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Effects of Clearance Size on the Dynamic Response of Planar Multi-body Systems with Differently Located Frictionless Revolute Clearance Joints

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Abstract

This paper numerically investigates the effects of clearance size of differently located revolute clearance joints without friction on the overall dynamic characteristics of a multi-body system. A typical planar slider-crank mechanism is used as a demonstration case in which the effects of clearance size of a revolute clearance joint between the crank and connecting rod (c-cr), and between the connecting rod and slider (s-cr) are separately investigated with comprehensive observations numerically presented. It is observed that, different joints in a multi-body system have different sensitivities to the clearance size. Therefore the dynamic behavior of one clearance revolute joint cannot be used as a general case for a mechanical system. Also the location of the clearance revolute joint and the clearance size play a crucial role in predicting accurately the dynamic responses of the system.

Keywords: Chaotic behavior, Contact-impact force, Dynamic response, Multi-body mechanical system, Periodic behavior, Poincare Map, Quasi-periodic behavior, revolute clearance joint.

1. Introduction

In traditional dynamic modeling of multi-body systems, the effect of clearance at the joints is routinely ignored in order to simplify the dynamic model. The increasing requirement for high-speed and precise machines, mechanisms and manipulators demands that the kinematic joints be treated in a realistic way. This is because in a real mechanical joint, a clearance which permits the relative motion between the connected bodies as well as the components assemblage is always present.

There is a significant amount of literature available which discuses theoretical and experimental analysis of imperfect kinematic joints in a variety of planar and spatial mechanical systems with rigid or flexible links (Muvengei *et al.* 2011b). Many of these works focus on the planar systems in which only one kinematic joint is modeled as an imperfect joint. Although, the results from such experimental and analytical models have been shown to provide important insights on the behavior of mechanical systems with imperfect joints, the models do not allow for study of the interactions of multiple kinematic imperfect joints. Furthermore a real mechanical system does not have only one real joint, but practically all joints are real. This led several researchers such as Flores (2004) and Cheriyan (2006) to strongly recommend for their work to be extended to include multi-body mechanical systems with multiple imperfect joints, and with a variety of joints such as prismatic and universal joints. Few recent papers by Erkaya and Uzmay (2009; 2010) and by Flores (2010a) have considered the nonlinear dynamic analysis of multi-body systems with two imperfect joints. However in these research papers, only mechanisms with rigid links have been considered and the interaction effects of the imperfect joints on the overall response of a multi-body system were not investigated. Also, Erkaya and Uzmay (2009; 2010) modeled the clearance in the journal bearing as a massless imaginary link whose length is equal to the clearance size. This assumption is not valid especially at large clearances because the journal and bearing will not be in contact at all times.

Majority of the research works on the study of clearance joints have assumed that differently located revolute joints in a multi-body system when modeled as clearance joints affect the dynamic behavior of the system in a similar manner. Hence the inherent observations derived from investigating the dynamic behavior of one clearance revolute joint have been used as general cases for all the joints in the mechanical system. Therefore, the primary objective of this research work is to investigate numerically if the clearance size of differently located revolute clearance joints in a multi-body system affect the behavior of the system in a similar dynamic manner or not. This work will provide inherent information which can be of great use in the analysis of multi-body systems with clearance joints especially as it regards to the effective design and control tasks of these systems. This study will also form a base towards the investigation of dynamic interaction of multiple revolute clearance joints in a multi-body mechanical system.

2. Equation of Motion of Multi-body Systems

The methodology adopted in this work to derive the dynamic equations of multi-body systems follows closely that of Nikravesh (1988) in which the generalized Cartesian coordinates and the Newton-Euler's approach are utilized. The methodology presented is implemented in a MATLAB code, which is capable of automatically generating and solving the equations of motion for the multi-body systems.

Computationally, the dynamic analysis of a multi-body mechanical system with real or ideal joints involves solving Equation (1) for the acceleration vector $\ddot{q}$ and vector of Langrage multipliers λ (Shabana 1994). Then, in each integration time step, the accelerations vector, together with velocities vector are integrated in order to obtain the system velocities and positions for the next time step. This procedure is repeated until the final analysis time is reached.

$$\begin{pmatrix} M & C_q^T \\ C_q & 0 \end{pmatrix} \begin{pmatrix} \ddot{q} \\ \lambda \end{pmatrix} = \begin{pmatrix} Q_e \\ -(C_q \dot{q})_q \dot{q} - 2C_{qt} \dot{q} - C_{tt} \end{pmatrix} \tag{1}$$

M is the mass matrix, C_q is the Jacobi matrix and Q_e is the vector containing the known external forces. The code developed in this work was able to derive automatically the overall matrices in Equation (1) and subsequently solve for the vectors of acceleration and Langrage multipliers, and finally integrate the vector for velocity and acceleration to get the system positions and velocities for the next time step.

