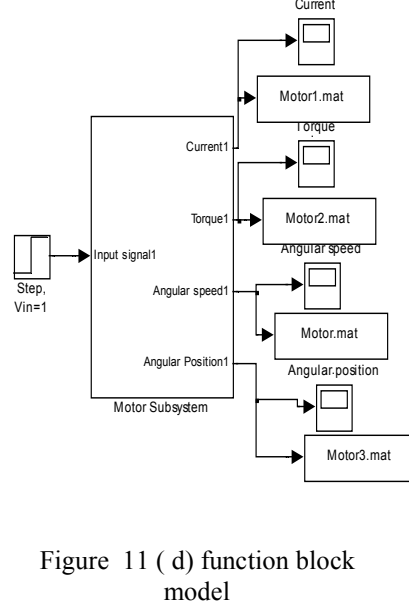


Figure 11 (d) function block model

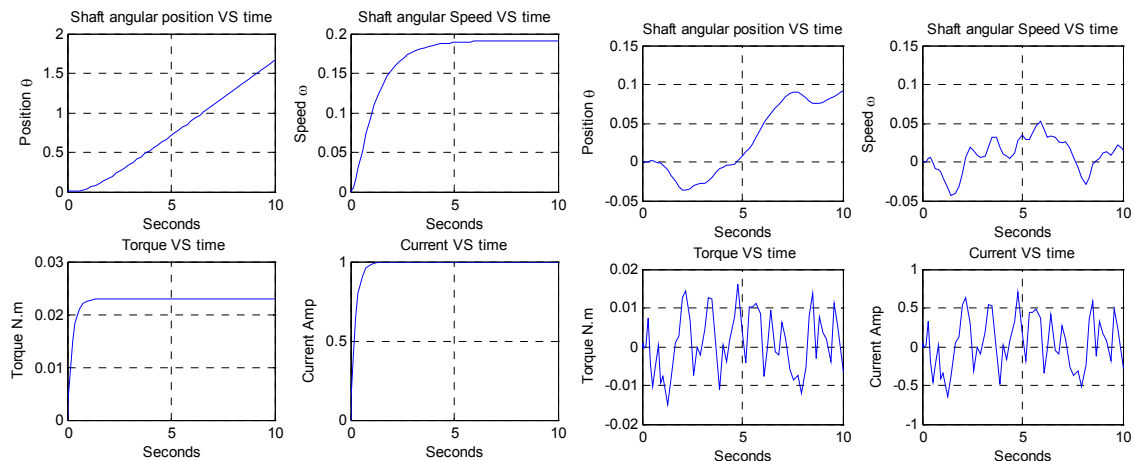


Figure 11 (e) DC motor position, time, angular speed/time, current/time and torque/time, both for step and random response curves



#### 5.4.2 Overall Ball and beam open loop system simulation and analysis

As shown in Figure 12(a), building overall Ball and beam open loop system simulink model, can be accomplished by connecting ,in series, both open loop transfer functions of DC motor and ball and beam subsystems, given by Eqs.(18) .

all three proposed DC motor simulink models, can be merged in one function block model shown in Figure 12(b), that can be connected to ball and beam subsystem model, and any DC motor model can be activated by using simulink manual switches, based on all this, the open loop simulink model of ball and beam system will have the form shown in Figure 12(c). Depending on required accuracy, selected control system type, algorithm and design requirement, the input signal can be connected to any of three DC motor models. This proposed open loop ball and beam model gives designer readings (numerical and graphical) about both sub-systems' dynamics, ball and beam as well as, DC motor subsystems.

By defining in MATLAB, both DC motor and ball and system parameters, and running the simulink model will return the following response curves of DC motor : angular position/time, angular speed/time, current/time and torque/time response curves, and the following response curves of ball system: ball position/time, ball speed/time, ball acceleration/time responses, applying random input signal ( to mimic ball position and corresponding shaft angle response) will result in all these response curves are shown in Figure 12(d)

$$G_{open}(s) = \frac{K_t / n}{(L_a s + R_a)(J_{equiv} s^2 + b_{equiv} s) + (L_a s + R_a)(T) + K_b K_t} \quad \text{and} \quad \Theta(s) = \frac{X(s)}{\Theta(s)} = \frac{g}{1 + \frac{2}{5} \left( \frac{R}{r} \right)^2 s^2} \quad (18)$$

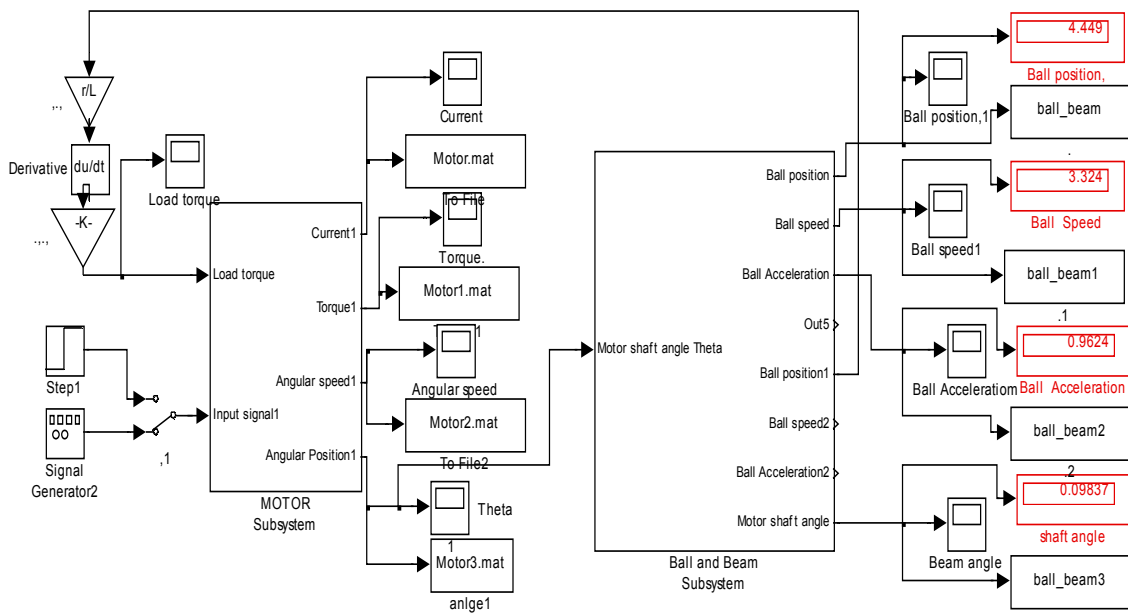


Figure 12(a) Open loop simulink model of ball and beam system.

