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Mastermind with a Deceptive Code-Maker

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Introduction

Mastermind is an extremely addictive 'code breaking' game for two players - here one player creates a secret code (code-maker) and the other (code-breaker) attempts to determine the secret code based on a set of hints/responses. Under correct (truthful) responses from the code-maker, the code-breaker can easily decode the message in five moves or fewer (e.g., Knuth's algorithm). We consider an interesting modification where the code-breaker is uncertain about the correctness of the code-maker's responses (e.g., allowing a deceptive/untrustworthy code-maker). We investigate the effects of a deceptive code-maker on the average and maximum number of turns.

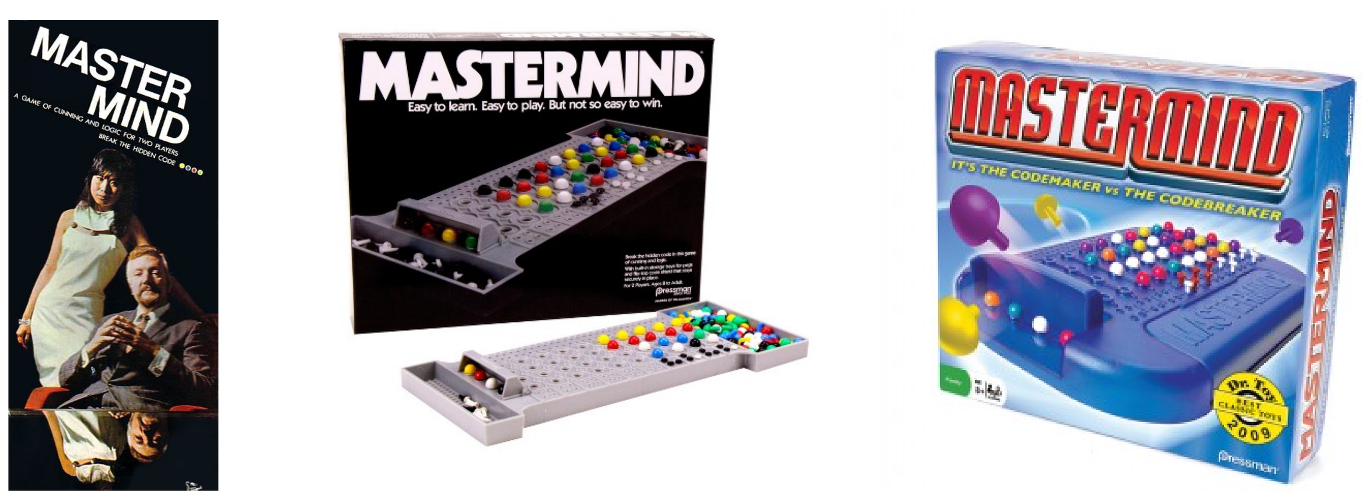

Figure 1: Mastermind box art and game boards since 1970 (game launch)

Mastermind and Rules

Mastermind is a code-breaking game for two players.

- The *code-maker* chooses a secret code of four pegs, e.g., $[5,4,3,3]$, of six possible repeatable colors – $1,2,3,4,5,6$.

  (Note - there are $4^6 = 1296$ possible secret codes.)
- The *code-breaker* tries to break the code by making guesses, i.e., submitting one code.

Following each guess, the code-maker answers using up to four pegs of two colors:

- A black peg means that a guess peg matches both color and position of a code peg.
- A white peg means that a guess peg matches the color but not the position of a code peg.

Examples/Walk Through

Following an incorrect guess (guess/score) the code-breaker removes from S (the set of all possible solutions) any element that would not give the same response if it (guess/score) were the secret code.

**:	[*, *, *, *]	(B,W)	#S
1 :	[1, 1, 1, 1]	(0, 0)	625
2 :	[2, 2, 2, 2]	(0, 0)	256
3 :	[3, 3, 3, 3]	(2, 0)	54
4 :	[3, 3, 4, 4]	(0, 3)	4
5 :	[4, 5, 3, 3]	(2, 2)	1
6 :	[5, 4, 3, 3]	(4, 0)	

**:	[*, *, *, *]	(B,W)	#S
1 :	[1, 1, 2, 4]	(0, 1)	276
2 :	[2, 2, 3, 2]	(1, 0)	54
3 :	[2, 5, 5, 5]	(0, 1)	6
4 :	[5, 3, 3, 1]	(2, 1)	1
5 :	[5, 4, 3, 3]	(4, 0)	

**:	[*, *, *, *]	(B,W)	#S
1 :	[3, 4, 5, 6]	(1, 2)	132
2 :	[1, 3, 4, 6]	(0, 2)	38
3 :	[2, 4, 3, 5]	(2, 1)	3
4 :	[5, 4, 3, 3]	(4, 0)	

Figure 2: Example games with guesses based on the consistency approach.