Since the system of the motion equations shown in Equation (1) does not use explicitly the position and velocity equations associated with the kinematic constraints, chances are that the original constraint equations will be violated during simulation. Due to simplicity and easiness of computational implementation, the Baumgarte Stabilization Method (Baumgarte 1972) is employed in this work to control the position and velocity constraint violations brought about by direct integration of Equation (1). By using the Baumgarte's approach, the equations of motion for a dynamic system subjected to holonomic constraints are represented as,

$$\begin{pmatrix} M & C_q^T \\ C_q & 0 \end{pmatrix} \begin{pmatrix} \ddot{q} \\ \lambda \end{pmatrix} = \begin{pmatrix} Q_e \\ Q_d - 2\alpha \dot{C} - \beta^2 C \end{pmatrix} \tag{2}$$

Where C is the vector of constraint equations and α and β are termed as feedback parameters which should be arbitrary chosen.

2.1. Kinematic Model of a Revolute Joint with Clearance

Figure 1 shows two bodies *i* and *j* connected with a revolute joint with clearance. Part of body *i* is the bearing while part of body *j* is the journal.

The eccentricity vector which connects the centers of the bearing and the journal is given as,

$$\vec{e} = r_{Pj} - r_{pi} = (R_j + A_j u_{Pj}) - (R_i + A_i u_{Pi}) \tag{3}$$

The magnitude of the eccentricity vector is

$$e = \sqrt{\vec{e}^T \vec{e}} \tag{4}$$

The penetration depth due to the impact between the journal and the bearing can be shown to be,

$$\delta = e - c \tag{5}$$

Where c is the radial clearance at the joint which is the difference between the radius of the bearing and the radius of the journal of the clearance joint. The contact points on bodies *i* and *j* during indentation are C_i and C_j respectively as shown in Figure 1.

$$\vec{n} = \frac{\vec{e}}{e} \tag{6}$$

The velocity of the contact points in the global coordinate system is,

$$\dot{r}_{Ci} = \dot{R}_i + \dot{A}_i u_{P_i} + R_B \dot{\vec{n}} \tag{7}$$

$$\dot{r}_{Cj} = \dot{R}_j + \dot{A}_j u_{P_j} + R_J \dot{\vec{n}} \tag{8}$$

Where $\vec{n}$ is the unit vector in the direction of indentation due to impact between the journal and the bearing, and R_B and R_J are the bearing and journal radii respectively.

The components of the relative velocity of the contact points in the normal and tangential plane of collision are given as,

$$\vec{v}_N = (\dot{r}_{Cj} - \dot{r}_{Ci})\vec{n} \tag{9}$$

$$\vec{v}_T = (\dot{r}_{Cj} - \dot{r}_{Ci})\vec{t} \tag{10}$$

Where $\vec{t}$ is a unit vector obtained by rotating $\vec{n}$ anticlockwise by 90^0.

2.2. Dynamic Model of a Revolute Joint with Clearance

When the journal makes contact with the bearing, then impacts occur and contact-impact forces are created at the joint. Closer inspection of Equation (5) shows that:

- When the journal is not in contact with the bearing, e<c and the indentation has a negative value. In this case, the journal is in free-flight motion inside the bearing, and no impact-contact forces are created.
- When contact between the journal and the bearing is established, the indentation has a value equal or greater than zero. In this case, impact-contact forces at the joint are established.

Therefore the computational algorithm developed for dynamic analysis of a system with revolute clearance joint should ensure that impact-contact forces are applied when the depth of penetration is greater or equal to zero.

Since there are velocity components in the normal and tangential directions of the collision between the journal and the bearings as given in Equation (9) and (10) then forces are generated in these two directions. The force normal to the direction of collision F_N can be evaluated using the contact force laws, such as Hertz, Lankarani-Nikravesh, Dubowsky-Freudenstein or ESDU-78035 contact models, while the force tangential to the direction of collision F_T which is the frictional force is evaluated using the appropriate frictional laws.

In this paper, it is assumed that no frictional forces are generated during the collision of the bearing and the journal; however friction will be included in further work. Since the direction of the normal unit vector is used as the working direction for the contact forces, then the contact forces at body *i* is;

$$F_{Ni} = F_N \vec{n} \tag{11}$$

From the Newton's third law of motion, the contact reaction force at body *j* will be;

$$F_{Nj} = -F_{Ni} \tag{12}$$

These forces which act at the contact points are transferred to the center of masses of bodies *i* and *j* as shown in Figure 2. This transfer of forces from contact forces to the center of masses contributes to the moments given as,

$$M_i = (x_{Ci} - x_i)F_{NiY} - (y_{Ci} - y_i)F_{NiX} \tag{13}$$

$$M_j = (x_{Ci} - x_i)F_{NjY} - (y_{Ci} - y_i)F_{NjX} \tag{14}$$

Once these forces and moments are known and added to the generalized vector of external forces Qe in Equation (2), then the description of the revolute joint with clearance is complete. No kinematic constraint was used when modeling the real joint; instead force constraints have been used.