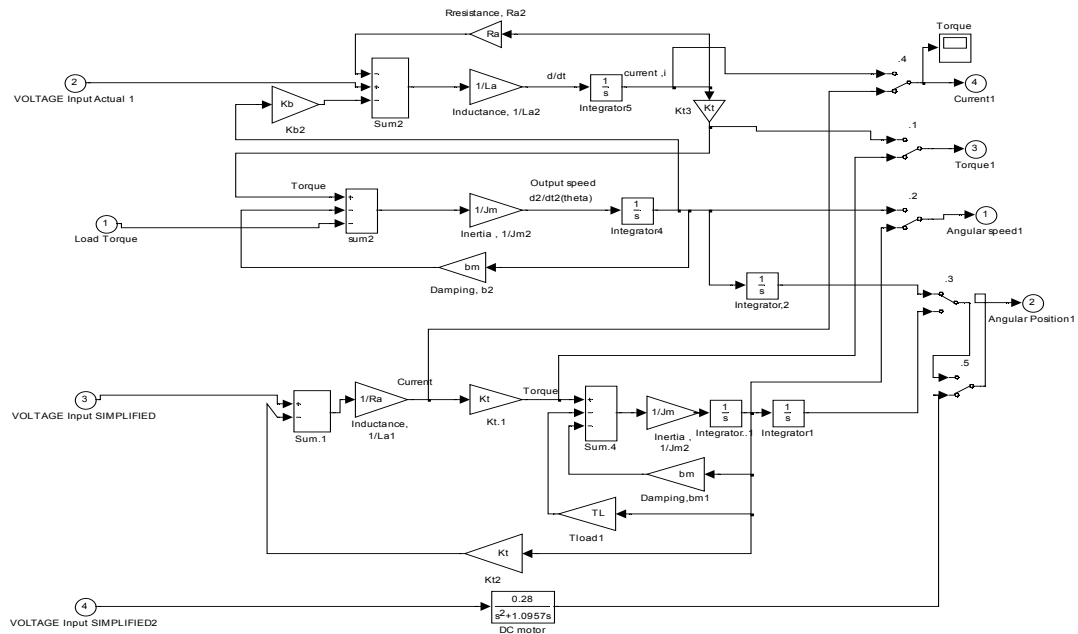


Figure 12 (b) three actuator models subsystem

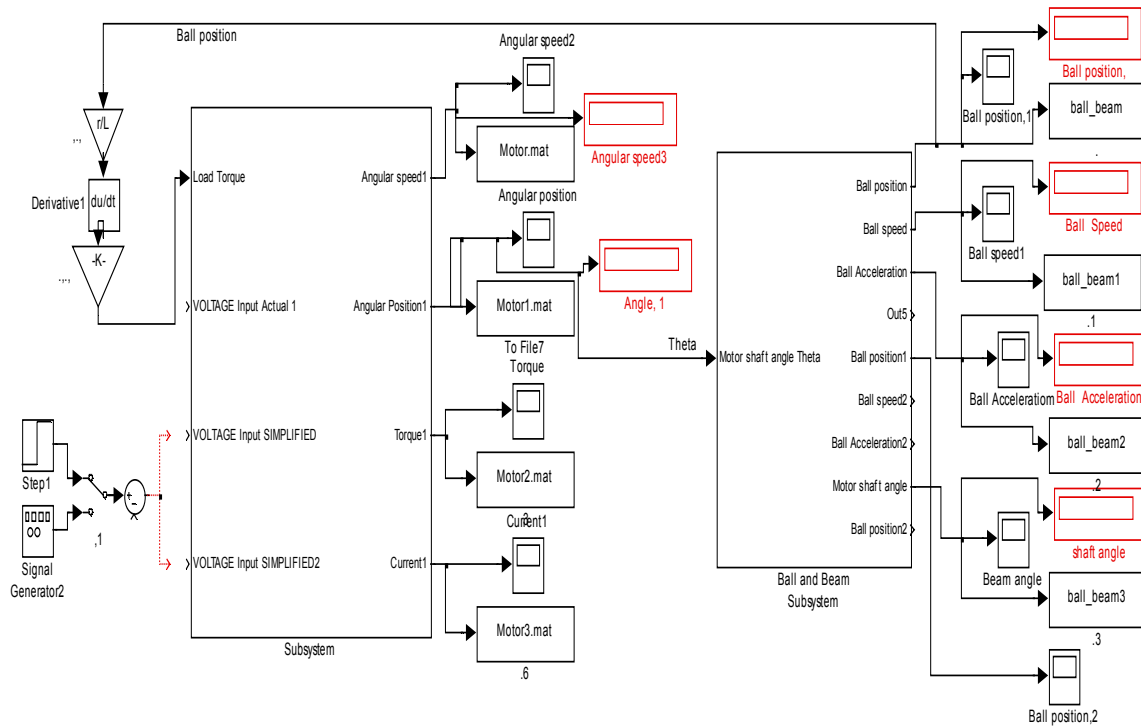


Figure 12 (c) Open loop simulink model of ball and beam system

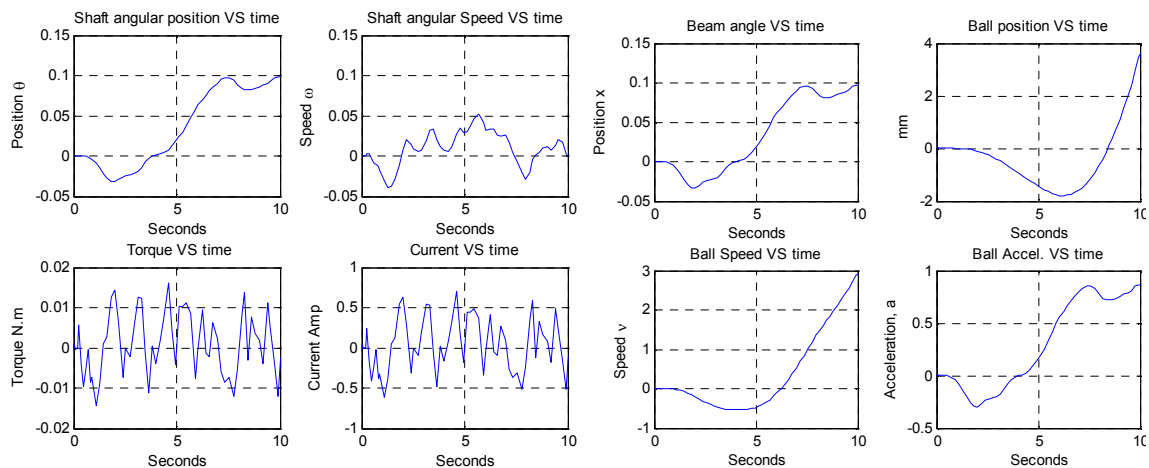


Figure 12 (d) angular position/time, angular speed/time, current/time and torque/time response curves of used DC motor and ball position/time, ball speed/time, ball acceleration/time responses, both for random input.