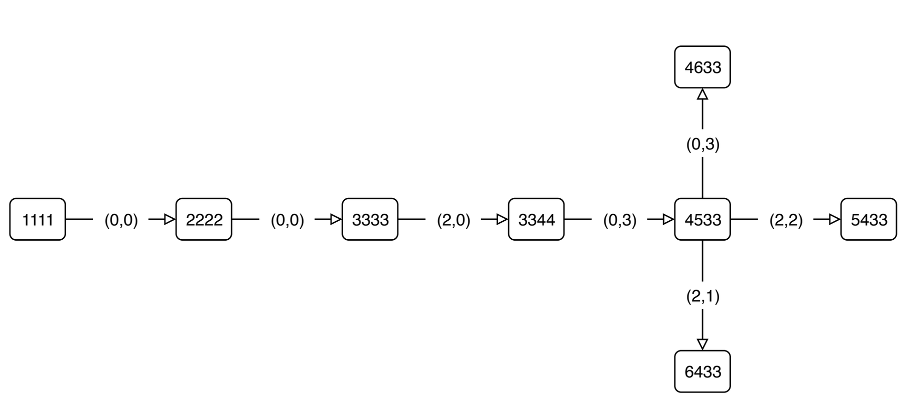

Figure 3: Tree diagram of game play with secret code $[5,4,3,3]$, initial guess $[1,1,1,1]$, and future guesses determined based on the consistency approach.

**:	[*, *, *, *]	(B,W)	#S
1 :	[1, 1, 1, 1]	(0, 0)	625
2 :	[2, 2, 2, 2]	(0, 0)	256
3 :	[3, 3, 3, 3]	(2, 0)	54
4 :	[3, 3, 4, 4]	(0, 3)	4
5 :	[4, 5, 3, 3]	(2, 2)	1
6 :	[5, 4, 3, 3]	(4, 0)	

**:	[*, *, *, *]	(B,W)	#S
1 :	[1, 1, 2, 4]	(0, 1)	276
2 :	[2, 3, 3, 3]	(2, 0)	237
3 :	[4, 3, 5, 3]	(1, 3)	1
4 :	[5, 4, 3, 3]	(4, 0)	

**:	[*, *, *, *]	(B,W)	#S
1 :	[3, 4, 5, 6]	(1, 2)	132
2 :	[1, 3, 4, 6]	(0, 2)	38
3 :	[2, 4, 3, 5]	(1, 3)	1
4 :	[5, 4, 3, 3]	(4, 0)	

Figure 4: Example games with guesses based on the frequency approach.

Next Guess Based on Consistency

In the consistency approach, a player (or program) selects the first entry from the set of possible solutions as their next guess (the set of possible solutions is updated after each guess).

Initial Guess	1	2	3	4	5	6	7	8	9	EL	ML
[1,1,1,1]	1	4	25	108	305	602	196	49	6	5.74	9
[1,1,2,2]	1	12	71	253	588	286	78	7	0	5.02	8
[1,1,2,4]	1	12	71	253	286	78	7	0	0	5.02	8
[1,2,3,4]	1	13	73	256	465	360	110	16	2	5.14	9
[3,4,5,6]	1	13	92	413	593	163	21	0	0	4.66	7

Figure 5: Game stats cycling through all possible secret codes ($4^6 = 1296$ games). EL = Expected Length (mean number of rounds) and ML = Maximum Length (maximum number of rounds)

Mastermind with a Liar - Consistency Approach

**:	[*, *, *, *]	(B,W)	#S
1 :	[1, 1, 1, 1]	(0, 0)	625
2 :	[2, 2, 2, 2]	(1, 0)	256
3 :	[2, 3, 3, 3]	(2, 0)	27
4 :	[2, 3, 4, 4]	(0, 2)	0
	Lie Detected!	R1?	
	Lie Detected!	R2?	
5 :	[4, 5, 3, 3]	(2, 2)	1
6 :	[5, 4, 3, 3]	(4, 0)	

**:	[*, *, *, *]	(B,W)	#S
1 :	[1, 1, 1, 1]	(0, 0)	625
2 :	[2, 2, 2, 2]	(0, 1)	0
	Lie Detected!	R1?	
	Lie Detected!	R2?	
3 :	[2, 2, 2, 2]	(0, 0)	256
4 :	[3, 3, 3, 3]	(2, 0)	54
5 :	[3, 3, 4, 4]	(0, 3)	4
6 :	[4, 5, 3, 3]	(2, 2)	1
7 :	[5, 4, 3, 3]	(4, 0)	