2.3. Contact Force Models

Once the journal makes contact with the bearing, forces normal to the direction of contact are created. In this work the nonlinear continuous contact force models between two colliding bodies will be used since they represent the physical nature of the contacting surfaces. These contact force modes include; Hertz, Lankarani-Nikravesh, Dubowsky-Freudenstein and ESDU-78035 contact force models. A detailed description of these contact-force models is available in (Muvengei *et al.* 2011b).

It has been shown by several researchers such as (Flores *et al*. 2006;Muvengei *et al.* 2011a) that the two cylindrical contact models (that is, Dubowsky-Freudenstein and ESDU-78035 contact models) do not present any added advantage compared to the elastic spherical contact model (that is, the Hertz contact model). However, the cylindrical models are nonlinear and implicit functions, and therefore they require an iterative procedure such as Newton-Raphson algorithm to solve them which is computationally time consuming. In this work therefore, Lankarani-Nikravesh contact force model represented in Equation (15) is used to evaluate the force normal to the direction of collision since it accounts for energy dissipation during the impact process.

$$F_N = K\delta^n\left[1+\frac{3(1-c_e^2)\dot{\delta}}{4\dot{\delta}^{(-)}}\right] \tag{15}$$

Exponent n=1.5 for metallic surfaces and the generalized stiffness K which depends on the material properties and the shape of the contacting surfaces is given as;

$$K = \frac{4}{3(\sigma_1+\sigma_2)}\left[\frac{R_1R_2}{R_1+R_2}\right]^{\frac{1}{2}} \tag{16}$$

Where R_1 and R_2 are the radii of the contacting spheres (the radius is negative for concave surfaces and positive for convex surfaces), and $\sigma_{i's}$ are material parameters given by,

$$\sigma_i = \frac{1-\upsilon_i^2}{E_i} \quad \text{for i=1, 2}$$

With E_i and υ_i being the Young's modulus and the Poisson's ratio of each contacting sphere.

3. Results and Discussion

This section contains extensive results obtained from computational simulations of a slider-crank mechanism with a revolute clearance joint. This study takes into account two main functional parameters of the slider-crank mechanism, that is, the location of the considered clearance joint and the clearance size.

3.1 Description of the Slider-Crank Mechanism

A typical slider-crank mechanism as shown in Figure 3 is used as a demonstrative example to study the parametric effect of revolute joint clearance on the dynamic response of a multi-body mechanical system.

The slider-crank mechanism considered has the following parameters: Length of crank L_{OA}=0.05m, length of the coupler link L_{AB}=0.12m, mass of the crank m_2=0.3kg, mass of the coupler m_3=0.21kg, mass of the slider m_4=0.14kg, moment of inertia of crank I_2=0.00001kg.m^2 and moment of inertia of coupler I_3=0.00025kg.m^2. In addition all the links are assumed to be uniform such that their centers of gravity are at their geometric centers. The following are other parameters used for the different contact models: Nominal bearing diameter d=10mm, Length of the cylindrical contact between the journal and the bearing L=20mm, Coefficient of restitution c_e=0.9, Young's modulus E=207MPa, Poisson's ratio υ=0.3 and reporting time step is 0.000001s.

The dynamic response of the slider-crank mechanism is presented by plotting the variations with time of the slider velocity, slider acceleration and torque required to maintain constant speed of the crank. The behavior of the revolute clearance joints is also illustrated by using the slider velocity and the slider acceleration to plot the phase portraits at different test scenarios.

3.2 Influence of the Clearance Size

In this subsection, the influence of the clearance size at c-cr joint and also at s-cr joint on the dynamic behavior of the slider-crank mechanism is investigated and comparisons presented. The range of the clearances used at each joint is 0.01mm, 0.1mm, 0.3mm and 0.5mm, and the crank rotates uniformly at 5000rev/min.