### 5.5 Control system selection, modeling, design, and simulation

Due to fact that it is double integrator (two poles at the origin) system and is open-loop unstable, in order to stabilize the system a special control system is to be designed. Ball and beam system, provides two outputs measurements and correspondingly two sensors one to measure ball translational position,  $x$ , and other to measure motor shaft angular position,  $\theta$ . Correspondingly, the control system design requires two control loops design, inner for motor angle control, to make motor follow the input angle and other outer loop to control the ball position on the beam, to make ball position follow the reference input position. These controllers can be designed as one or separately, mainly if the motor system is required to be faster than the beam controller.

#### 5.5.1 Proportional Derivative, PD- controller:

The transfer function of PD-controller is given by Eq.(19), Rearranging, we have the form given by Eq.(20):

$$G_{PD}(s) = K_p + K_D s \quad (19)$$

$$G_{PD}(s) = K_p + K_D s = K_D [s + (K_p / K_D)] = K_D (s + Z_{PD}) \quad (20)$$

The PD-controller is equivalent to the addition of a simple zero at  $Z_{PD} = K_p / K_D$ . The addition of zeros has the effect of improving the transient response

**Systems design with prefilter:** Prefilter is defined as a transfer function  $G_p(s)$  that filters the input signal  $R(s)$  prior to calculating the error signal. Adding a control system to plant, will result in the addition of poles and/or zeros, that will effect the response, mainly the added zero, will significantly inversely effect the response and should be cancelled by prefilter, therefore the required prefilter transfer function to cancel the zero is given by (21).

$$G_{Prefilter}(s) = Z_o / (s + Z_o) \quad (21)$$

#### 5.5.2 Inner Motor PD controller loop with deadbeat response

Often, the goal for a control system is to achieve a fast response to a step command with minimal overshoot. Deadbeat response means the response that proceeds rapidly to the desired level and holds at that level with minimal overshoot. The characteristics of deadbeat response include; Zero steady state error, Fast response, (short rise time and settling time), percent overshoot greater or equal to 0.1% and less or equal 2% and minimal undershoot, less than  $\pm 2\%$  error band. In controller design for deadbeat response approach, the controller's coefficients, (controller gains, poles and zeros), depend on the physical parameters of the system. To determine the optimal coefficients, that yield the optimal deadbeat response, the system's overall equivalent closed-loop transfer function,  $T(s)$  is compared with standard, of corresponding order, and normalized transfer function, (particularly the characteristic equations are compared). These coefficients were selected to achieve deadbeat response and minimize settling time  $T_s$  and rise time,  $T_R$  to 100% of the desired command. The desired Standard second order closed-loop transfer function for achieving desired deadbeat response specifications is given by Eq.(22):

$$T(s) = \frac{\omega_n^2}{s^2 + \alpha \omega_n s + \omega_n^2} \quad (22)$$

Based on [18], the following derivation and design for PD controller with desired deadbeat response specifications in terms of desired output angular position. Considering that system dynamics and disturbance torques depends on application shape and dimensions (robot arm, conveyer etc), the mechanical DC motor part, will have the form:

$$K_t * i_a = T_a + T_\omega + T_{load} + T_f$$

The coulomb friction can be found at steady state, to be: ( $K_t * i_a - b * \omega = T_f$ ). In the following calculation the disturbance torque,  $T$ , is all torques including coulomb friction, and given by:  $T = T_{load} + T_f$ . Applying PD controller with deadbeat response design for output desired output angular position, the open-loop transfer function of the PMDC, is given by:

$$G_{open}(s) = \frac{K_t / n}{(L_a s + R_a)(J_m s^2 + b_m s) + (L_a s + R_a)(T) + K_b K_t}$$

Manipulating for forward and closed loop transfer functions, including disturbance torques, sensor and gear ratio, gives:

$$G_{open}(s) = \frac{K_t}{(L_a s + R_a)(J_m s^2 + b_m s) + (L_a s + R_a)(T) + K_b K_t} = \frac{nK_{pot} K_t}{L_a J_m s^3 + (R_a J_m + b_m L_a) s^2 + (R_a b_m + L_a T) s + (R_a T + K_b K_t)}$$

$$G_{open}(s) = \frac{nK_{pot} K_t}{T J_m s^3 + \{ (T_a) R_a + (T) \} s^2 + (R_a T + K_b K_t) T} = \frac{nK_{pot} K_t K_D s + nK_{pot} K_t K_P}{L_a J_m s^3 + (R_a J_m + b_m L_a) s^2 + (R_a b_m + L_a T) s + (R_a T + K_b K_t)}$$

$$G_{closed}(s) = \frac{nK_{pot} K_t K_D s + nK_{pot} K_t K_P}{L_a J_m s^3 + (R_a J_m + b_m L_a) s^2 + (R_a b_m + L_a T + nK_{pot} K_t K_D) s + R_a T + K_b K_t + nK_{pot} K_t K_P}$$

$$G_{closed}(s) = \frac{L_a J_m}{s^3 + \frac{(R_a J_m + b_m L_a)}{L_a J_m} s^2 + \frac{(R_a b_m + L_a T + nK_{pot} K_t K_D)}{L_a J_m} s + \frac{(R_a T + K_b K_t + nK_{pot} K_t K_P)}{L_a J_m}}$$