**:	[*, *, *, *]	(B,W)	#S
1 :	[1, 1, 1, 1]	(0, 0)	625
2 :	[2, 2, 2, 2]	(0, 0)	256
3 :	[3, 3, 3, 3]	(1, 0)	108
4 :	[3, 4, 4, 4]	(1, 1)	24
5 :	[5, 3, 4, 5]	(1, 2)	6
6 :	[5, 4, 3, 6]	(3, 0)	0
	Lie Detected!	R1?	
7 :	[5, 4, 3, 1]	(3, 0)	0
	Lie Detected!	R2?	
8 :	[5, 4, 3, 2]	(3, 0)	0
	Lie Detected!	R3?	
9 :	[5, 4, 3, 3]	(4, 0)	

When to Lie/Best Lie - Consistency Approach

Lie/Lie Round	1	2	3	4
(0,0)	(6.76,10)	(7.61,10)	(7.34,10)	(6.97,10)
(0,1)	(6.76,10)	(6.92,10)	(7.20,11)	(7.14,12)
(0,2)	(6.76,10)	(6.67,10)	(6.94,11)	(7.10,10)
(0,3)	(6.76,10)	(6.75,10)	(6.82,10)	(6.87,10)
(0,4)	(6.76,10)	(6.76,10)	(6.75,10)	(6.67,10)
(1,1)	(6.76,10)	(6.67,10)	(6.95,10)	(7.17,10)
(1,2)	(6.76,10)	(6.65,10)	(6.79,10)	(6.95,10)
(1,3)	(6.76,10)	(6.76,10)	(6.75,10)	(6.70,10)
(2,0)	(5.92,9)	(6.36,9)	(6.74,10)	(6.97,10)
(2,1)	(6.76,10)	(6.64,10)	(6.78,10)	(6.85,10)
(2,2)	(6.76,10)	(6.75,10)	(6.70,10)	(6.59,10)
(3,0)	(6.20,9)	(6.36,9)	(6.48,9)	(6.47,9)

Figure 6: Game stats with an initial guess of $[1,1,1,1]$ and future guesses determined via the consistency approach. Games cycled through all possible secret codes, lies, and round of lie. Individual cells represent (EL, ML).

Next Guess Based on Frequency

By determining the frequency of values, $[1,2,3,4,5,6]$, occurring in the set of possible solutions a player selects the guess that most closes aligns with the maximum frequencies of values.

Initial Guess	1	2	3	4	5	6	7	8	9	EL	ML
[1,1,1,1]	1	4	25	108	305	602	196	49	6	5.76	9
[1,1,2,2]	1	12	74	266	588	283	66	6	0	4.98	8
[1,1,2,4]	1	13	70	292	586	300	34	0	0	4.91	7
[1,2,3,4]	1	13	84	341	535	277	44	1	0	4.86	8
[3,4,5,6]	1	13	84	347	553	264	33	1	0	4.83	8

Figure 7: Game stats cycling through all possible secret codes ($4^6 = 1296$ games). EL = Expected Length (mean number of rounds) and ML = Maximum Length (maximum number of rounds)

When to Lie/Best Lie - Frequency Approach

Lie/Lie Round	1	2	3	4
(0,0)	(6.76,10)	(6.66,10)	(6.20,10)	(5.62,9)
(0,1)	(5.88,8)	(6.31,9)	(6.36,9)	(5.70, 10)
(0,2)	(5.88,9)	(6.04,9)	(6.16,9)	(5.70,10)
(0,3)	(5.78,8)	(5.86,9)	(5.89,8)	(5.56,9)
(0,4)	(5.82,9)	(5.82,9)	(5.75,9)	(5.48,9)
(1,1)	(5.59,8)	(5.95,8)	(6.18,10)	(5.75,10)
(1,2)	(5.78,9)	(5.91,9)	(5.98,9)	(5.65,9)
(1,3)	(5.82,9)	(5.80,9)	(5.74,9)	(5.47,9)
(2,0)	(5.76,8)	(5.88,8)	(6.05,9)	(5.77,10)
(2,1)	(5.76,8)	(5.84,8)	(5.88,9)	(5.63,9)
(2,2)	(5.82,9)	(5.77,8)	(5.69,8)	(5.38,8)
(3,0)	(5.81,9)	(5.67,8)	(5.64,8)	(5.34,8)

Figure 8: Game stats with an initial guess of $[3,4,5,6]$ and future guesses determined via the frequency approach. Games cycled through all possible secret codes, lies, and round of lie. Individual cells represent (EL, ML).