Figures 4 and 6 show that increasing the clearance size of a revolute joint, the mechanism experiences increased peaks of the slider acceleration. This implies that at higher joint clearances, higher impacts followed by rebounds take place, instead of continuous or permanent contact between the journal and the bearing walls. When the clearance is small, the system response tends to be closer to the ideal response as evident in Figures 4(a) and 6(a). This implies that at smaller clearance, the journal and the bearing of the joint experience a smaller number of impacts and the journal follows the bearing wall. This is also evident in the phase portraits presented in Figures 5 for c-cr from which it is clearly observed that increasing the radial clearance of this joint from 0.01mm to 0.5mm, the behavior of the system changes from periodic to quasi-periodic. This quasi-periodic behavior is evident in the phase portrait diagram because at large clearance sizes (0.3mm and 0.5mm), cycles of the mechanism fill up the diagram in a fully predictable manner. When the clearance is small (0.01mm and 0.1mm), the behavior of the system tends to be periodic since the cycles of the mechanism follow the same path in the phase portrait diagram. However, increasing the clearance size beyond 0.5mm the behavior of the system changed from quasi-periodic to chaotic. At large clearance sizes such as 0.3mm and 0.5mm, the three types of journal motion inside the bearing are clearly observed, namely, the free-flight motion, the impact mode and the permanent or continuous contact mode.

However, as seen in Figure 7 increasing the radial clearance of the s-cr joint from 0.01mm to 0.5mm, the behavior of the system changes from periodic to quasi-periodic, and then to chaotic. The chaotic behavior is evident in Figure 7(d) since at a joint clearance of 0.5mm, different cycles of the mechanism lead to different curves in the phase portrait diagram in an unpredictable manner. These observations are consistent with those made by (Flores *et al.* 2010b, Flores *et al.* 2007) who investigated the effect of clearance size of a gudgeon joint (s-cr joint) on the behavior of a slider-crank mechanism. This behavior is different from the one witnessed when only c-cr joint was modeled as a revolute clearance joint, in which at a clearance of 0.5mm the behavior of the system was still quasi-periodic. This shows that the dynamics of the revolute clearance s-cr joint in the slider-crank mechanism is more sensitive to the clearance size as compared to that of revolute clearance c-cr joint. This confirms the already made observation that, dynamic response peaks of the mechanism when s-cr joint is modeled as a real joint are higher than the peaks produced when c-cr joint is modeled as a real joint with the same radial clearances. Although, the clearances of different joints in a system show almost the same effects on the dynamic response of the system, it has been shown that the joints will have different sensitivities to the clearance size. Therefore in order to design effective controllers for eliminating fully the chaotic behaviors brought about by the non-linearities of joints with clearances, the dynamic effect of each joint on the system should be understood, that is, the effects of clearance sizes in one joint cannot be used as a general case in a mechanical system.

4. Conclusions

From the numerical simulations presented in this work, it can be concluded that the dynamic response of a multi-body mechanical system with revolute clearance joint depends on the location of the joint and the clearance size of the joint. It is clear that the dynamics of the revolute clearance joint in a mechanical system is quite sensitive to the clearance size such that by slightly changing the value of the clearance size, the response of the system can shift from chaotic to periodic behavior. However, the degree of sensitivity to the clearance size varies from one joint to another. For instance in a slider-crank mechanism, the joint between the slider and connecting rod is more sensitive to the clearance size than the joint between the crank and the connecting rod. This explains why many researchers have emphasized on modeling the joint between the slider and connecting rod as a clearance joint. Therefore in order to design effective controllers for eliminating fully the chaotic behaviors brought about by the non-linearities of joints with clearances, the dynamic effect of each joint on the system should be understood. This is because the effects of the clearance sizes in one clearance joint cannot be used as a general case in a mechanical system.

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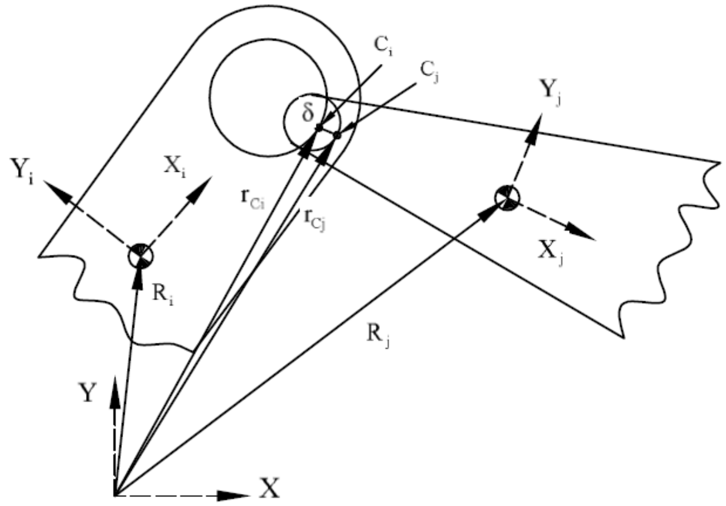