Comparing with standard normalized third order system and manipulating, for  $K_D$  and  $K_P$ , given by Eq.(22)

$$\text{by: } T(s) = \frac{\omega_n^3}{s^3 + \alpha \omega_n s^2 + \beta \omega_n^2 s + \omega_n^3} \quad \beta \omega_n^2 = \frac{(R_a b_m + L_a T + nK_{pot} K_t K_D)}{L_a J_m} \Leftrightarrow K_D = \frac{\beta \omega_n^2 L_a J_m - (R_a b_m + L_a T)}{nK_{pot} K_t}$$

$$\omega_n^3 = \frac{(R_a T + K_b K_t + nK_{pot} K_t K_P)}{L_a J_m} \Leftrightarrow K_P = \frac{\omega_n^3 L_a J_m - (R_a T + K_b K_t)}{nK_{pot} K_t}$$

For design consideration, and based on required design accuracy we can use the simplified second order model, assuming  $L_a = 0$ , and manipulating for closed loop with PD controller, gives:

$$G_{angle}(s) = \frac{\theta(s)}{V_{in}(s)} = \frac{K_t}{\left\{ [L_a J_m s^3 + (R_a J_m + b_m L_a) s^2 + (R_a b_m + K_t K_b) s] \right\}} \quad G_{angle}(s) = \frac{\frac{K_t}{R_a J_a}}{s \left[ s + \frac{1}{J_m} \left( b_m + \frac{K_t K_b}{R_a} \right) \right]}$$

$$G_{angle}(s) = \frac{\theta(s)}{V_{in}(s)} = \frac{nK_t}{s \left[ (R_a J_m) s^2 + (R_a b_m + K_t K_b) \right]} \quad G_{forward}(s) = \frac{\theta(s)}{V_{in}(s)} = \frac{K_t K_P + K_t K_D s}{s \left[ (R_a J_m) s + (R_a b_m + K_t K_b) \right]}$$

$$G_{closed}(s) = \frac{\theta(s)}{V_{in}(s)} = \frac{nK_t K_P + nK_t K_D s}{s \left[ (R_a J_{equiv}) s + (R_a b_{equiv} + K_t K_b + nK_{pot} K_t K_D) \right] + nK_{pot} K_t K_P}$$

Comparing with standard normalized second order system and manipulating, for  $K_D$  and  $K_P$ , gives :

$$G_{closed}(s) = \frac{\frac{nK_t K_P + nK_t K_D s}{(R_a J_{equiv})}}{s^2 + \left( \frac{R_a b_{equiv} + K_t K_b + nK_{pot} K_t K_D}{R_a J_{equiv}} \right) s + \frac{nK_{pot} K_t K_P}{(R_a J_{equiv})}} = \frac{\omega_n^2}{s^2 + \alpha \omega_n s + \omega_n^2}$$

$$\alpha \omega_n = \frac{R_a b_{equiv} + K_t K_b + nK_{pot} K_t K_D}{R_a J_{equiv}} \quad \omega_n^2 = \frac{\alpha \omega_n R_a J_{equiv} - (R_a b_{equiv} + K_t K_b)}{nK_{pot} K_t}$$

$$\omega_n^2 = \frac{nK_{pot} K_t K_P}{(R_a J_{equiv})} \Leftrightarrow K_P = \frac{R_a J_{equiv} \omega_n^2}{nK_{pot} K_t}$$

$$Z_{PD} = \frac{K_P}{K_D}$$

Referring to [18] The controller gains  $K_P$  and  $K_D$  depend on the physical parameters of the actuator drives, to

determine gains that yield optimal deadbeat response, the overall closed loop transfer function  $T(s)$  is compared with standard second order transfer function given by (22), and knowing that parameters  $\alpha$  and  $\omega_n$  are known coefficients of system with deadbeat response given by [13],  $\alpha = 1.82$  and  $\omega_n T_n = 4.82$ ,  $T_n = 2$  and gives the following:  $\omega_n T_n = 4.82$ ,  $\omega_n = 4.82/2 = 2.41$ . In [18] to simplify and accelerate mechatronics design process, a new MATLAB built-in function with specific purpose, which is design and verification for desired deadbeat response specification, particularly for desired settling time. The presented built-in function is named deadbeat(a), the input argument a, can have the value of 1 or 2. The input argument 2 for controlling electric motor system output angular displacement with PD-controller design for deadbeat response with desired settling time, this new built in function determines the coefficients (gains pole and zero) that yield the optimal deadbeat response for desired output speed or angle control, as well as performance specifications in terms of  $M_p$ ,  $E_{ss}$ ,  $T_s$ , final output value.

### 5.5.3 Inner Motor PD Position control loop with position and rate feedback

Considering the output position control, applying position and rate feedback, (see Figure 13(b)), to have the motor follow the desired output angle, the input voltage to DC motor will be given by Eq.(23). Rearranging, we have Eq.(24):

$$V_i = K_p [\theta_i - \theta_o] - K_D s \theta_o \quad (23)$$

$$V_i = K_p \theta_i - K_p \theta_o - K_D s \theta_o \quad (24)$$

$$V_i = K_p \theta_i - (K_p + K_D s) \theta_o$$

Finding the overall equivalent closed loop transfer function, and comparing with corresponding standard transfer function (second or third order), and by comparison, we can find the gains  $K_p$  and  $K_D$  depending on motor parameters and desired damping ratio,  $\zeta$  and settling time,  $T_s$ . Standard second order system, in terms of  $\zeta$  and  $\omega_n$ , is given by Eq.(25):

$$T(s) = \frac{\theta_{out}}{\theta_{desired}} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (25)$$

Assuming the constraints for this system are to be, desired damping ratio of 0.707, or overshoot and Peak time of  $T_p = 0.1$  second, we find undamped natural frequency  $\omega_n$ , and correspondingly  $K_p$ ,  $K_D$

$$\xi = \frac{-\ln(\%OS / 100)}{\sqrt{\pi^2 + \ln^2(\%OS / 100)}}, \quad T_p = \frac{\pi}{\omega_n \sqrt{1 - \xi^2}}$$

From, the DC motor transfer function, depending on desired accuracy, and correspondingly applied mathematical model, and by substituting the value of  $V_i$ , we have:

$$V_i = s \left[ \left( \frac{sJ_m R_a}{n K_b} \right) + n K_t \right] \theta_o(s)$$

$$V_i = K_p \theta_i - (K_p + K_D s) \theta_o \Leftrightarrow \theta_o s \left[ \left( \frac{sJ_m R_a}{n K_b} \right) + n K_t \right] = K_p \theta_i(s) - (K_p + K_D s) \theta_o(s)$$

$$\theta_o(s) s \left[ \left( \frac{sJ_m R_a}{n K_b} \right) + n K_t \right] + (K_p + K_D s) \theta_o(s) = K_p \theta_i(s)$$

$$\theta_o(s) \left( s \left[ \left( \frac{sJ_m R_a}{n K_b} \right) + n K_t \right] + (K_p + K_D s) \right) = K_p \theta_i(s)$$

$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{1}{\left( \frac{s^2 J_m R_a}{n K_b} + (n K_t + K_D) s + K_p \right)}$$

By comparison with standard second order system, we find PD controller gains  $K_p$ ,  $K_D$

### 5.5.4 Inner Motor PD Position control with only position feedback

Considering the output position control, applying only position feedback with proportional and derivative terms, as shown in Fig 13(a). The input voltage to DC motor will be given by

$$V_i = K_p [\theta_i - \theta_o] - K_D s \theta_o$$

To find the gains  $K_p$  and  $K_D$ , apply the same procedure, for desired damping ratio,  $\zeta$  and settling time,  $T_s$ .

### 5.5.5 Inner Motor PID Position control loop with only position feedback

Considering the output position control, applying only position feedback with proportional, derivative and integral terms, as shown in Fig 13(b). The input voltage to motor will be given by:

$$V_i = K_p [\theta_i - \theta_o] + s K_D \theta_o + \frac{K_I [\theta_i - \theta_o]}{s}$$

The gains  $K_p$ ,  $K_D$  and  $K_I$ , are found by applying the same procedure, for desired damping ratio,  $\zeta$  and settling

time,  $T_s$ .

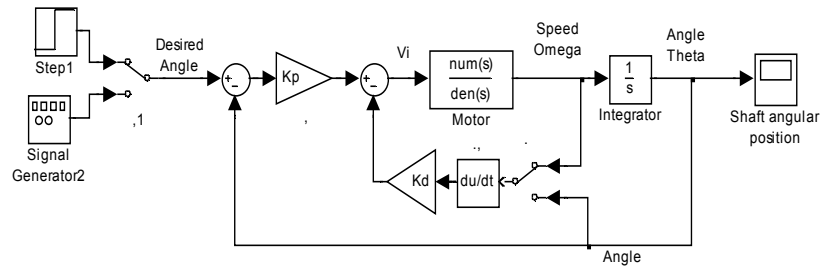


Figure 13(a) Motor PD Position control with position and/or rate feedback ( to switch use manual switch)

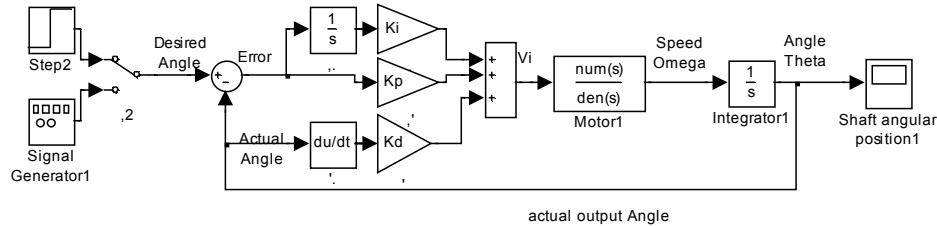


Figure 13 (b) Motor PID Position control with position feedback

#### 5.5.6 Outer Ball and beam PD, PID Position control with only position feedback

The same procedure can be applied, to calculate controller gains, for controlling beam angle, when applying PD or PID, this is shown in Fig 13(c)

##### 5.5.6.1 Outer ball and beam PD Position control loop with position and rate feedback

The input beam angle,  $\alpha$ , to ball and beam system, applying PD controller, will be given by:

$$\alpha = K_p [x_i - x_o] - K_D \frac{dx_o}{dt}$$

Taking Laplace transform,

$$\alpha = K_p [X_i(s) - X_o(s)] - K_D s(s)$$

Substituting  $\alpha$ , ball and beam transfer function, as shown below, we have

$$\frac{X(s)}{\alpha(s)} = \frac{(5/7)g}{s^2} \Rightarrow \frac{X(s)}{\alpha(s)} = \frac{7s^2}{(5)(9.8)} X_o(s) \Rightarrow \frac{X(s)}{\alpha(s)} = \frac{s^2}{7} X_o(s)$$

$$\frac{s^2}{7} X_o(s) = K_p [X_i(s) - X_o(s)] - K_D s(s)$$

$$\frac{X_o(s)}{X_i(s)} = \frac{7K_p}{s^2 + 7K_D s + 7K_p}$$

This is the overall equivalent closed loop transfer function of ball and beam, comparing it with standard second order transfer function, and by comparison, we can find the gains  $K_p$  and  $K_D$  depending on plant parameters and desired damping ratio,  $\zeta$  and settling time,  $T_s$ . as shown next:

$$T(s) = \frac{X_o(s)}{X_i(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{7K_p}{s^2 + 7K_D s + 7K_p}$$

$$K_p = \frac{\omega_n^2}{7} \Leftrightarrow K_D = \frac{2\zeta\omega_n}{7}$$

Assuming the constrains for this system are to be , desired damping ratio of 0.707, or overshoot and Peak time of  $T_p=1$  second, we find undamped natural frequency  $\omega_n$ , and correspondingly  $K_p$ ,  $K_D$

$$\zeta = \frac{-\ln(\%OS / 100)}{\sqrt{\pi^2 + \ln^2(\%OS / 100)}}, T_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

Solving we have:  $\omega_n = 4.4422$ ,  $K_p = 2.8190 \Leftrightarrow K_D = 0.89728$