Figure 1: Penetration depth due to impact between the bearing and the journal

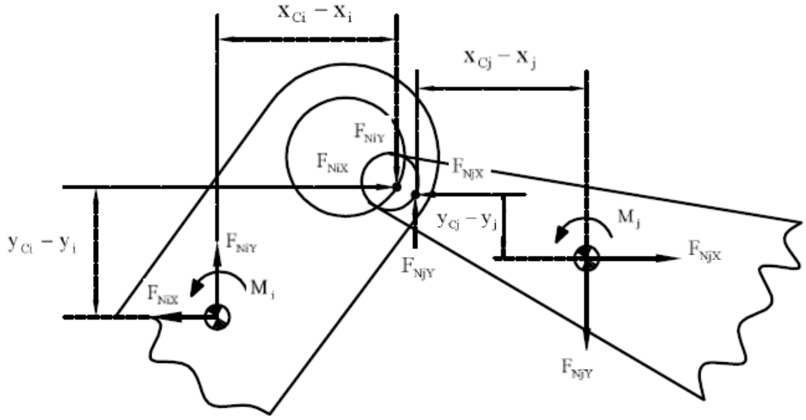

Figure 2: Transfer of impact forces to the center of masses of the bodies

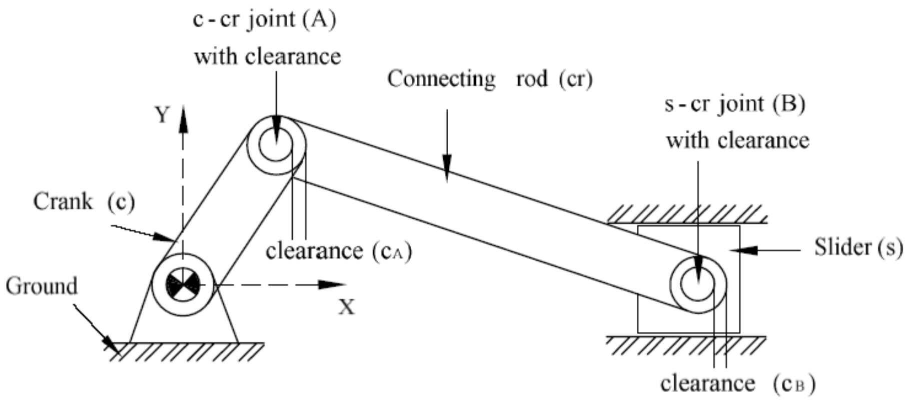

Figure 3: Slider-crank mechanism

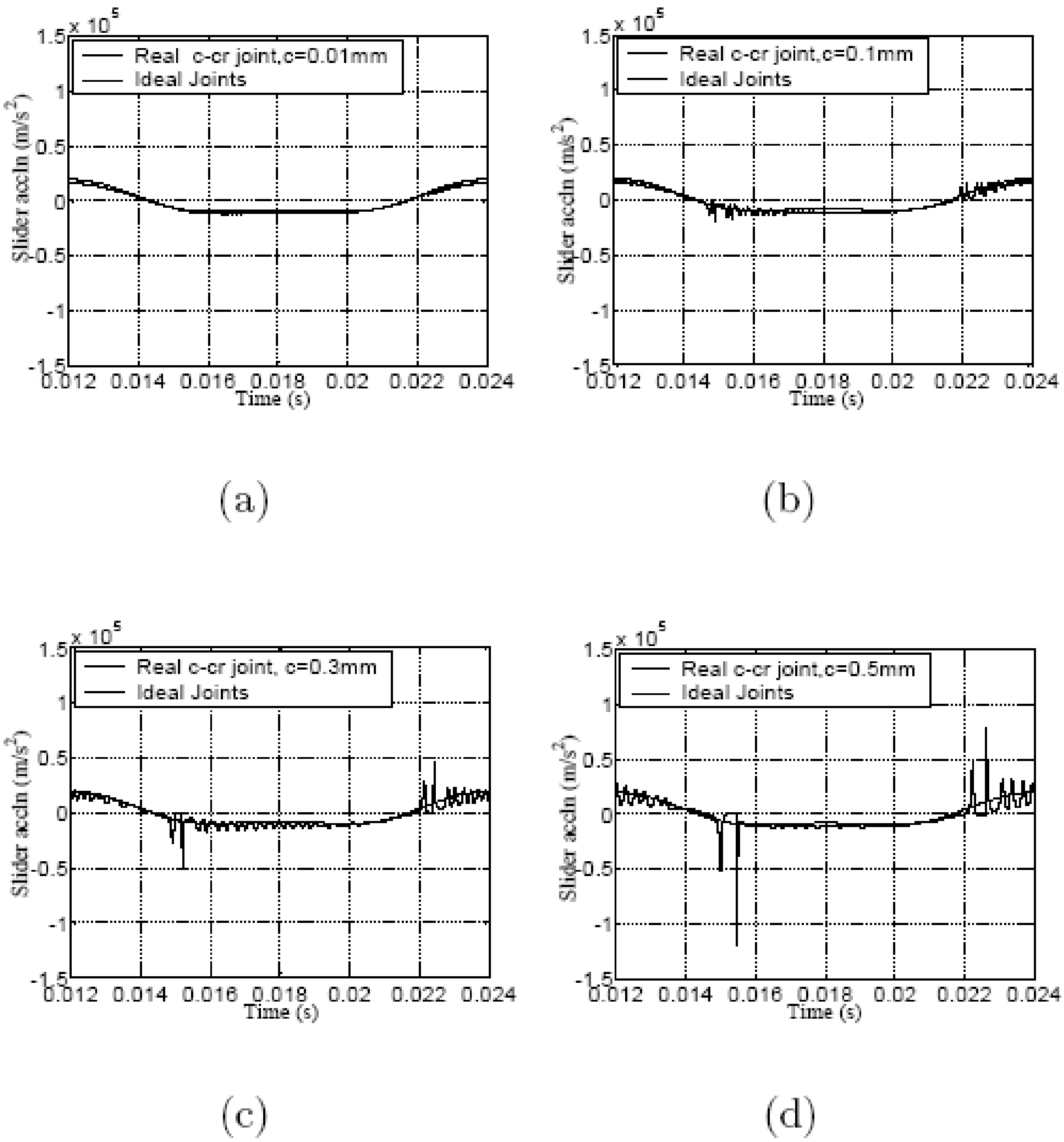

Figure 4. Slider acceleration responses for different clearance sizes at c-cr joint (a) 0.01mm (b) 0.1mm (c) 0.3mm (d) 0.5mm