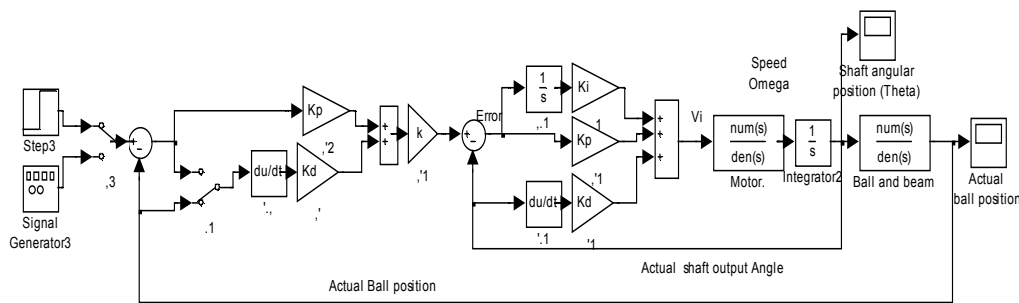


Figure 13 (c) Ball and beam position control with position feedback

### 5.5.7 Sensor sub-system modeling

There is a relationship between the beam angle,  $\alpha$ , and DC motor output position angle,  $\theta$ , where the distance traveled by the beam at radius equals to  $L$ , is equal to distance traveled by gear at radius,  $r$ , that is:  $r\theta = L\alpha \Leftrightarrow \alpha = (r/L)\theta$ , Where: The beam angle  $\alpha = (r/L)\theta$ , and the motor angle  $\theta = (L/r)\alpha$ . The transfer function between the ball position and rotor angle can be obtained based on basic transfer function given by Eq.(2) and on relation given by (11), and given as shown next:

$$\frac{Y(s)}{V_{in}(s)} = \frac{b}{s^2} = \frac{g}{s^2} \quad \text{and} \quad \frac{V(s)}{V_{in}(s)} = \frac{r}{L} \frac{g}{s^2} = \frac{r * g}{L} \frac{1}{s^2}$$

The position of the ball on the beam can be measured using special sensors, e.g. linear Potentiometer, in [19] a linear resistive wire is used as a position sensor and the conductive ball moving along it acting as a voltage divider. Linear resistive wire has a relative large resistance of about 2.15 ohm/m. A series resistor is added to the circuit in order to divide the voltage to avoid unwanted heat. According to the position ( $X$ ) of the ball, the voltage will linearly vary from 0.1 V to 0.74 V. Schematic of the resistive wire circuit is shown in Figure 14.

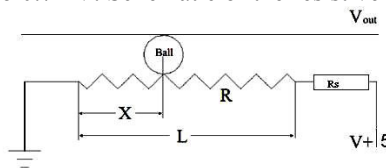


Figure 14 Schematic of the linear resistive wire circuit "position sensor" [19]

$$K_{sc} = \frac{\text{(Voltage change)}}{\text{(Position change)}} = \frac{(0.74 - 0.1)}{(L)} = \frac{(0.64)}{(40)} = 0.016$$

For voltage range of 10 volts;  $K_{se} = 1 / 40 = 0.025$

**5.6 Electronics and interfaces simulation and testing: simulation in Proteus;** To test and evaluate the selection and integration of circuit design, programming and interface components, simulation using ISIS-Professional Proteus is used. The control program written in C language, with the help of MikroC program is converted to Hex. File and downloaded on the simulated PIC-microcontroller and circuit, the simulation is shown in Figure 18, after testing, evaluating and optimizing various aspects, the final simulation results show the correctness of written program, interfaces, and microcontroller, all these can be used to build the physical circuit.

### 5.7 Ball and beam overall system simulation and analysis.

Based on the specification of requirements and design, the subsystems models and the whole system model, are to be tested and analyzed. If the specification are not satisfied, modifications can be made, if the specifications are satisfied the model can be Optimized. The primary basic simulink model, with PID control algorithm for both loops is shown in Figure 15, these two controllers will be replaced with corresponding proposed control algorithm to achieve optimal response.

Switching model, using manual switch, to nonlinear PMDC motor model, applying two simulink built-in PD controllers for inner and outer loops, subjecting system to step input, and applying simulink block tuning; will result in ball position/time, speed/time, ball acceleration/time, beam angle/time responses curves shown in Figure 16(a)), when system is subjected to random input, will result in ball and beam response curves shown in Figure 16(b). Switching model to simplified PMDC motor model, and subjecting the system to step input, will result in response curves shown in Figure 16(c)), comparing response curves, show that the overshoot using simplified motor model is less, as well as setting time, but generally system takes 10 second response to reach desired position which is large for such system.

Applying Outer Ball and beam PD Position control with only position feedback, Figure 17(a) Switching model to simplified PMDC motor model, defining parameters, running the model, will result in response ball position/time, ball speed/time, ball acceleration/time and beam angle/time response curves response curves



shown in 17(b). Refining the PD Position control design, for both loops, will result in more acceptable response curves response curves shown in 17(c), the system reaches desired output at 2.3 seconds.

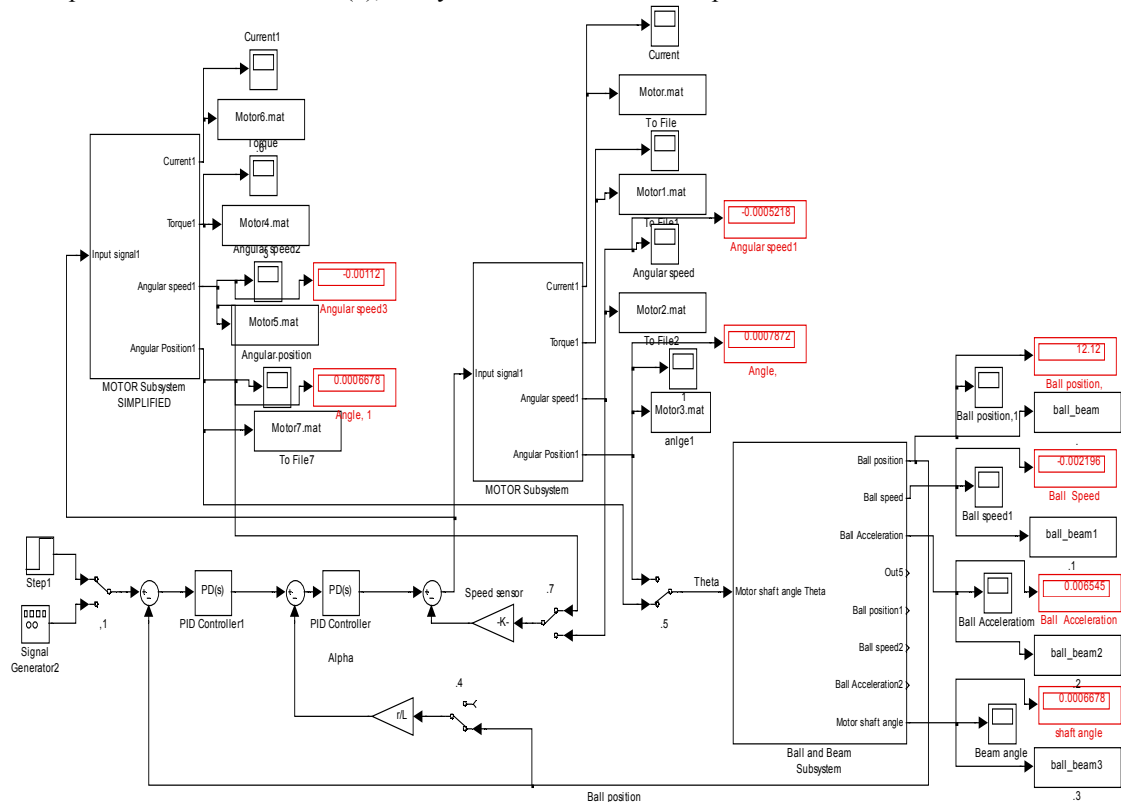


Figure 15 The primary simulink model, with PID controller.

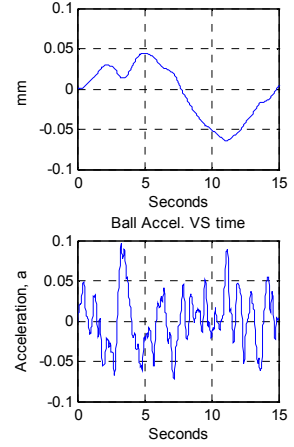
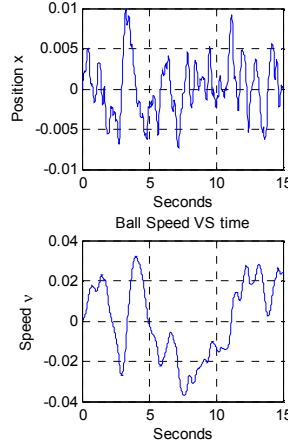
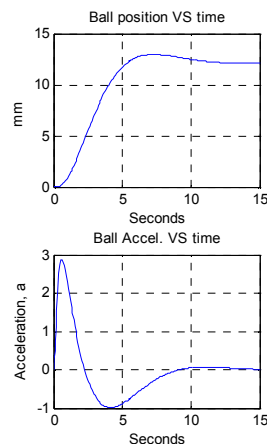
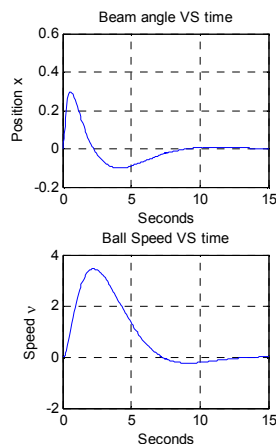


Figure 16 (a)

Figure 16 (b)

Figure 17 Ball and beam system step and random input response, using actual PMDC model.