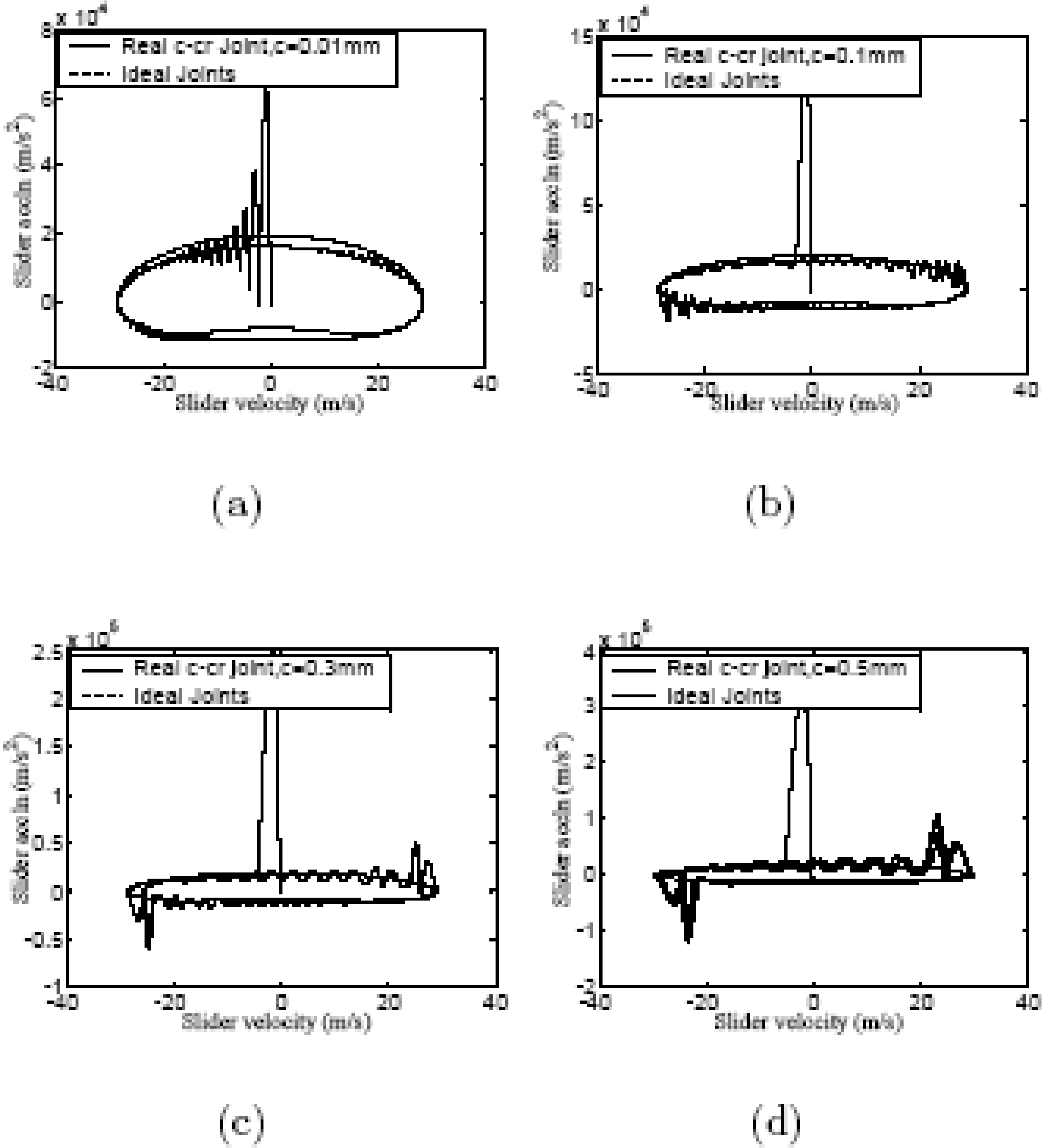

Figure 5. Phase portraits for different clearance sizes at c-cr joint (a) 0.01mm (b) 0.1mm (c) 0.3mm (d)0.5mm