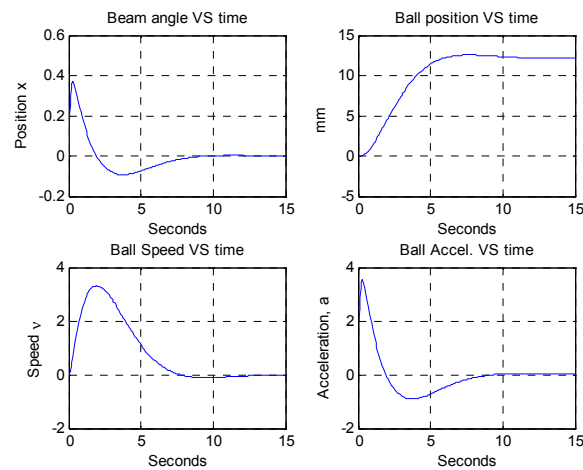


Figure 16 (c)) Ball and beam system step response, using simplified PMDC model

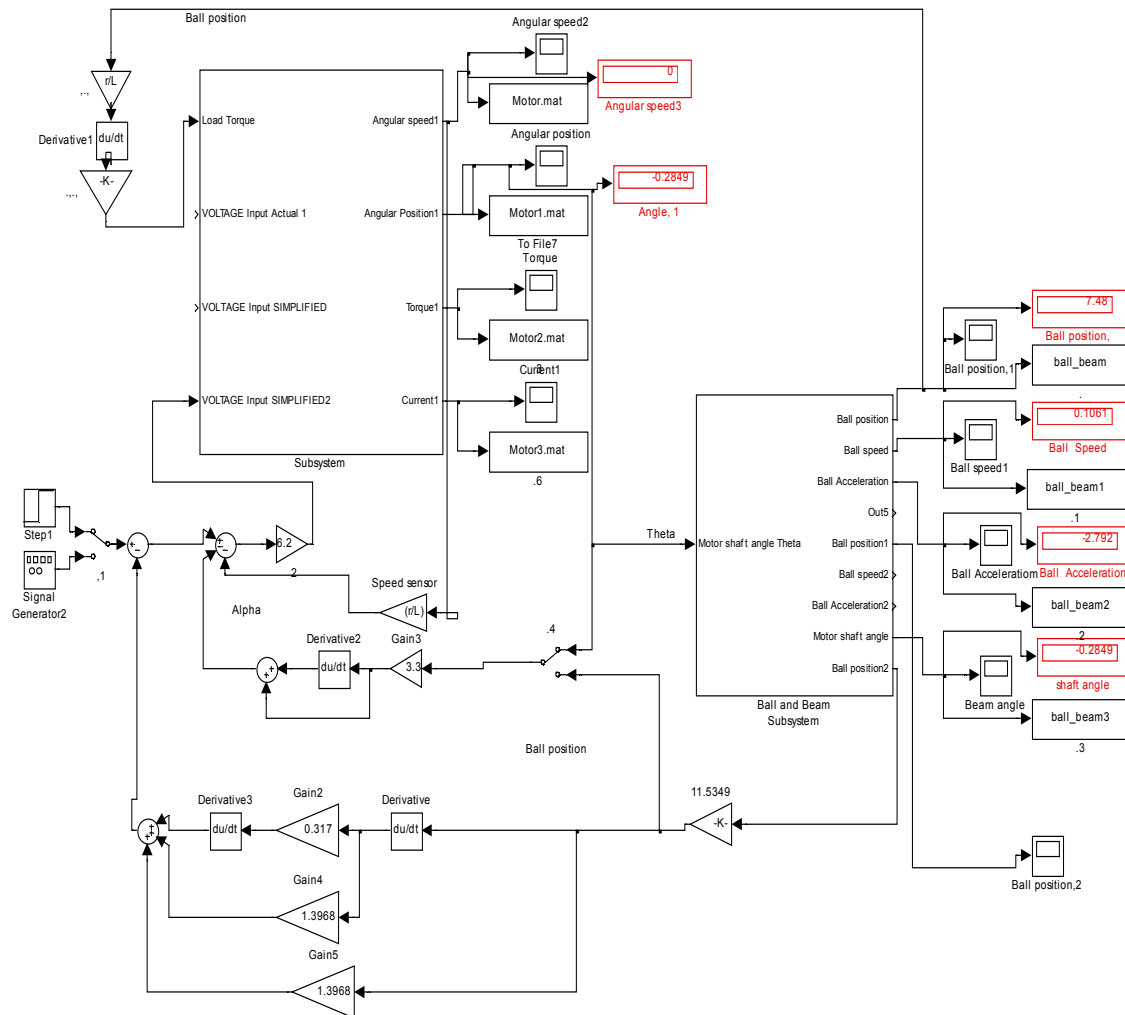


Figure 16 (a) Ball and beam PD Position control with only position feedback

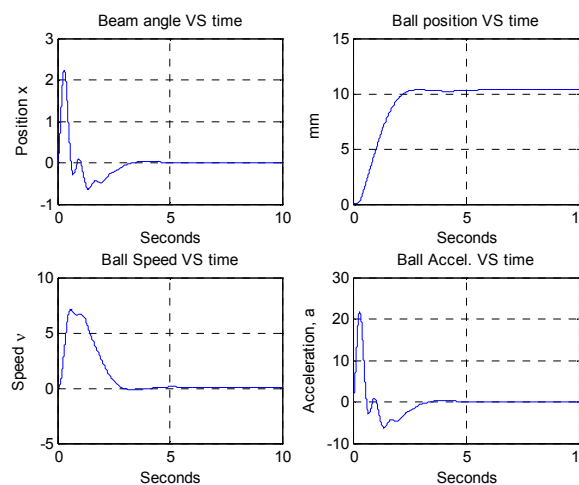


Figure 17(b) Ball and beam system step response

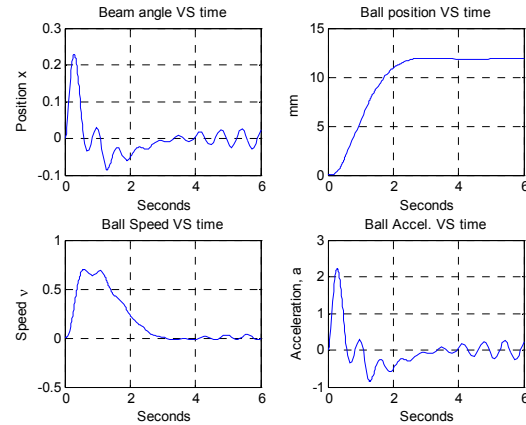


Figure 17 (c) Ball and beam system step response.

## 6. Prototyping, testing, evaluation and optimization.

There is no single model which can ever flawlessly reproduce reality, there will always be errors called as unmodeled errors between behavior of a product model and the actual product. In order to take into account the unmodeled errors and enhance precision, performance in the design process, the mechatronics design approach includes prototyping phase. Prototyping development may be carried out in the following two forms; Virtual Prototype and Physical Prototype, in this paper, only virtual prototype is built, Virtual Prototype is computer model of a product presented in a virtual environment with, ideally, all information and properties included, to visualize text, analyze and evaluate machine movement and logical operations. Different sources introduce different virtual prototypes for evaluating the overall system selection, integration and design, in [22], introduced model shown in Figure 18(a), the software used is MATLAB/Simulink.

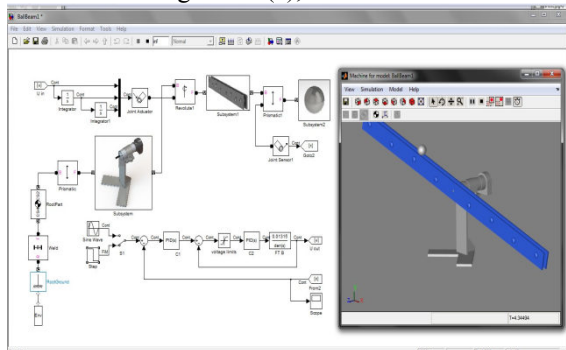


Figure 18(a) overall system virtual prototype

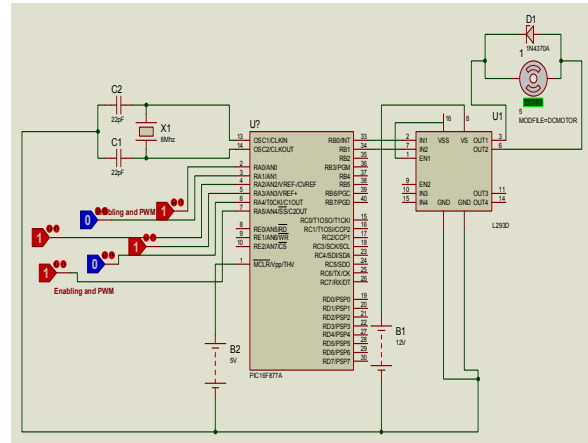


Figure 18 Electronics simulation in Proteus software

## 7. Conclusion & Future work

This paper proposes the conception and development of ball and beam system based on mechatronics design approach, where realization of design is achieved applying simultaneous, parallel, design approach and consideration. A complete system's components' selection, design and integration as well as modeling, simulation and analysis are presented. The simulation results for PD controller shown. Depending on desired accuracy, different mathematical and simulink model forms of the DC motor and plant have been developed. The proposed mechatronics design and models are intended for research purposes as well as for application in educational process. The proposed models were created and verified using MATLAB simulink, Proteus software. This paper is part (I) of authors' research study on Mechatronics design of ball and beam system, in part (II) of this research, authors intend built and test physical prototype of this design, and to propose a general MATLAB/Simulink model with its function block parameters window, that can be used to control ball and beam system using different control strategies, as well as to simplify and accelerate the selection, design and verification processes

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