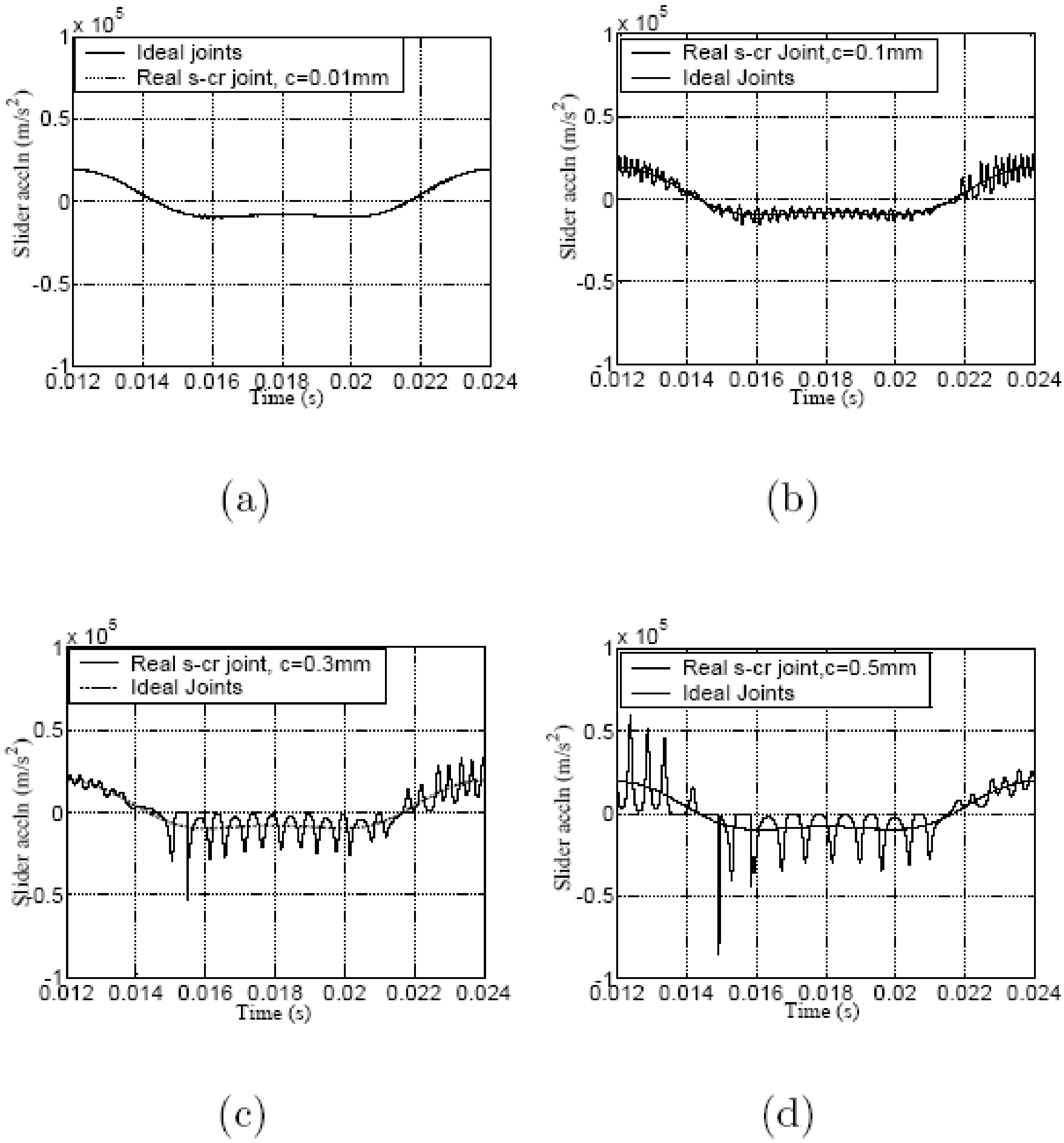

Figure 6. Slider acceleration responses for different clearance sizes at s-cr joint (a) 0.01mm (b) 0.1mm (c) 0.3mm (d) 0.5mm

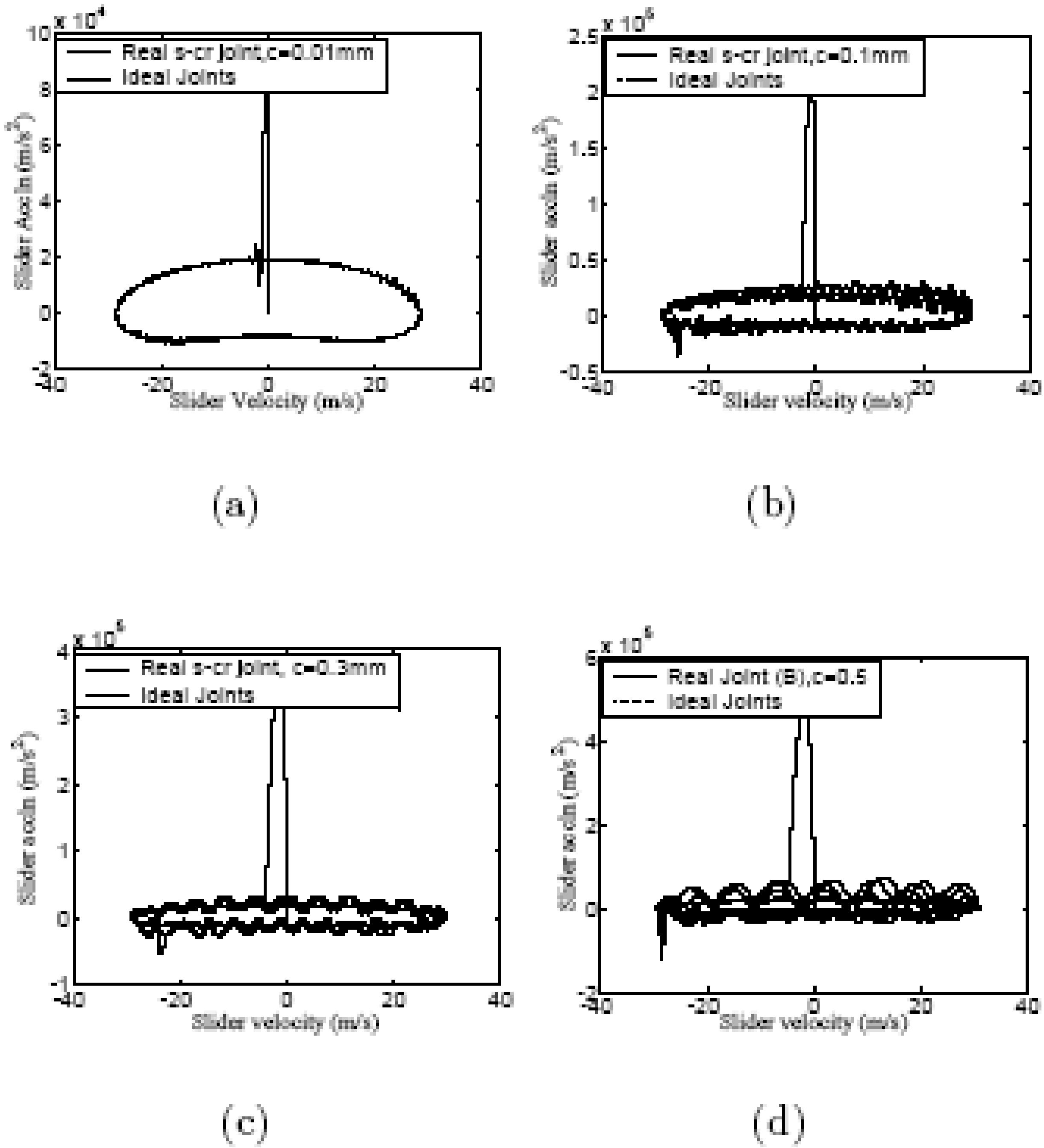

Figure 7. Phase portraits for different clearance sizes at s-cr joint (a) 0.01mm (b) 0.1mm (c) 0.3mm (d)0.5mm